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CALCULATION OF CHARACTERISTICS AND THE DESIGN OF TRIODES*

By

Yuziro Kusunose

(Electrotechnical Laboratory, Ministry of Communications, Tokyo, Japan)

Summary—*The primary object of the paper is to present a simple method of designing triode vacuum tubes. It comprises three parts; the first part deals with the calculation of characteristics and constants of the triode from the electrode structures, special considerations being taken for applying the formulas already established for tubes of complicated structures. In the second part the derivation of various working conditions of a triode from its static characteristic is treated, in which the writer worked out a graphical representation of d-c and a-c components of the anode working current at various working voltages. The resulting dynamic characteristic diagram is applicable to any type of triodes in evaluating the working voltages, currents and powers, whether the tube be used as an amplifier, oscillator, or modulator. The third part presents the designing procedure for a typical case in which the use of the triode is indicated and its power output is given. First the working points are determined on the dynamic characteristic diagram so as to give the required working condition; then all the quantities arising in the operation are known in their relative amounts. The type of the tube is selected between the high-impedance type and the low-impedance one, which necessitates technical and economical considerations. Then all the quantities are known in numerical values and characteristics of the tube are determined. The electrode structures are calculated to give the required characteristics.*

The present paper, being an abridgment of a paper published in Japan,[1] is primarily intended for presenting a simple method of designing triodes.

Part 1: Computation of Characteristics from Electrode Structures

The well-known space-charge equation[2]

$$i_a = Ge_a^{3/2} \tag{1}$$

applies to diodes in actual cases, when it has a cathode of uniform potential all over the surface or when the anode voltage is very high compared with the filament voltage. This equation may also be applied, in ordinary cases, if the anode voltage is referred to the neutral point of the cathode, or when the filament is lighted by alternating current.

* Dewey decimal classification: R 130. Original manuscript received by the Institute, April 9, 1929.

[1] Y. Kusunose, "Calculation on Vacuum Tubes and the Design of Triodes," Researches of the Electrotechnical Laboratory, No. 237; September, 1928, (written in English). Published by Kōseikai, No. 1/1 Yūrakucho, Tokyo, Japan. (Price about $1.70).

[2] I. Langmuir, *Phys. Rev.*, 2, 450, 1913.

When the cathode consists of a filament heated by direct current at a terminal voltage e_f, the above equation must be corrected for the non-uniformity of anode-to-cathode potential difference along the length of the filament, and the modified forms[3] are as follows:
for $e_a < e_f$:

$$i_a = \tfrac{2}{5}G\frac{1}{e_f}e_a^{5/2} = \tfrac{2}{5}Ge_f^{3/2}\left(\frac{e_a}{e_f}\right)^{5/2} \tag{2}$$

and for $e_a > e_f$:

$$i_a = \tfrac{2}{5}G\frac{1}{e_f}\left[e_a^{5/2} - (e_a - e_f)^{5/2}\right] = \tfrac{2}{5}Ge_f^{3/2}\left[\left(\frac{e_a}{e_f}\right)^{5/2} - \left(\frac{e_a}{e_f} - 1\right)^{5/2}\right] \tag{3}$$

The two equations may be reduced to a single one, as

$$i_a = \tfrac{2}{5}Ge_f^{3/2} \cdot f\left(\frac{e_a}{e_f}\right). \tag{4}$$

The constant G which is to be called "perveance" is determined from the electrode configurations, and in case of cylindrical anode,

$$G = 2.33 \times 10^{-6}\frac{A}{x_a^2} \tag{5}$$

in which A = effective anode area; x_a = distance of the anode surface from the axis of the cathode, or radius of the anode in this case.

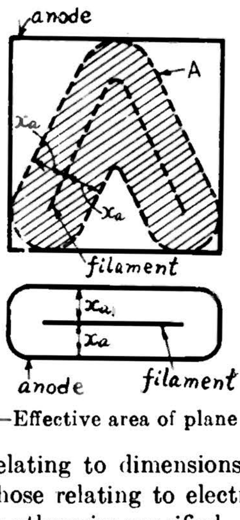

Fig. 1—Effective area of plane anode.

All the quantities relating to dimensions are to be expressed in centimeter units, and those relating to electricity in volts, amperes, ohms, watts, etc., unless otherwise specified.

[3] Van der Bijl, "Thermionic Vacuum Tube," p. 64, 1920.

The writer has found that the above expression of G is equally applicable to anodes of plane forms, if the effective anode area is taken as that area comprised in the breadth $2x_a$ along the filament length projected on the anode, as illustrated in Fig. 1.*

Triode Characteristics

In a triode, an anode voltage e_a and a grid voltage e_g produce as their combined effect a certain strength of electric field around the cathode, and the same electric field can be produced if a single anode exists at the grid suface and is at the potential[4]

$$e_g' = \frac{e_a + ke_g}{1+k} \tag{6}$$

where k is the amplification constant.

The electron current is mainly governed by the electrostatic action

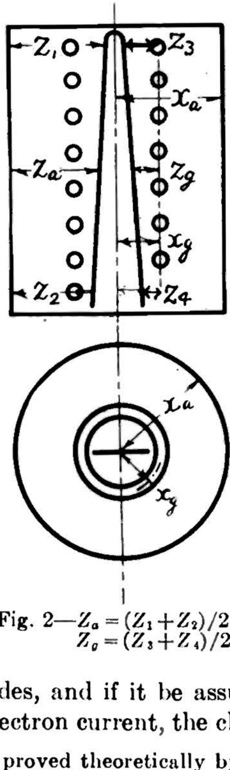

Fig. 2—$Z_a = (Z_1 + Z_2)/2$
$Z_g = (Z_3 + Z_4)/2$

of the system of electrodes, and if it be assumed that the two cases above cited give equal electron current, the characteristic of the triode

* This has lately been proved theoretically by N. Kato and S. Koizumi.

[4] W. H. Eccles, "Continuous Wave Wireless Telegraphy," Part I, p. 338; 1921.

will be expressed by the following equations which are obtained by replacing e_a in (1) and (4) by e_g':

$$i = Ge_g'^{3/2} \tag{7}$$

$$i = \tfrac{2}{5}Ge_f^{3/2} \cdot f\left(\frac{e_g'}{e_f}\right) \tag{8}$$

in which $f(e_g'/e_f)$ may be obtained from Table I.

TABLE I

e_g'/e_f	$f(e_g'/e_f)$
0	0
0.25	0.031
0.50	0.177
0.75	0.414
1.00	1.00
1.5	2.57
2.0	4.65
2.5	7.13
3.0	9.94
4.0	16.51
5.0	24.1
6.0	32.5
8.0	53.0
10	74.0
15	138
20	211
40	622

In this case

$$G = 2.33 \times 10^{-6} \frac{\text{grid surface area}}{x_g^2}$$

and for cylindrical electrodes, grid surface area $= 2\pi x_g l$ and anode area $= A = 2\pi x_a l$, l being the effective length of the electrodes.

Hence

$$G = 2.33 \times 10^{-6} \frac{A}{x_a x_g}$$

in which x_a and x_g are the distances of the anode and grid surfaces respectively from the cathode axis. But the writer found by experiment that in case of triodes having cylindrical anode and grid, and a cathode of other than a single axially-spanned filament, such as V-shaped, the values of x_a and x_g in the expression of G should be replaced by the mean shortest distances from the respective electrodes to the cathode z_a and z_g which should be taken in such a way as illustrated in Fig. 2.†

Then generally

$$G = 2.33 \times 10^{-6} \frac{A}{z_a z_g}. \tag{9}$$

† This can be verified by conformal transformation and integration as worked out by S. Koizumi.

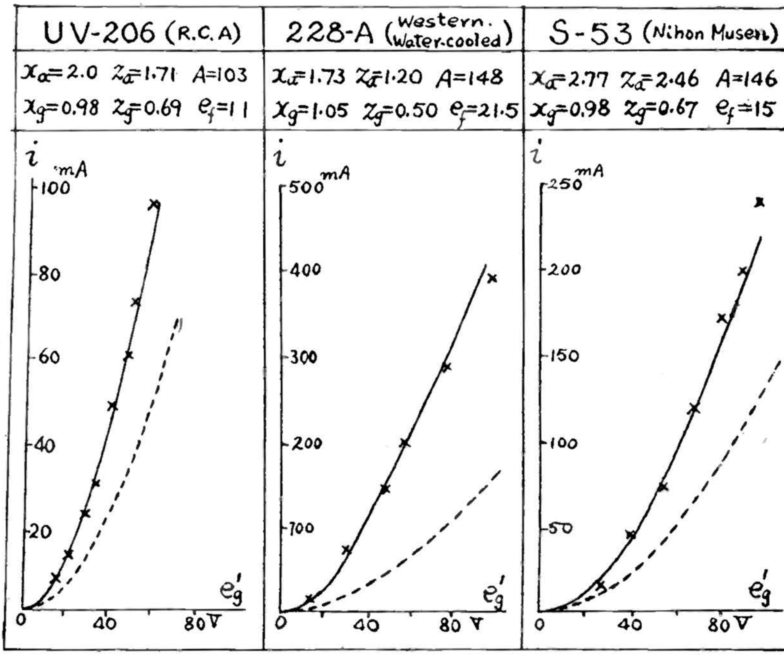

Fig. 3—Calculated characteristics of tubes with cylindrical anode and *V*-shaped filament.

