BEAM TETRODE CHARACTERISTICS*

The Effect of Electron Deflections

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SUMMARY.—In this paper the anode-current—anode-voltage characteristic of a beam tetrode is deduced on the assumption that the electrons entering the screen-to-anode space are not all projected normally to the screen plane, but that they possess a distribution-in-angle given by a particular continuous function, for convenience given in terms of the sine of angle of entry. This supplements previous treatments of the problem, in which it has been assumed that above a maximum angle the relative number of entering electrons falls abruptly to zero, i.e. the distribution is represented by a discontinuous function.

It is shown that, although this discontinuous feature is lacking in the newly assumed distribution, " regions of instability " yielding sharp knees are still obtained, due to the action of space charge.

Introduction

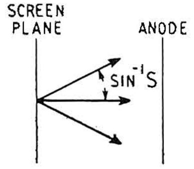
IT has been recognised for some time that the anode-current—anode-voltage characteristics of beam tetrodes are greatly modified by the fact that the electrons which enter the screen-to-anode space have previously been deflected at the grid and screen wires [1–5]. The main consequence of such deflections is that the forward electron velocities are diminished, so that the strongly deflected electrons are brought to rest and then reflected back to the screen from some plane intermediate between the screen and anode where the potential is sufficiently low, but still positive. If the current were carried by undeflected electrons such reflection could occur only at a " virtual cathode," where the potential is zero, but the "virtual cathode" theory leads to tetrode knee-voltages which are much too high.

In this paper attention is restricted to the plane parallel electrode arrangement, with an infinitely long and infinitely wide emitting cathode, so that if a section of the beam is taken at a plane parallel to the cathode, the area of the beam is infinitely large. In this case edge effects can be neglected. It is assumed that the electron trajectories at entry into the screen-to-anode space are inclined to the normal to the screen plane, and it is further necessary to make some assumption regarding the distribution-in-angle of the current projected into this space. For the calculations which follow a modified expression for the distribution function has been adopted; this gives the current density distribution as a function of the sine s of the angle of entry; thus

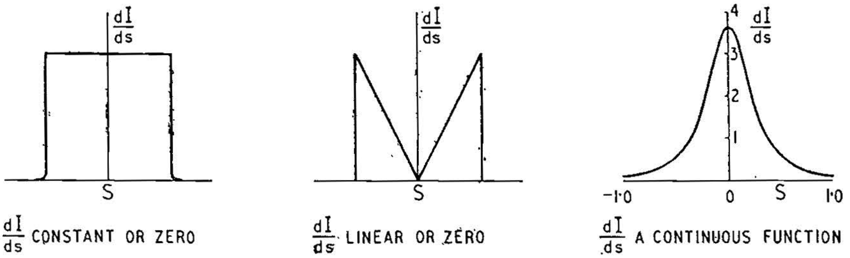
$$dI = I_T i(s)\, ds$$

where $I_T i(s)$ is the current density comprised between values of the sine of the angle of entry ranging from s to $s + ds$.

I_T is the total current density, i.e. the current flowing across unit area of the screen plane.



(a)



$\frac{dI}{ds}$ CONSTANT OR ZERO (b)

$\frac{dI}{ds}$ LINEAR OR ZERO (c)

$\frac{dI}{ds}$ A CONTINUOUS FUNCTION (d)

Fig. 1. *Various assumed distributions plotted against sine of angle of entry.*

dI/ds [i.e. $I_T i(s)$] is the distribution function as shown in Fig. 1. When $i(s)$ is known, the potential throughout the screen-to-anode space can be precisely evaluated. This has already been done, for several simple assump-

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tions regarding $i(s)$, by Strutt and van der Ziel, by G. B. Walker and by the writer. It is the object of the present paper to extend these calculations to the important case where $i(s)$ is a continuous, instead of a discontinuous function, and to show that "regions of instability"[4] yielding sharp knees, are still obtained.

The Current reaching the Anode

Let it be supposed that the screen plane is at a potential V_1 above the cathode, with the anode at a potential V_2, and that there is a potential minimum, of magnitude V_m, somewhere between the screen and anode.

[In what follows we shall write

$$V/V_1 = W\,;\ \sqrt{V/V_1} = w\,;\ \sqrt{V_m/V_1} = w_m\,;$$
$$\sqrt{V_2/V_1} = w_2\,;\ \text{and}\ \sqrt{V} = v]$$

The condition that an electron should reach the anode is that $V_1 s^2$ should be less than V_m, i.e. s lies between $-w_m$ and $+w_m$. The primary current incident on the anode is therefore

$$I_A = I_T \int_{-w_m}^{+w_m} i(s)\,ds \quad \ldots \quad (2)$$

This equation for I_A gives the anode current as a function of w_m; for fixed conditions of grid and screen voltage this characteristic should be independent of the screen-to-anode gap. For small values of I_T, or for short gaps, a potential minimum will not exist, but if the anode voltage is lower than the screen voltage the anode voltage V_2 will be the lowest in the screen-to-anode space, and hence w_2 should replace w_m as the limits in the integral. With regard to the form of $i(s)$ it may be shown that when the deflections at the grid and screen wires are cumulative, as in a tetrode, the basic I_A—w_m characteristic approaches the full value gradually, and therefore the function $i(s)$ should represent a continuous curve[5].

In order to reproduce this the following function has been chosen for $i(s)$

$$i(s) = \frac{1}{2\tan^{-1}k}\cdot\frac{k}{1 + k^2 s^2} \quad \ldots \quad (3)$$

where k is an arbitrary constant.

This yields on integration with respect to s, with limits $-w_m$, $+w_m$

$$I_A = I_T\left[\frac{\tan^{-1} k w_m}{\tan^{-1} k}\right] \quad \ldots \quad (4)$$

These relations are shown in Figs. 1(d) and 2.

The Space Charge Conditions between Screen and Anode

If the space-charge density is known throughout the screen-to-anode space we can find the potential distribution by effecting a solution of Poisson's equation.

Now if dI is the current density comprised between s and $s + ds$ the corresponding contribution $d\rho$ to the negative space-charge density ρ is dI/u, where u is the forward electron velocity.

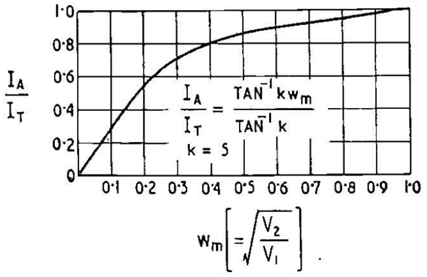

Fig. 2. *I_A/I_T plotted againt w_m for assumed distribution.*

At a plane where the potential is V (E.S.U.) the value of u is

$$\sqrt{2\epsilon/m}\,\sqrt{V - V_1 s^2}$$

where ϵ = charge of electron in E.S.U.

m = mass of electron

But $dI = I_T\, i(s)\, ds$

$$\therefore \quad d\rho = \frac{I_T\, i(s)\, ds}{\sqrt{2\epsilon/m}\,\sqrt{w^2 - s^2}} \quad \ldots \quad (5)$$

If sufficient current density is projected across the screen plane a potential minimum will be formed. On the anode side of the potential-minimum plane there are present only electrons for which $V_1 s^2$ is less than V_m, but on the screen side of the potential-minimum plane the circumstances are more complicated, since there are present both forward and returning electrons. The forward electrons, crossing a plane where the potential is V, comprise those for which $V_1 s^2 < V$, while the returning electrons comprise those for which $V_1 s^2$ lies between V_m and V (the electrons for which $V_1 s^2 < V_m$ go on to the anode).