Calculated with $G=2.33\times10^{-6}$ $x\,\dfrac{A}{Z_aZ_g}$ ————

Observed points x. $x\,\dfrac{A}{X_aX_g}$ — — — —

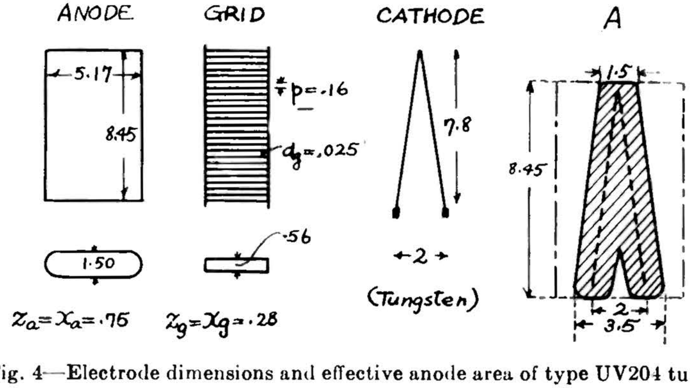

Fig. 4—Electrode dimensions and effective anode area of type UV204 tube.

This expression may also be applied to tubes with plane forms of anode and grid and in this case evaluation of the effective anode area A should be made as described before (Fig. 1).

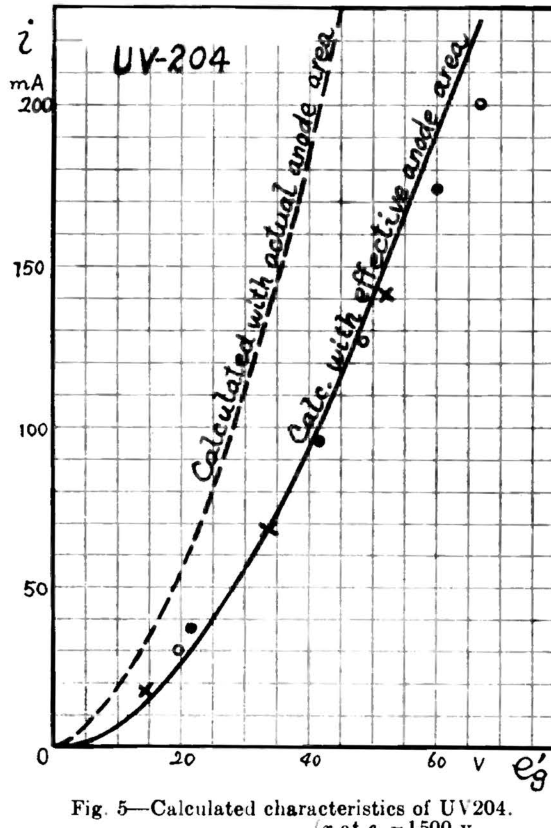

Fig. 5—Calculated characteristics of UV204.

Observed points $\left\{\begin{array}{l} x \text{ at } e_a = 1500 \text{ v} \\ \bullet \text{ " " } = 2000 \text{ "} \\ 0 \text{ " " } = 2500 \text{ "} \end{array}\right.$

Example 1: Calculation of characteristics of triodes by equation (8), one with the value of G obtained from expression (9) (full lines in Fig. 3), and the other with $G = 2.33 \times 10^{-6}\ A/x_a x_g$ (dotted lines in Fig. 3). Observed characteristics (points in Fig. 3) show the validity of taking the former expression of G.

Example 2: Evaluation of the effective anode area of a plane anode. A triode type UV204 (manufacturer: Tokyo Denki Co.) has dimensions as shown in Fig. 4. The effective anode area obtained after the principle described above is $A = 2 \times (8.45 \times (1.5 + 3.5)/2) = 42.3$ and for this area $G = 0.446 \times 10^{-3}$. If, on the other hand, actual area of the anode surface is taken $A' = 2\times(8.45\times5.17) = 87.3$ and for this area $G' = 0.965\times10^{-3}$. The characteristics are calculated by equation (8) for both and are plotted in Fig. 5. Observed data shows the validity of taking $A = 42.3$.

The depiction of the anode-current grid-voltage characteristic curves may be simplified in the following manner. First obtain the numerical relation between i and (e_g'/e_f), and from it the relation of i to e_g' for a given value of e_f. Then assume $e_a = 0$, or $e_g' = \{k/(1+k)\}e_g$, and i versus e_g curve can be drawn and this corresponds to the characteristic for the anode voltage zero. In order to obtain a characteristic for any anode voltage e_a, this curve for $e_a = 0$ should be shifted horizontally toward the negative direction of the grid voltage by an amount of e_a/k in the scale of grid voltage.

It should be remembered that a slightest error in the calculated value of k causes a considerable amount of error on the resulting values of i, especially at high anode voltages.

Example 3: Computation of characteristics of a triode type 211A (Western Electric Co.). Dimensions are shown in Fig. 6.

Calculations:

$$A = 2\times\left(5.7\times\frac{2.9+3.1}{2}\right) = 34.2$$

$$Z_a = x_a = 0.68 \qquad Z_g = x_g = 0.25$$

$$G = 2.33\times\frac{34.2}{0.25\times0.68}\times10^{-6} = 0.469\times10^{-}$$

$$e_f = 9.0 \qquad 2/5Ge_f^{3/2} = 0.0051$$

$$i = 0.0051f\left(\frac{e_g'}{e_f}\right).$$

Characteristic curve representing the relation between i and e_g' is shown in Fig. 7, together with some observed points.

TABLE II

(e_g'/e_f)	$f(e_g'/e_f)$	e_g'	i
0	0	0	0
2	4.65	18	0.024
4	16.51	36	0.084
6	32.5	54	0.166
8	53.0	72	0.270

Amplification constant calculated by equation (13) given later is $k = 12.7$

$$\left(L_g = 8.3 \qquad \frac{x_a}{x_g} = 2.7 \qquad c = 1.8\right)$$

while observed value is $k = 12.8$ at $e_a = 1{,}000$ and $e_g = -45$.

To draw i versus e_g characteristic curves, first assume $e_a = 0$; then $e_g' = \{k/(1+k)\}e_g$ or $e_g = 1.08\, e_g'$ and Table III shows the result of calculation of the characteristic for $e_a = 0$, which is depicted in Fig. 8.

TABLE III

$e_a = 0$

i	e_g'	e_g
0	0	0
0.024	18	19.5
0.084	36	39
0.166	54	58
0.270	72	78

For other anode voltages, this curve is to be shifted to the negative direction by amounts as shown in Table IV.

TABLE IV

e_a	e_a/k
500	39
1,000	79
1,500	118

The resulting characteristic curves are drawn in Fig. 8 and are consistent with the observed data.

The electron current i is the sum of anode and grid currents,

$$i = i_a + i_g. \tag{10}$$

As long as the grid is negative $i_g = 0$ and $i_a = i$, but when grid becomes positive, grid current begins to flow. In ordinary tubes, and at

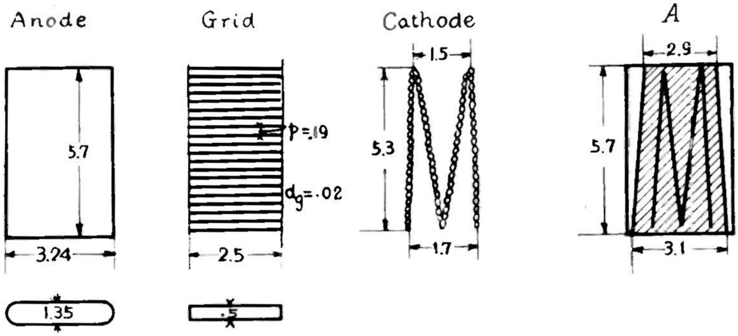

Fig. 6—Dimensions and effective anode area of type 211A tube.

low grid voltages where no remarkable effect of secondary emission appears, the following formula[5] is applicable for estimation of the grid current:

$$i_g = \gamma a \sqrt{e_g/e_a} \qquad i_a = \frac{\gamma a \sqrt{e_g/e_a}}{1 + \gamma a \sqrt{e_g/e_a}}\, i \tag{11}$$

[5] H. Lange, *Z. f. Hochfreq.*, 31, 105; April, 1928.

and hence

$$i_a = \frac{i}{1+\gamma a\sqrt{e_g/e_a}} \tag{12}$$

in which

$$\gamma = \sqrt{\log\frac{x_a}{r_c}/\log\frac{x_g}{r_c}}$$

(γ_c = radius of the cathode) are usually of the order: $\gamma = 1\sim 2 \doteqdot 1.5$ and a = ratio of the grid conductor projected area to the grid surface area.