Hence the value of ρ at a plane lying on the screen side of the potential minimum is given by

$$\sqrt{2\epsilon/m\,V_1}\,.\,\rho = I_T\left[\int_{-w}^{+w}\frac{i(s)\,ds}{\sqrt{w^2 - s^2}} + \int_{-w}^{-w_m}\frac{i(s)\,ds}{\sqrt{w^2 - s^2}} + \int_{w_m}^{w}\frac{i(s)\,ds}{\sqrt{w^2 - s^2}}\right] \quad (6)$$

If $i(s)$ is an even function, which is the practical case, this simplifies to

$$\sqrt{2\epsilon/m\,V_1}\,.\,\rho = 2I_T\left[2\int_0^w \frac{i(s)\,ds}{\sqrt{w^2-s^2}} - \int_0^{w_m}\frac{i(s)\,ds}{\sqrt{w^2-s^2}}\right] \quad .. \quad .. \quad (7)$$

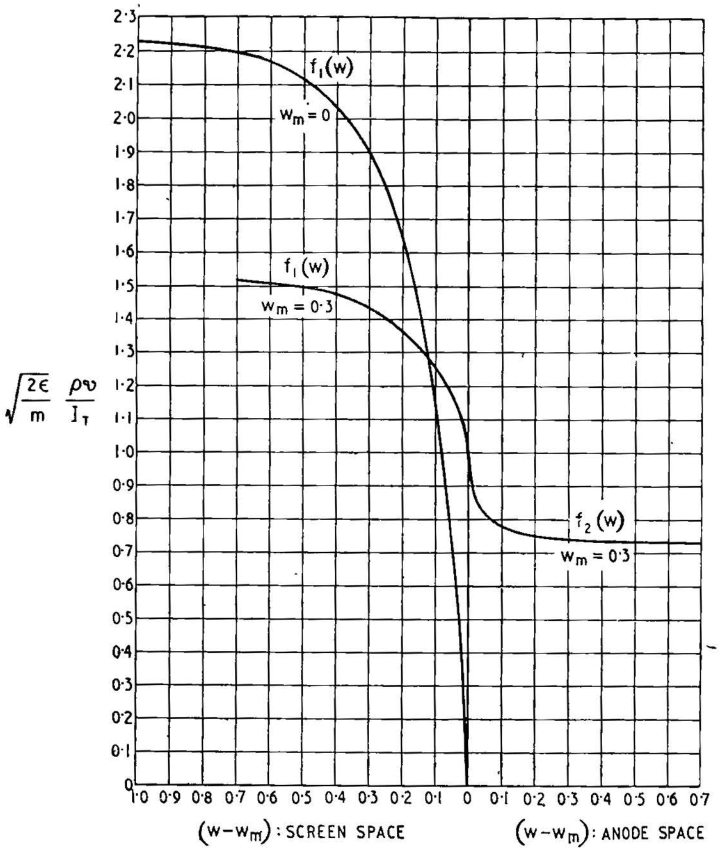

Fig. 3. $\sqrt{2\epsilon/m}\,\rho v/I_T$ *plotted against* $(w - w_m)$.

In a similar manner the space charge density at a plane on the anode side of the potential minimum is given by the expression

$$\sqrt{2/\epsilon m\,V_1}\,.\,\rho = 2I_T\int_0^{w_m}\frac{i(s)\,ds}{\sqrt{w^2-s^2}} \qquad (8)$$

It is interesting to consider the product ρv instead of the space charge density ρ; thus if each side of the above expressions is multiplied by w, we have, since

$$\sqrt{V_1}\,.\,w = v$$

$$\sqrt{2\epsilon/m}\,.\,\rho v = 2I_T\left[2w\int_0^w\frac{i(s)\,ds}{\sqrt{w^2-s^2}} - w\int_0^{w_m}\frac{i(s)\,ds}{\sqrt{w^2-s^2}}\right] \quad .. \quad .. \quad .. \quad .. \quad (9)$$

and

$$\sqrt{2\epsilon/m}\,.\,\rho v = 2I_T\left[w\int_0^{w_m}\frac{i(s)\,ds}{\sqrt{w^2-s^2}}\right] \quad .. \quad .. \quad (10)$$

on the screen and anode sides of the potential minimum, respectively.

When $i(s) = \dfrac{1}{2\tan^{-1}k}\cdot\dfrac{k}{1+k^2s^2}$ the integrals can be evaluated (see Appendix) and give the following results :—

(1) *On screen side of potential minimum*

$$\sqrt{2\epsilon/m}\,.\,\rho v = I_T\frac{\sin\alpha}{\tan^{-1}k}[\pi - (\psi+\chi)]$$
$$= I_T f_1(w), \text{ say} \quad .. \quad .. \quad (11)$$

(2) *On anode side of potential minimum*

$$\sqrt{2\epsilon/m}\,.\,\rho v = I_T\frac{\sin\alpha}{\tan^{-1}k}[\psi+\chi]$$
$$= I_T f_2(w), \text{ say} \quad .. \quad .. \quad (12)$$

where

$$\alpha = \tan^{-1}kw\,; \quad \psi = \sin^{-1}w_m/w\,;$$
$$\chi = \tan^{-1}\left[\frac{\sin 2\psi\,\tan^2(\alpha/2)}{1-\cos 2\psi\,\tan^2(\alpha/2)}\right]$$

The courses of $f_1(w)$ and of $f_2(w)$ are shown in Fig. 3, with k chosen to be 5.

Determination of the Potential Distribution, knowing ρv

Poisson's Equation is $d^2V/dx^2 = 4\pi\rho$ where ρ is the density of negative space charge in E.S.U., and x is measured in cm. If both sides are multiplied by $2dV/dx$ (i.e. by $4v\,dv/dx$) we get

$$dV/dx\,(dV/dx)^2 = 16\pi\,\rho v\,dv/dx$$
$$= 16\pi\,V_1^{\frac{1}{2}}\,\rho v\,dw/dx$$

$$\therefore \quad (dV/dx)^2 = 16\pi\,V_1^{\frac{1}{2}}\int_{w_m}^{w}\rho v\,.\,dw$$

or since

$$W = V/V_1$$

$$(dW/dx)^2 = \frac{16\pi}{V_1^{3/2}} \int_{w_m}^{w} \rho v \, . \, dw$$

Now ρv is of the form $\dfrac{I_T}{\sqrt{2\epsilon/m}} f(w)$

$$\therefore \quad (dW/dx)^2 = \frac{16\pi \, I_T}{\sqrt{2\epsilon/m} \; V_1^{3/2}} \int_{w_m}^{w} f(w) \, dw$$

In order to simplify this, we may note that the current which flows in a diode of gap x_a with anode maintained at a potential V_1 is

$$\frac{\sqrt{2\epsilon/m} \; V_1^{3/2}}{9\pi \, x_a^2} \text{ E.S.U/cm}^2 = I_d, \text{ say,}$$

If then I_T is put equal to $J_T \, . \, I_d$

$$(dW/dx)^2 = 16/9 \; J_T/x_a^2 \int_{w_m}^{w} f(w) \, dw$$

and

$$\therefore \quad \frac{x - x_m}{x_a} = \frac{3}{2\sqrt{J_T}} \int_{w_m}^{w} \frac{w \, dw}{\sqrt{\int_{w_m}^{w} f(w) \, dw}} \quad \ldots \quad \ldots \quad (13)$$

From this the potential distribution against x can immediately be derived. Thus if the distance from the potential minimum to the screen is x_1

$$x_1/x_a = \frac{3}{2\sqrt{J_T}} \int_{w_m}^{1} \frac{w \, dw}{\sqrt{\int_{w_m}^{w} f(w) \, dw}} \quad \ldots \quad (14)$$

the upper limit being unity, since $w = 1$ at the screen.

Again, when the total gap x_a from screen to anode is fixed, the potential minimum to anode distance x_2 must equal $(x_a - x_1)$ The value of the upper limit w must then be found which gives this value for x_2; thus the anode voltage V_2 is such that

$$x_2/x_a = \frac{x_a - x_1}{x_a} = \frac{3}{2\sqrt{J_T}} \int_{w_m}^{w_2} \frac{w \, dw}{\sqrt{\int_{w_m}^{w} f_2(w) \, dw}} \quad \ldots \quad \ldots \quad (15)$$

The integrals must be evaluated by numerical methods; when this is done, there is at our disposal complete means for finding the anode-current—anode-voltage characteristics.