At $e_a = 0$, the electron current is totally absorbed in the grid if the grid voltage is positive.

When the grid voltage becomes so high as to approach the anode voltage, the characteristics are severely affected by secondary emissions from the electrodes, and grid current usually increases very rapidly while anode current falls off as shown in Fig. 9.

Example 4: Calculation of grid current.

TABLE V

Tube	Observed data				Calculated grid current
	e_a	e_g	i_a	i_g	$i_g = 1.5\, a\sqrt{e_g/e_a}\cdot i_a$
102-D (a = 0.156, k = 30)	100	5.2	0.0046	0.0003	0.00025
	100	8.3	0.0074	0.0004	0.00050
	148	5.2	0.0065	0.0003	0.00029
	148	8.3	0.0100	0.0004	0.00055
UV-199 (a = 0.108, k = 6.6)	20	6	0.0023	0.0002	0.00020
	20	16	0.0040	0.0005	0.00058
	60	10	0.0038	0.0003	0.00025
	100	16	0.0042	0.0003	0.00027
UV-201A (a = 0.105, k = 7)	20	5	0.0025	0.0001	0.0002
	20	15	0.0095	0.0013	0.0013
	60	5	0.0065	0.0001	0.0003
	60	10	0.0113	0.0004	0.0007
UV-204A (a = 0.16, k = 25)	1,000	30	0.172	0.011	0.006
	1,000	60	0.274	0.019	0.009
	5,000	21	0.220	0.009	0.006
MT-9 (a = 0.245, k = 90)	1,000	80	0.305	0.027	0.031
	3,000	40	0.223	0.012	0.0094
	5,000	20	0.230	0.003	0.0033

Amplification Constant

Notations regarding the electrode dimensions are illustrated in Fig. 10.

For $r_g \ll p \ll x_g$, and for plane-electrode tubes,[6]

[6] W. Schottkey, *Archiv f. Elektrot.*, 8, 21, 1919.

$$k = \frac{2\pi x_g}{p} \cdot \frac{\dfrac{x_a}{x_g} - 1}{\log e \dfrac{p}{2\pi r_g}} \qquad (k-1)$$

and for cylindrical-electrode tubes,[7]

$$k = \frac{2\pi x_g}{p} \cdot \frac{\log \dfrac{x_a}{x_g}}{\log \dfrac{p}{2\pi r_g}}. \qquad (k-2)$$

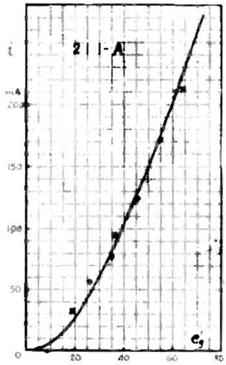

Fig. 7—Calculated characteristic of 211A.

Observed points $\begin{cases} x \text{ at } e_a = 500 \text{ v} \\ \bullet \;\text{“ “} = 1000 \text{ “} \\ 0 \;\text{“ “} = 1500 \text{ “} \end{cases}$

If the condition $r_g \ll p \ll x_g$ is not fulfilled, the more general formula for cylindrical-electrode tubes is to be applied,[8]

$$k = \frac{\dfrac{2\pi x_g}{p} \log \dfrac{x_a}{x_g} - \log \cosh \dfrac{2\pi r_g}{p}}{\log \cosh \dfrac{2\pi r_g}{p} - \log \sinh \dfrac{2\pi r_g}{p}}. \qquad (k-3)$$

The formulas for cylindrical electrodes were originally derived for grid wires parallel to the filament, but they are equally applicable to grids of a spiral or a system of parallel wires spanned crossways to the length of the filament.

[7] M. Abraham, *Archiv f. Elektrot.*, 8, 42, 1919. R. W. King, *Phys. Rev.*, 15, 256, 1920.

[8] F. B. Vogdes. *Phys. Rev.*, 24, 683, 1924; 25, 255, 1925.

Example 5: Calculation of amplification constant of a triode type ABT (made at the Laboratory), which has dimensions as shown in **Fig. 11**.

$$x_a = 0.20 \qquad x_g = 0.14 \qquad p = 0.12 \qquad r_g = 0.0085$$

$$\frac{2\pi x_g}{p} = 7.3 \qquad \frac{2\pi r_g}{p} = 0.44 \qquad \log\frac{x_a}{x_g} = 0.155$$

By $(k-2)$:

$$k = 7.3 \times \frac{0.155}{0.357} = 3.2$$

By $(k-3)$:

$$k = \frac{7.3 \times 0.155 - 0.040}{0.040 + 0.345} = 2.8.$$

Observed value of $k = 2.9$.

In an actual triode in which anode and grid are plane, the cathode is not actually plane but is a filament arranged on a plane surface and

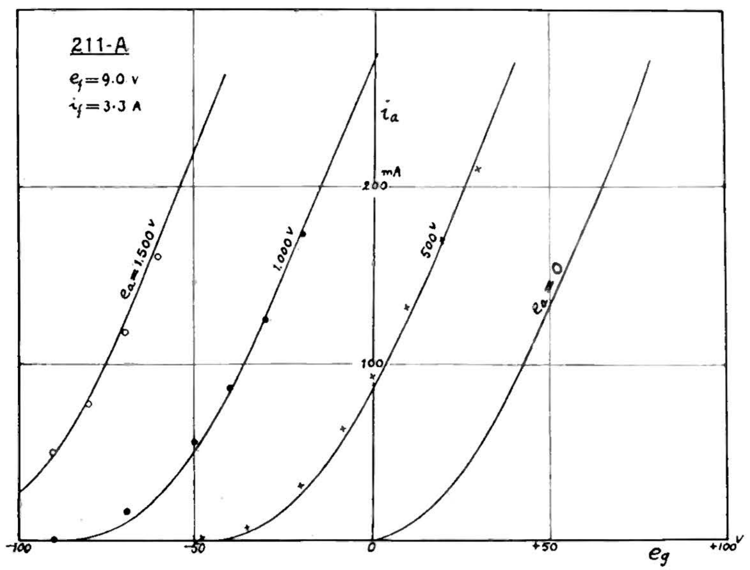

Fig. 8—Characteristics of 211A.

Calculated——
Observed at $e_a = \begin{cases} 1500 & 0 \\ 1000 & \bullet \\ 500 & x \end{cases}$

accordingly does not strictly conform to the condition from which the formula $(k-1)$ has been derived. But by trying the formulas on actual cases, the writer found that the formula for plane $(k-1)$ is applicable for a tube with W-shaped filaments, while it gives somewhat larger value for tubes with V-shaped filaments. The formulas for cylinder $(k-2)$, if applied to these tubes, give lower values than observed. Comparing the two formulas, we find that they are different only in

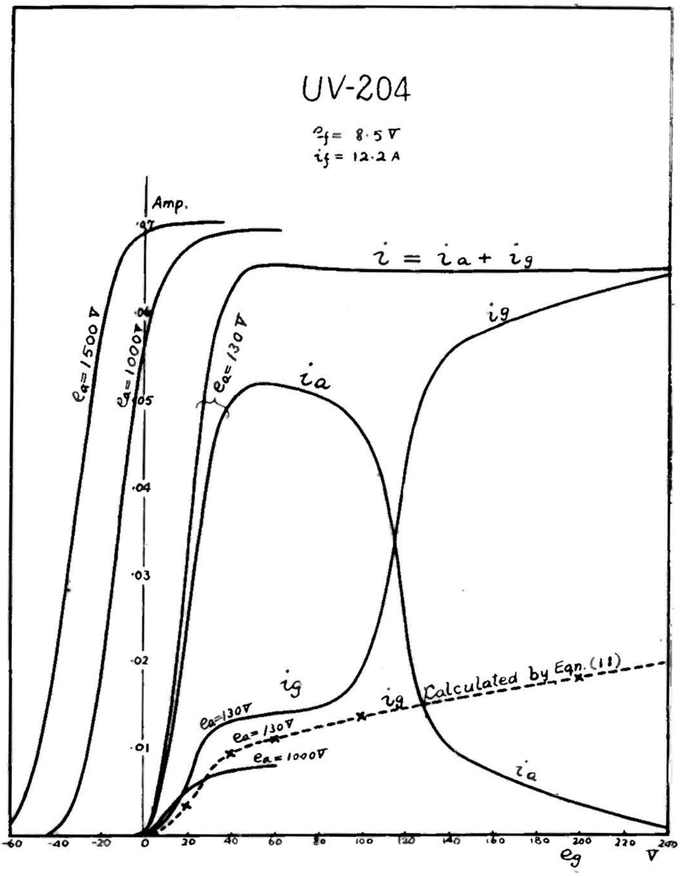

Fig. 9—Observed characteristics of UV204 showing the effect of secondary emission.

terms of x_a/x_g, and the writer suggests that for approximate calculation of k for plane-electrode tubes, the formula for cylindrical electrodes may be applied by multiplying a factor c which depends on x_a/x_g and the shape of the filament as given in Table VI.