In order to do this, we first plot w_m against $\sqrt{J_T} \, x_1/x_a$ the values being computed from Equ. (14), This gives the boundary curve shown in Fig. 4, and indicates the relative position of the potential minimum plane. The course of w for any assigned w_m is next calculated from Equ. (13), with $f(w) = f_2(w)$, and plotted on the same graph against $\sqrt{J_T} \, x/x_a$. This is done for a suitable set of values of w_m. Now draw a vertical line displaced $\sqrt{J_T}$ from the origin; this will cut the family of w curves, and will give the values of w_2 corresponding to the values of w_m. Since, however, each w_m corresponds to a specific value of J_A/J_T we can plot J_A/J_T as a function of w_2 (Fig. 5), or better still, plot J_A against V_2/V_1. Fig. 6 shows J_A against V_2/V_1 for $J_T = 2$ and $J_T = 4$, and these are the computed shapes of the anode-current — anode-voltage characteristics.

In some ways the computed curves differ notably from the measured characteristics. It would be found in practice that the initial rising part of the $J_T = 4$ curve would approximately coincide with the initial rising part of the $J_T = 2$ curve. The writer does not think that this difficulty can

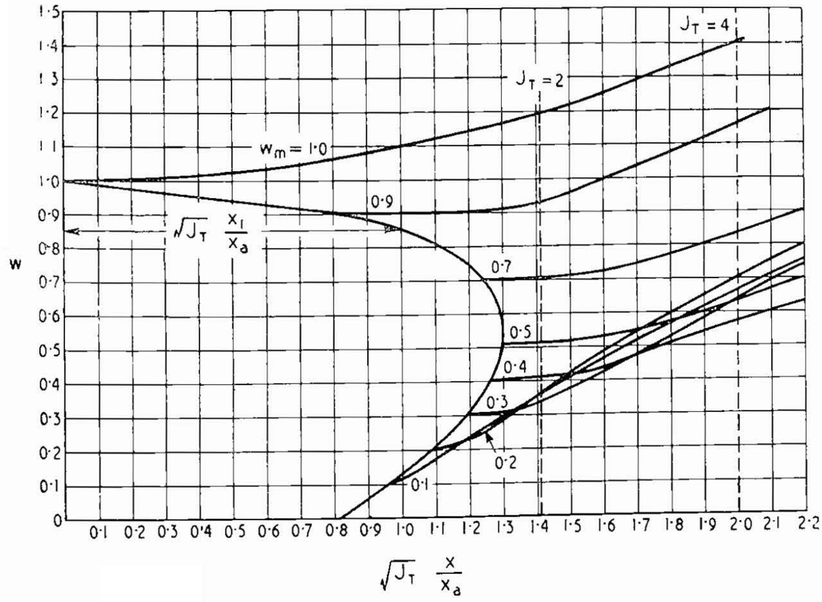

Fig. 4. *w plotted against* $\sqrt{J_T} \, x/x_a$ *for various values of* w_m.

be overcome as long as attention is confined to beams which are of large cross-sectional area in relation to the length. The negative resistance region, from W_σ to W_K, also is not so well marked in practice, although the space charge due to secondary

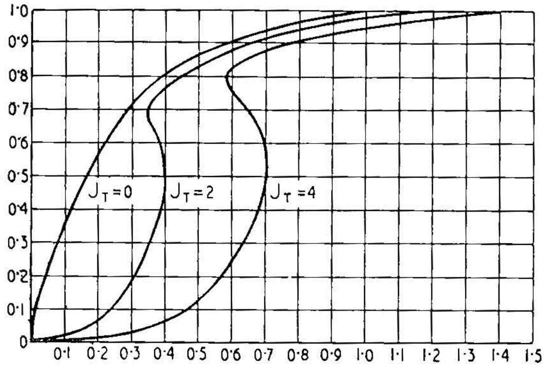

Fig. 5. *J_A/J_T plotted against w_2.*

electrons from the anode helps to reduce the effect.

If we identify W_σ the beginning of " the region of instability," as the beam tetrode " knee voltage " (since here, with increasing anode voltage, the anode current will shoot sharply up to a greater value), the calculations once more strikingly confirm that there is a large reduction in the knee voltage if the electrons are projected into the screen to anode space over a range of angles, as compared with the case when they all projected normally through the screen plane. The curves of Fig. 6 also verify the thesis of this paper that " knees " are obtainable even if the $i(s)$ characteristic is itself free from kinks.

Simplification where k is large

When k is sufficiently large, the equation for χ reduces to

$$\chi = \tan^{-1}\left[\frac{\psi}{\sin^2\psi + 1/kw}\right]$$

and if $w \gg w_m \,.\, kw_m$ this becomes

$$\chi = \tan^{-1} kw_m.$$

w can be large in comparison with w_m, so that ψ can be neglected in the expression $(\psi + \chi)$. It is then found that as $(w - w_m)$ is increased $\sqrt{2\epsilon/m}\,\rho v$ rapidly tends to the constant values I_A and $(2I_T - I_A)$ respectively on the anode and screen sides of the potential minimum (see Fig. 3). The calculations then give

$$x_1/x_a = \frac{(1 + 2w_m)(1 - w_m)^{\frac{1}{2}}}{\sqrt{2J_T - J_A}}$$

$$x_2/x_a = \frac{(w_2 + 2w_m)(w_2 - w_m)^{\frac{1}{2}}}{\sqrt{J_A}}$$

so that

$$1 = \frac{(1 + 2w_m)(1 - w_m)^{\frac{1}{2}}}{\sqrt{2J_T - J_A}} + \frac{(w_2 + 2w_m)(w_2 - w_m)^{\frac{1}{2}}}{\sqrt{J_A}}$$

In these equations we may put

$$J_A = J_T\,(2/\pi)\tan^{-1} kw$$

and on plotting obtain the anode-current—anode-voltage characteristic as shown in Fig. 7. The region of negative slope extends from W_b back to W_K with an abrupt change at Wb intermediately. If W_2 lies between W_K and $W_{\sigma\sigma}$ there are seen to be *three* possible values of J_A for an assigned value of W_2. As k tends to infinity* the upper parts of the curve become coincident, so that the distinction is no longer graphically evident. The condition that $k = \infty$ represents the case in which a " virtual cathode " is formed between screen and the anode (when I_A is less than I_T). Fig. 7 shows that, as already stated, the corresponding " virtual cathode " knee-voltage, given by W_σ is extremely high in comparison with the practical values obtained by measurement, which are normally much

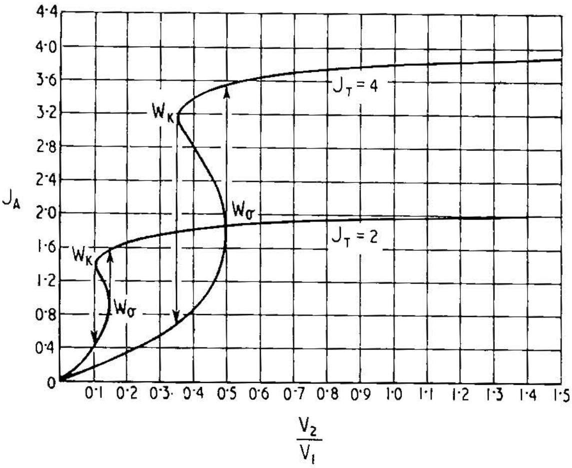

Fig. 6. *Computed anode-current—anode-voltage characteristics.*

more in agreement with the value of W_σ given by $k \simeq 5$.

* Or, if $I_A = I_T$ abruptly above some value of w_m, as in the distributions hitherto studied.