TABLE VI

VALUES OF C

Cylindrical-electrode tubes;	$C=1$
Plane-electrode tubes;	
with grid of a mesh:	$C=1$
with grid of parallel wires:	

x_a/x_g	C	
	W-shaped filament	V-shaped filament
7	1.0	1.0
2	1.5	1.3
3	1.9	1.6
4	2.4	1.9
5	2.8	2.1

The formula $(k-3)$ is a more general form of $(k-2)$, and this is to be taken as a standard formula. In this formula $(\log \cosh 2\pi r_g/p)$

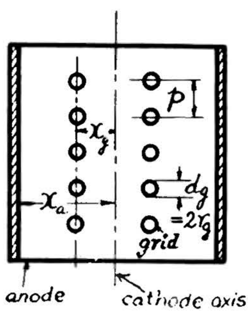

Fig. 10

is of the order of 0.1 and is small compared with $(2\pi x_g/p \log x_a/x_g)$ which is usually of the order of 10 to 100, and so the formula may be reduced to a simpler form:

$$k=\frac{c\dfrac{2\pi x_g}{p}\log\dfrac{x_a}{x_g}}{\log \coth\dfrac{2\pi r_g}{p}} \qquad (k-4)$$

Example 6: Calculation of amplification constant of a triode type UV203 (Tokyo Denki Co.) which is a plane-electrode tube with a cathode of W-shaped filament as shown in Fig. 12.

$$x_a=0.49 \qquad x_g=0.19 \qquad p=0.12 \qquad r_g=0.0085$$

$$\frac{2\pi x_g}{p}=9.95 \qquad \frac{2\pi r_g}{p}=0.445 \qquad \frac{p}{2\pi r_g}=2.25 \qquad \log\frac{x_a}{x_g}=0.412\,.$$

By equation $(k-1)$,

$$k=9.95\times\frac{1.60}{2.34\times 0.353}=19.6$$

and by equation $(k-2)$,

$$k=9.95\times\frac{0.412}{0.353}=11.6.$$

Equation $(k-4)$ gives

$$k=\frac{9.95\times 0.412\times 1.74}{0.467}=15.3 \quad (c=1.74)$$

and the observed value $k=15.9$.

Example 7: Calculation of amplification constant of a triode type UV201A (R. C. A.), dimensions of which are given in Fig. 13.

$$x_a=0.34 \qquad x_g=0.17 \qquad p=0.102 \qquad r_g=0.0053$$

$$\frac{2\pi x_g}{p}=10.5 \qquad \frac{2\pi r_g}{p}=0.33 \qquad \frac{p}{2\pi r_g}=3.03 \qquad \log\frac{x_a}{x_g}=0.301$$

By equation $(k-1)$,

$$k=10.5\times\frac{1.0}{2.30\times 0.482}=9.4$$

by $(k-2)$,

$$k=10.5\times\frac{0.301}{1.022}=3.1$$

and by $(k-4)$, as the filament is V-shaped $c=1.3$ for $x_a/x_g=2.0$ and

$$k=\frac{10.5\times 0.301\times 1.30}{0.505}=8.1.$$

Observed value, $k=8.2$.

In the above formulas grid is considered to be formed of parallel wires only. But in actual cases supports are usually added and in some cases grid is constructed of wires weaved into a mesh.

The writer brought out the following deductions in order to make the formula equally applicable to these cases.

In formula $(k-4)$, the quantity $2\pi x_g/p$ means the "total active length of grid wires per unit axial length of the grid" in a cylindrical-electrode tube, because $2\pi x_g$ is the length of one turn of grid wire in circular form and $1/p$ is the number of turns per unit length. This quantity is to be denoted by L_g, thus for a grid of closely-wound spiral wire

$$L_g=\frac{2\pi x_g}{p}.$$

Example 8: Amplification constant of a triode type RE-84 (Telefunken) which has a grid of spiral wire of wide pitch of winding. (Fig. 14.) If the value of $L_g=2\pi x_g/p$ is taken, i.e. if the grid is assumed to be formed of circular rings spaced at the pitch p,

$$x_a = 0.263 \quad x_g = 0.125 \quad p = 0.25 \quad r_g = 0.025 \quad \frac{2\pi x_g}{p} = 3.14$$

$$k = \frac{3.14 \times 0.323}{0.25} = 3.96.$$

If, on the other hand, actual length of the spiral grid wire per unit axial length of the grid as defined above is taken,

$$L_g = \sqrt{1 + \left(\frac{2\pi x_g}{p}\right)^2} = 3.30$$

$$k = \frac{3.30 \times 0.323}{0.25} = 4.4.$$

The observed value $k = 4.5$ proves that the latter is more consistent.

For a grid consisting of a square mesh of very thin wires

$$L_g = \frac{4\pi x_g}{p}$$

but when grid wire is not of small diameter, effect of overlapping of the wires should be considered. At each crossing point of two wires, active

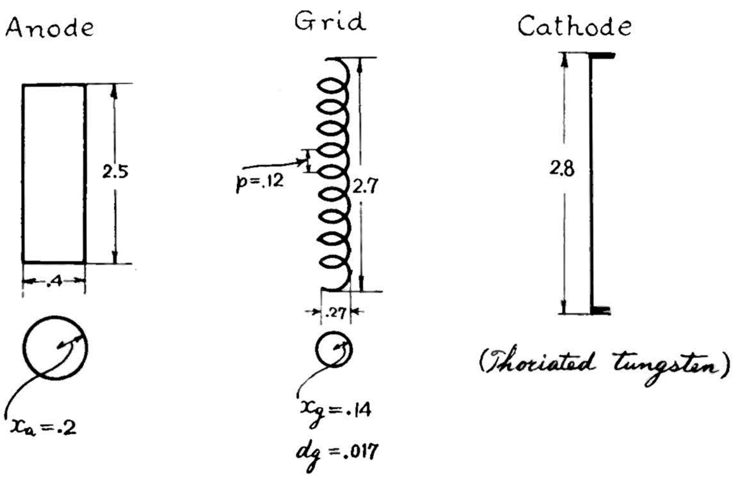

Fig. 11—Dimensions of type *ABT* tube.

length of one wire is reduced by an amount equal to the diameter of the wire, and as the number of crossing points per unit axial length of the grid is $2\pi x_g/p^2$ (Fig. 15),

$$L_g = \frac{4\pi x_g}{p} - \frac{2\pi x_g}{p^2} \cdot d_g = \frac{4\pi x_g}{p}\left(1 - \frac{r_g}{p}\right).$$

Effect of grid supports should be considered in the same way and for a grid of parallel wires,

$$L_g = \frac{2\pi x_g}{p} + s - \frac{sl}{p}$$

in which s is the number of grid supporting wires which are active in influencing on the space charge, and sl/p is the amount of reduction of

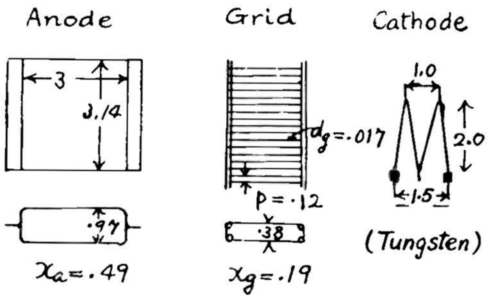

Fig. 12—Dimensions of type UV203 tube.

effective length of the grid wires due to the grid support covering a portion of each grid wire (Fig. 16).

For a grid made of a mesh, the effect of supports may not be considered, as the total length of the wires is very large, and moreover the

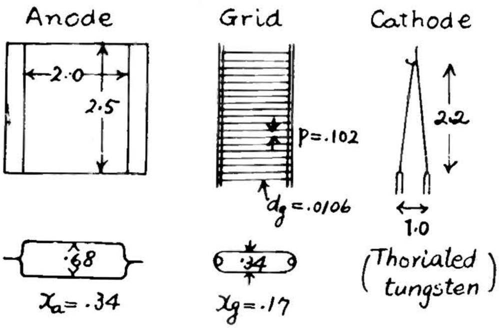

Fig. 13—Dimensions of type UV201A tube.

increase of L_g due to the supports is usually canceled by the reduction of it due to the support covering a portion of the mesh.