Acknowledgement

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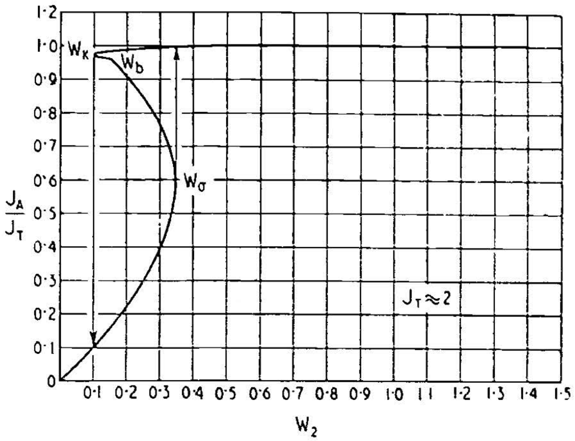

Fig. 7. *Anode-current—anode-volts characteristic when k is large.*

APPENDIX

Expressions (10) and (11) are derived from integrals of the form

$$\int_0^{w_m} \frac{i(s)\ ds}{\sqrt{w^2 - s^2}} \qquad \text{where } w > w_m$$

These occur, for example, in equations (7), (8), (9) and (10).

Let $i(s)$ be developed as a Fourier Integral for an even function.

$$i(s) = \int_0^{\infty} b(\lambda) \cos \lambda s\, d\lambda$$

where $b(\lambda) = 1/\pi \int_{-\infty}^{+\infty} i(z) \cos \lambda z\, dz$

Thus if $i(s) = \frac{ck}{1 + k^2 s^2}$ $\quad b(\lambda) = c\epsilon^{-\lambda/k}$

Substitute $s = w \sin \phi$

then $\frac{b(\lambda) \cos \lambda s\, ds\, d\lambda}{\sqrt{w^2 - s^2}}$

$$= c\, d\lambda\, \epsilon^{-\lambda/k} \cos \left[\lambda w \sin\phi\right] d\phi$$

Now $\cos (\lambda w \sin \phi)$

$$= J_0(\lambda w) + 2 \sum_1^{\infty} J_{2n}(\lambda w) \cos 2n\phi$$

where J_0 J_{2n} are the Bessel Functions.

Integrate $\cos [\lambda w \sin \phi] d\phi$ from ϕ = zero up to $\phi = \sin^{-1} w_m/w$ i.e. up to ψ

$$\int_0^{\psi} \cos [\lambda w \sin\phi]\, d\phi = J_0(\lambda w) \,.\, \psi + \sum_1^{\infty} J_{2n} (\lambda w) \frac{\sin 2n\psi}{n}$$

We next integrate

$$c\epsilon^{-\lambda/k} \left[J_0(\lambda w)\psi + \sum_1^{\infty} J_{2n}(\lambda w) \frac{\sin 2n\psi}{n} \right] d\lambda$$

from $\lambda = 0$ to $\lambda = \infty$

using the standard result $\int_0^{\infty} \epsilon^{-\lambda/k} J_{2n}(\lambda w) d\lambda$

$$= \frac{k}{\sqrt{1 + k^2 w^2}} \left[\frac{\sqrt{1 + k^2 w^2} - 1}{kw} \right]^{2n}$$

Then $\int_0^{w_m} \frac{i(s)\ ds}{\sqrt{w^2 - s^2}}$

$$= ck \cos \alpha \left\{ \psi + \sum_1^{\infty} \tan^{2n} (\alpha/2) \frac{\sin 2n\psi}{n} \right\}$$

$$= ck \cos \alpha \left\{ \psi + \chi \right\}$$

where

$$\tan \alpha = kw \,;\, \chi = \tan^{-1} \left[\frac{\sin 2\psi \,.\, \tan^2 (\alpha/2)}{1 - \cos 2\psi \,.\, \tan^2 (\alpha/2)} \right]$$

c is chosen to meet the requirement that the total current $= I_T$, when Equ. (2) is integrated with limits for s equal to $+ 1, - 1$.

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