The quantity $2\pi r_g/p$ in formula $(k-4)$ signifies π-times d_g/p or "π-times the ratio of the grid conductor projected area to the grid surface area," and this ratio of areas being denoted by a,

$$\pi a = \pi \frac{\text{grid wire projected area per unit length of the grid}}{2\pi x_g}.$$

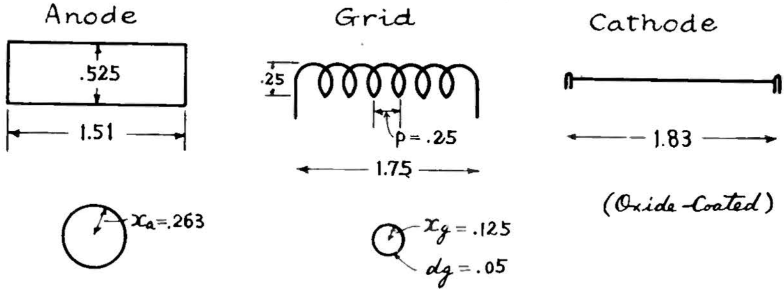

Fig. 14—Dimensions of type RE84 tube.

For a grid of a mesh or of a system of parallel wires,

$$\pi a = \frac{L_g d_g}{2x_g} = \frac{L_g r_g}{x_g}.$$

If the parallel wire grid has supports,

$$\pi a = \frac{(L_g - s)d_g + st}{2x_g}.$$

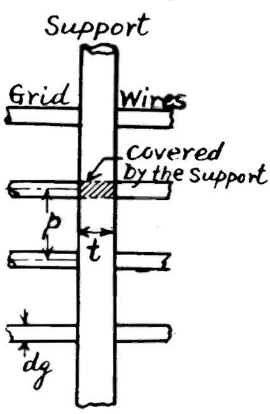

Fig. 15

Example 9: A triode type TA08/10 (Philips' Lamp Works) has a grid supported by a thick wire. (Fig. 17.)

$$x_a = 0.64 \quad x_g = 0.24 \quad p = 0.112 \quad r_g = 0.014 \quad \log \frac{x_a}{x_g} = 0.436.$$

Without considering the support:

$$L_g=\frac{2\pi x_g}{p}=13.2 \qquad \pi a=\frac{2\pi r_g}{p}=0.79 \qquad k=31.6.$$

Considering the support: $s=1, t=0.092$

$$L_g=\frac{2\pi x_g}{p}+s\left(1-\frac{t}{p}\right)=13.4 \qquad \pi a=\frac{(L_g-s)d_g+st}{2x_g}=0.93 \qquad k=43.$$

Observed value: $k=46$.

Example 10: A triode type MT-4 (Marconi Co.) has a grid of a mesh as shown in Fig. 18.

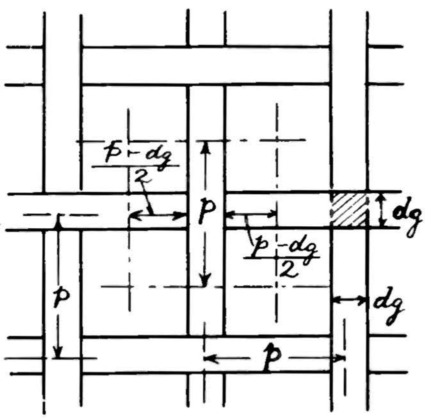

Fig. 16

$$x_a=2.17 \quad x_g=0.93 \quad p=0.13 \quad r_g=0.011 \quad \text{og}\frac{x_a}{x_g}=0.368.$$

Without considering the overlapping of the grid wires,

$$L_g=\frac{4\pi x_g}{p}=90 \qquad \pi a=\frac{L_g r_g}{x_g}=1.06 \qquad k=\frac{90\times 0.368}{0.105}=315$$

and considering the overlapping;

$$L_g=\frac{4\pi x_g}{p}\left(1-\frac{r_g}{p}\right)=90\times 0.91=81 \quad \pi a=\frac{L_g r_g}{x_g}=0.96 \quad k=\frac{81\times 0.368}{0.129}=230.$$

Observed value: $k=210$.

The formula for the amplification constant finally attains the form;

$$k=\frac{c\,L_g \log\dfrac{x_a}{x_g}}{\log \coth \pi a}. \tag{13}$$

The estimation of L_g and πa is summarized in Table VII.

TABLE VII

Grid construction	L_g	πa
Wires parallel to the filament	n	$\frac{nd_g}{2x_g}$ or $\frac{L_g r_g}{x_g}$
Parallel wires or spiral, no support	$\frac{2\pi x_g}{p}$	$\frac{\pi d_g}{p}$ or $\frac{L_g r_g}{x_g}$
Ditto, with supports in the path of electrons	$\frac{2\pi x_g}{p}+s\left(1-\frac{t}{p}\right)$	$\frac{(L_g-s)d_g+st}{2x_g}$
Square mesh, with or without support	$\frac{4\pi x_g}{p}\left(1-\frac{r_g}{p}\right)$	$\frac{\pi d_g}{p}\left(2-\frac{d_g}{p}\right)$ or $\frac{L_g r_g}{x_g}$

These are equally applicable to tubes of either cylindrical or plane electrodes.

The amplification constant is mainly governed by grid construction and depends little on anode area and filament configuration. Main factors determining k are x_a/x_g, p/x_g, and r_g/p, and similar tubes of

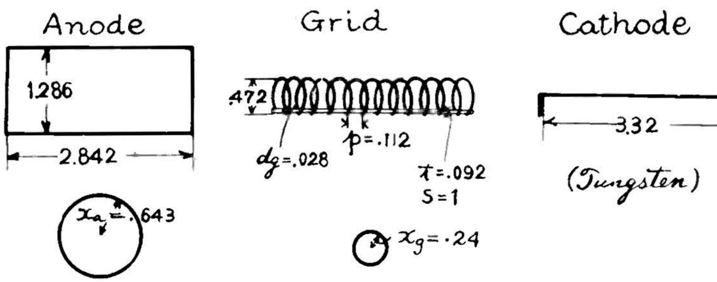

Fig. 17—Dimensions of type TA18/10 tube.

equal relative dimensions will give equal values of k whatever the size may be. k is affected by grid current at positive grid voltages and by the non-uniformity of grid action along the length of the cathode at very low anode currents, observed value of k being slightly reduced in both the cases.

Electrolytic Method of Determining k

By the amplification constant k is meant that the electrostatic influence of the grid upon the cathode is k-times that of the anode. Electrostatic field in a space assumes similar distribution as stream lines of electric current that would take place if the space were filled with conducting material. The amplification constant may thus be determined by means of a model as shown in Fig. 19, in which only a single section of grid wire is taken, as this is sufficient for the purpose. By taking the ratio of electric conductance of grid-to-cathode to that of anode-to-cathode, after the bridge is balanced, the amplification constant is known as

$$k = \frac{R_a}{R_g}.$$

Table VIII shows an example obtained by the simplest structure as shown in Fig. 19, and calculated values are given for comparison.

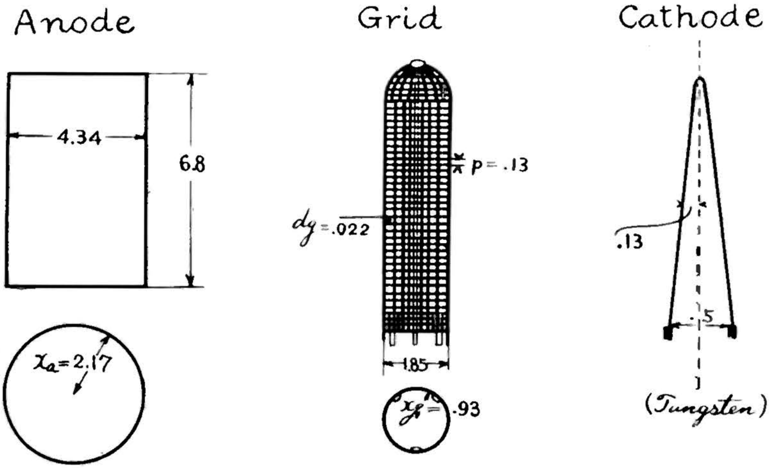

Fig. 18—Dimensions of type MT4 tube.

TABLE VIII

x_a	x_g	p	r_g	k obs.	k calc.
4.0	3.0	3.0	0.125	1.4	1.27
10.0	3.0	4.0	0.125	3.2	3.45
10.0	2.0	4.0	0.250	5.4	5.09
10.0	4.0	3.0	0.250	10.0	10.3
7.0	3.0	2.0	0.250	21.0	20.4

That the cathode configuration gives little effect on k could be evidenced by trying various forms and arrangements of the cathode conductor.

In designing a triode with very complicated structures that do not permit calculation, this method of predetermining the value of k will save much the effort of evacuation and readjustment of completed tubes.

Mutual Conductance and Anode Resistance

The mutual conductance g for given anode and grid voltages e_a and e_g can be obtained by differentiation of the characteristic equation, thus

$$g=\frac{\partial i_a}{\partial e_g}=\tfrac{2}{5}G\frac{1}{e_f}\cdot\frac{\partial}{\partial e_g}\left[e_g'^{5/2}-(e_g'-e_f)^{5/2}\right]$$

$$\doteqdot 1.5G\frac{k}{1+k}\left(\sqrt{e_g'}-\frac{1}{4}\cdot\frac{e_f}{\sqrt{e_g'}}\right) \qquad (14)$$

in which

$$e_g'=\frac{e_a+ke_g}{1+k}.$$

This is strictly applicable when grid current is not appreciable.

The mutual conductance is principally governed by the perveance G which is proportional to the anode effective area and is inversely proportional to the electrode distances.

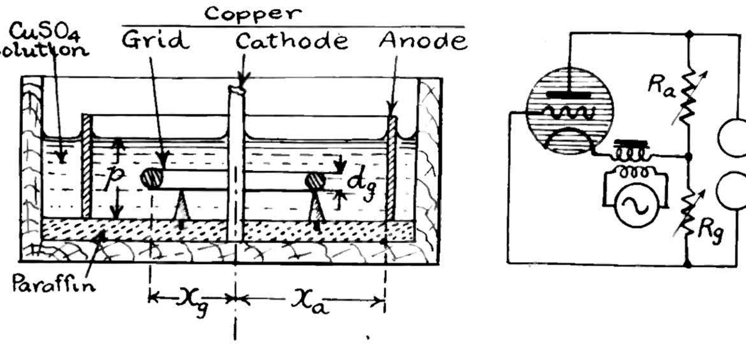

Fig. 19—Determination of the amplification constant by means of models.

The anode resistance r may also be obtained as $\partial e_a/\partial i_a$, but it is more convenient to calculate it from the known values of k and g as

$$r=\frac{k}{g} \qquad (15)$$

Example 11: Calculation of mutual conductance and anode resistance of a triode type T-100 (Marconi Co.).

From the dimensions shown in Fig. 20.

$x_a=1.15$ $x_g=0.44$ $p=0.095$ $r_g=0.0075$ $A=2\pi xal=23.9$

$k_{\text{calc.}}=52.7$ $Z_a=1.15-0.08=1.07$ $Z_g=0.44-0.08=0.36$

$e_f=10$ (rating) $G=0.127\times10^{-3}$ $1.5G\frac{k}{1+k}=0.187\times10^{-3}$.

(1) To obtain g and r for $e_a=1{,}500$ and $e_g=0$,

$e_g'=27.9$ $\sqrt{e_g'}=5.28$

$g=0.187\times10^{-3}\times4.81=0.00090$ $r=\frac{52.7}{0.00090}=585{,}000$.

observed values: $g=0.00089$, $r=540{,}000$.

(2) For $e_a=2{,}000$ and $e_g=-20$,

$$e_g'=17.7 \qquad \sqrt{e_g'}=4.20$$

$$g=0.000675 \qquad r=78{,}100$$

observed values: $g=0.000675$, $r=71{,}000$.

Saturating Part of the Characteristic

Saturation current I_s depends on the material of which the cathode is made, and for a given material it depends on the dimensions and working temperature of the cathode. General characteristics of the

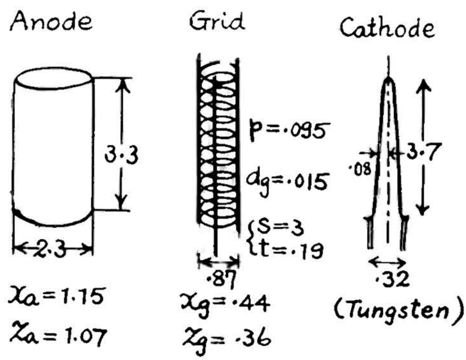

Fig. 20—Dimensions of type T100 tube.

three types of cathode, i.e. tungsten, thoriated tungsten, and oxide-coated cathode, were already published by many authorities and no particular mention is made here.

The point at which the anode current begins to saturate occurs at an equivalent voltage e_g' such as

$$e_s=\left(\frac{I_s}{G}\right)^{2/3}.$$

If the cathode consists of a uniformly emissive surface of equal potential all over it, the characteristic would sharply bend to saturate as soon as the voltage reaches the saturating voltage e_s. But in actual cases, slow bending of the characteristic curve is invariably observed, which is caused by the non-uniformity of filament temperature as well as the voltage drop along the filament. The expression of electron current considering the latter cause only is as follows;[3] in a range of the voltage tending to saturation: $e_s<e_g'<e_s+e_f$

$$i=\frac{1}{5}G\frac{1}{e_f}\left[e_s^{3/2}(5e_g'-3e_s)-2(e_g'-e_f)^{5/2}\right].$$

This expression does not fully represent the saturating part in actual cases, and the non-uniformity of emissive power of the cathode, especially due to the cooling of the ends, is by far the important factor. Even in an indirectly-heated tube the bending takes place very slowly.

Calculation of this part of the characteristic is very difficult as the temperature distribution must be known, and moreover the emissive power of the cathode is affected by various unexpected causes. The writer finds no need of calculating this part as it is of little practical importance for the designing.

In the original paper numerical examples of computation of characteristics on about fifty types of diodes and triodes were given, being compared with their observed characteristics.

Part II: Determination of Operations of a Triode from its Static Characteristics

Working Anode Current

The operations of a triode as an amplifier, oscillator, modulator, etc., can be determined from its static characteristics for a given circuit arrangement, and strict and elaborated theories in this respect have been published by many authors. The following discussion is intended only for a rough estimation of the operating conditions, simplicity being aimed at with an accuracy sufficient for the designing purpose.

The static characteristic of a triode comprises two parts:

1st Part: the anode current is determined from the approximate relation,

$$i_a=Ge_g'^{3/2}=\frac{G}{(1+k)^{3/2}}(e_a+ke_g)^{3/2}=G'(e_g+ke_g)^{3/2}. \tag{15}$$

2nd Part: anode current reaches saturation, and forms the upper limit of the characteristic.

The characteristic curve is assumed to have a shape as depicted in Fig. 21 with its sharp bending point at the saturating voltage E_s. When the characteristics are simplified in such a manner, all the tubes will have the similar curves, and if we express the anode current in the unit of the saturation current I_s, and all voltages in the unit of saturating voltage E_s, a simple and universal representation of the triode characteristic is obtained.

Let E_g = grid steady voltage,
E_a = anode steady voltage,
$\mathcal{E}_g$ = grid alternating voltage in maximum value,
$\mathcal{E}_a$ = anode alternating voltage in maximum value,

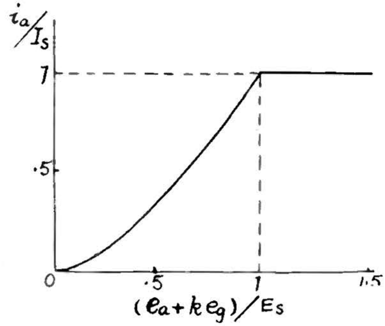

Fig. 21

positive directions being taken in the direction of direct current (Fig. 22). The load connected in the anode circuit is assumed to act as a resistance R at the operating frequency and to produce no potential drop by the d-c component of the anode current.

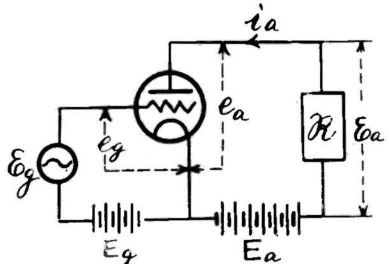

Fig. 22

In ordinary working conditions $\mathcal{E}_a$ and $\mathcal{E}_g$ are generally in opposite phase, and if the instantaneous value of the grid voltage is represented by

$$e_g = E_g + \mathcal{E}_g \cos \omega t$$

anode voltage will be

$$e_a = E_a - \mathcal{E}_a \cos \omega t$$

and for a given tube, instantaneous value of anode current will be determined by

$$e_a+ke_g=(E_a+kE_g)+(k\mathcal{E}_g-\mathcal{E}_a)\cos\omega t.$$

Thus in Fig. 23, a steady voltage E_a+kE_g determines the working point p on the characteristic and the alternating component $k\mathcal{E}_g-\mathcal{E}_a$ gives the amplitude of oscillation, the working point being the center.

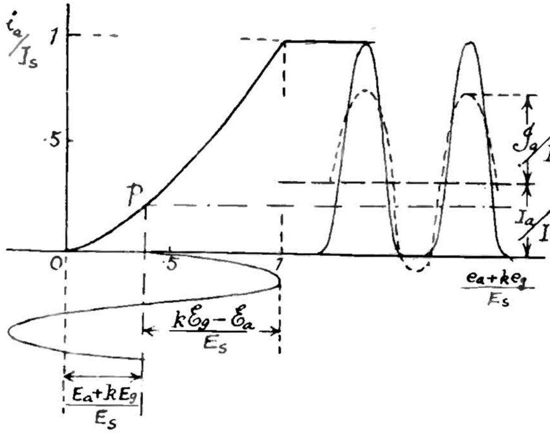

Fig. 23*

The variation of anode current due to this oscillating voltage is sinusoidal when the amplitude of voltage is very small, but becomes distorted when it is large.

Let I_a = mean value or d-c component of the anode current,

$\mathbf{I}_a$ = fundamental a-c component of the anode current, in maximum value, positive direction being taken as that of direct current.

For an infinitesimal amplitude of oscillation,

$$\mathbf{I}_a=\frac{di_a}{d(e_a+ke_g)}(k\mathcal{E}_g-\mathcal{E}_a)$$

and for a constant anode voltage,

$$\frac{di_a}{d(e_a+ke_g)}=\frac{1}{k}\cdot\frac{di_a}{de_g}=\frac{g}{k}=\frac{1}{r}$$

and

$$\mathbf{I}_a=\frac{k\mathcal{E}_g-\mathcal{E}_a}{r}$$

this gives the well-known fact that the triode circuit can be represented by an equivalent circuit as shown in Fig. 24, regarding the a-c components of working quantities.

* The symbol *Ia* in the illustrations is designated by **I***a* in the text.

For general cases, the writer obtained the wave forms of the anode current, and calculated the d-c and a-c components of the anode current by the Fourier analysis. (Fig. 23)

Dynamic Characteristic Diagram

The result of calculation of the d-c and a-c components of the anode current at various working voltages is summarized in a diagram shown in Fig. 25. This diagram is universally applicable to any particular

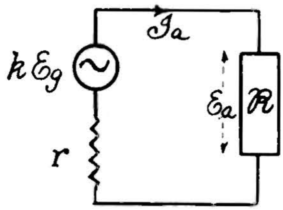

Fig. 24

triode as long as voltages are expressed in the unit of its saturating voltage E_s, and currents in the unit of the saturation current I_s. These quantities taken as units can be obtained from the static characteristics and E_s can also be calculated from the following relations:

$$E_s=\left(\frac{I_s}{G'}\right)^{2/3}. \qquad G'=\frac{G}{(1+k)^{2/3}}.$$

The dynamic characteristic diagram represents the surfaces of I_a/I_s and I_a/I_s as determined by the voltages $(E_a+kE_g)/E_s$ and $(k\mathcal{E}_g-\mathcal{E}_a)/E_s$, and comprises the following regions of practical importance, as designated in Fig. 26.

TABLE IX

Regions	Working point	Working range of anode current	Practical value
A	On the slope of the static characteristic	Entirely in the slope, not limited	Distortionless amplification
B	Ditto.	Limited by zero line.	Self-oscillation practicable and effectively subjected to modulation.
C	Ditto.	Limited by saturation.	Ditto, but with lower efficiency.
D	Ditto.	Limited by both zero-line and saturation.	Self-oscillation or amplification with large power.
E	At zero anode current with highly negative grid	Lower half-cycle entirely suppressed, upper peak not limited.	Power amplification of modulated waves.
F	Ditto.	Ditto, upper peak limited by saturation.	Self-oscillation or amplification of large power with high efficiency.

It should be remembered that this diagram has been derived under the following assumptions:

(a) static characteristic has sharp bending toward saturation,

(b) anode and grid voltages vary in sine wave form and just in opposite phase,

(c) grid current is not appreciable,

(d) load connected in the anode circuit does not produce appreciable voltage drop by direct current as well as by harmonic waves.

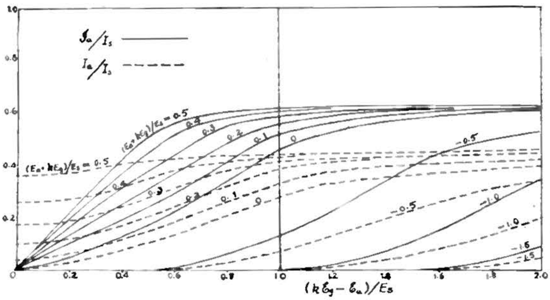

Fig. 25—Dynamic characteristics.

Thus the diagram is not strictly applicable when there is considerable grid current flowing, which takes place when the maximum grid voltage $E_g + \mathcal{E}_g$ becomes comparable with the minimum anode voltage $E_a - \mathcal{E}_a$. In such a case the anode alternating current attains a drooping characteristic at high excitation as shown in Fig. 27.

Experimental Verification

The dynamic characteristic diagram was obtained by experiment in the following way. A triode was used as a radio-frequency power amplifier in a circuit shown in Fig. 28. The static characteristics of the tube are shown in Fig. 29, from which the necessary quantities for the units were calculated as follows:

$$I_s = 0.071 \qquad E_s = 1.220 \qquad k = 25.$$

During the experiment, under constant filament current, E_a and E_g were varied and at each step of variation, I_0 and I_a were observed with variation of I' and C_g. Calculations were then made as follows,

$$\mathcal{E}_a = \frac{\mathrm{I}_0}{\omega C} \qquad \mathcal{R} = \frac{L}{CR} = \frac{1}{\omega^2 C^2 R} \qquad \mathrm{I}_a = \frac{\mathcal{E}_a}{\mathcal{R}} \qquad \mathcal{E}_g = \frac{\mathrm{I}'}{\omega C_g}.$$

In the actual case, $\omega = 3 \times 10^6 (\lambda = 628 \text{ m})$ and high-frequency resistance of the oscillatory circuit was 3 ohms so that $R = 3 + R'$. Hence

$(k\mathcal{E}_g - \mathcal{E}_a)/E_s$ could be calculated for each value of $(E_a+kE_g)/E_s$, and $\mathbf{I}_a/I_s$ and I_a/I_s were plotted against $(k\mathcal{E}_g - \mathcal{E}_a)/E_s$ to obtain the dynamic characteristic curves.

The resulting curves are shown in Fig. 30, which are in conformity with the theoretical diagram in Fig. 25 in the main.

Application of the Diagram

In Fig. 31, the dynamic characteristic diagram is shown with its left side extended to another diagram in which the ordinate gives

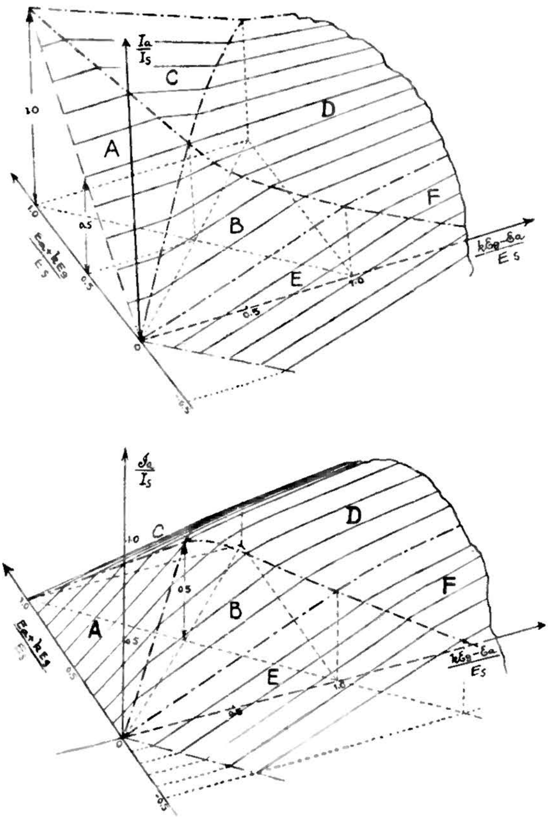

Fig. 26—Dynamic characteristic surfaces.

currents in the unit of I_s, and the abscissa gives voltages in the unit of E_s. In this part of the diagram a group of straight lines starting at the origin represent the external resistance R in the unit of the mean anode internal resistance $\bar{r}$ which is derived as $\bar{r}=E_s/I_s$, and hyperbolas give the contour lines of power, unit of which is to be taken as E_sI_s.

To explain the use of the diagram, assume that a triode having saturation current I_s, amplification constant k, and saturating voltage E_s, is to be used as an amplifier with an anode circuit impedance R, and its operating conditions are to be studied. First calculate the unit quantities $\bar{r}=E_s/I_s$ and E_sI_s from the given E_s and I_s. All the currents, voltages, resistances and powers are to be expressed in the units of I_s, E_s, $\bar{r}$ and E_sI_s, respectively. In the following explanations quantities expressed in this manner are taken instead of actual values.

Let the anode voltage be E_a and grid voltage be E_g. Calculate E_a+kE_g and the corresponding dynamic characteristic curves of I_a/I_s and I_a/I_s are found on the diagram such as the lines *oeg* and *def* in Fig. 32. Draw a line *oh* to represent the given external impedance. For a given grid exciting voltage $\mathcal{E}_g$, calculate $k\mathcal{E}_g$ and take this

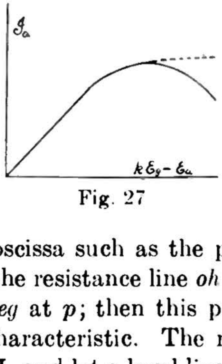

Fig. 27

quantity on the right abscissa such as the point i; from i a straight line is drawn parallel to the resistance line *oh* and let this line intersect with the dynamic line *oeg* at p; then this point p gives the working point on the dynamic characteristic. The reason is as follows: the ordinate of p represents I_a and let a level line through p intersect with the *oh* line at j and with the ordinate axis at k, then $\overline{ok}=\mathrm{I}_a$ and $\tan\angle jok=R$. Therefore $\overline{jk}=\mathrm{I}_aR=\mathcal{E}_a$, but $\overline{pj}=\overline{oi}=k\mathcal{E}_g$ and thus the abscissa of the point p is $\overline{pk}=\overline{pj}-\overline{jk}=k\mathcal{E}_g-\mathcal{E}_a$ and is in conformity with the dynamic characteristic curves which have been expressed by the abscissa of $k\mathcal{E}_g-\mathcal{E}_a$.

The area of the triangle *ojk* gives $\frac{1}{2}\mathcal{E}_a\mathrm{I}_a$ or $\frac{1}{2}\mathrm{I}_a^2R$ and represents the power output, which can be read by means of the power contour hyperbolas numbered at the top. The steady component I_a is obtained by drawing a line vertically from point p to intersect with the I_a/I_s dynamic line at a point q. Take $\overline{oa}$ equal to E_a. A level line through q intersects with the vertical line through a at l, and with the ordinate axis at m. Then $\overline{om}=I_a$, and the rectangular area *omla* is E_aI_a and represents the anode power input to the tube which can be read from the position of the point l on the contour hyperbolas numbered at the left end. The efficiency of power conversion can be calculated from the two power values as $\eta = \frac{1}{2} I_a^2 R / E_a I_a$.

When the triode is used as a self-excited oscillator,[9] the grid voltage $\mathcal{E}_g$ is in definite relation to the anode voltage $\mathcal{E}_a$ as determined from the circuit conditions. In an example shown in Fig. 33, I_a is determined from $\mathcal{E}_g$ by a dynamic line *ops*, while I_a and external impedance $R \doteq L/CR$ determine $\mathcal{E}_a = I_a(L/CR)$ and this induces an e.m.f. $\mathcal{E}_g \doteq \mathcal{E}_a(M/L)$ in the grid circuit. Hence $I_a/\mathcal{E}_g = CR/M$ is a straight line, the inclination of which is $\tan^{-1} CR/M$ as shown by a line *ot* in Fig. 33. Thus the dynamic characteristic curve *ops* gives the relation in which I_a is determined by $\mathcal{E}_g$ from the tube characteristics, while the excitation line *ot* gives the relation in which $\mathcal{E}_g$ is determined by

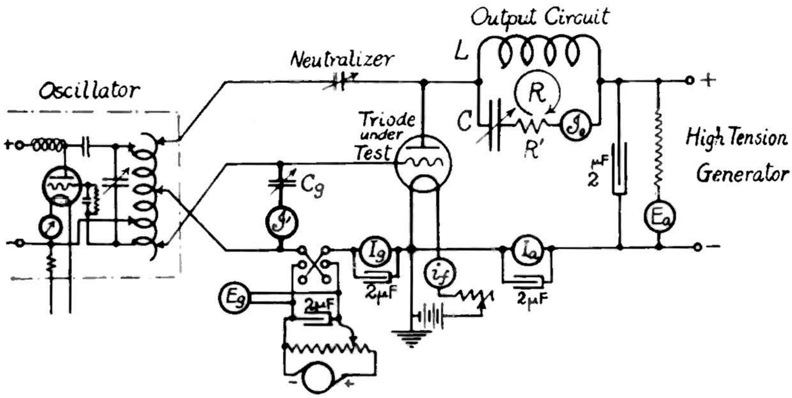

Fig. 28

I_a from circuit conditions. Then the point of intersection p shall be the working point of the self-oscillator. If the coupling becomes loose, the excitation line is shifted to left and finally will not intersect with the dynamic line, and in this case oscillation cannot be produced.

The regeneration can be explained in the same diagram.[9,10] When regeneration is applied, anode circuit is coupled to grid circuit but not so close as to produce oscillation, such as a line *ot′* in Fig. 33. If the regenerative connection is not made, for a certain grid voltage such as $\overline{ou}$, the anode current is $\overline{uv}$. But on regenerative condition, a line is to be drawn from the point u parallel to the line *ot′* and the intersecting point w on the characteristic gives the working point. The reason is that the level line from w intersecting with *ot′* at r and with the ordi-

[9] H. G. Möller, "Elektronenröhren und ihre technischen Anwendungen," 1920.

[10] E. H. Lange, *Phil. Mag.*, 750; October, 1925.

nate axis at z represents the grid excitation which is the sum of the originally impressed voltage $\overline{ou}=\overline{rw}$ and the voltage induced by the anode current $\overline{rz}$. The ratio of regenerative amplification to non-regenerative one may be expressed by $\overline{oz}/\overline{uv}$.

Now these ideas can be applied to the dynamic characteristic diagram as shown in Fig. 34. Comparing the figure with Fig. 33, resistance line *oh* corresponds to the ordinate axis of Fig. 33, and the abscissa representing $k\mathcal{E}_g$ corresponds to that of Fig. 33. The excitation line *ot* in Fig. 33 is obtained in Fig. 34 by taking the quantity $k\mathcal{E}_g/\mathrm{I}_a$ from the point h to the right on the level line as a point t, and by connecting this point to the origin; in other words $\{(k\mathcal{E}_g/\mathrm{I}_a)-R\}/\bar{r}$ shall be taken on right abscissa as $\overline{zt}$ in order to obtain the excitation line *ot*.

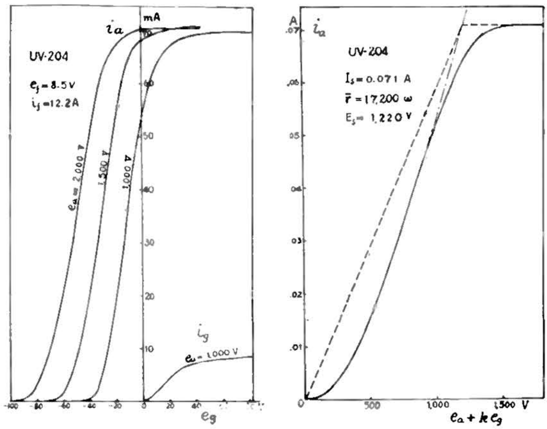

Fig. 29—Characteristics of type UV204 tube.

The point of intersection of the excitation line with the dynamic characteristic curve gives the working point p and all the other quantities can be derived in the same way as explained before. In Fig. 31 $(k\mathcal{E}_g/\mathrm{I}_a)_c$ means that the quantity is to be derived from the circuit conditions.

The modulation of a self-oscillating tube consists in altering the anode or grid steady voltage with audio frequency, hence producing the corresponding variations in the amplitude of oscillation. This means the variation of E_a+kE_g, other things being unvaried. Thus, for example, if anode voltage E_a is varied in sinusoidal wave form as shown in Fig. 35, E_a+kE_g will vary accordingly and the dynamic characteristic curve is shifted up and down. The excitation line *ot* remaining unvaried, the working point p must travel on the *ot* line and the oscillating current varies its amplitude in accordance with the variation of the anode voltage.

Applications of the diagram have been further illustrated by numerical examples in the original paper. The diagram gives a bird's-eye view of various working conditions.

Part III: The Design of Triodes

Working Points on the Dynamic Characteristics

The design of a triode to satisfy specified conditions can be accomplished in the following way, which shows a typical case where the use of the triode is indicated and its power output is given.

Optimum working points can always be found on the dynamic characteristic diagram for any particular use of a triode. These optimum conditions which the writer conceives to be appropriate are given below.

TABLE X

Use of Triode	$(E_a+kE_g)/E_s$	I_a/I_s	R/r
Low power amplifier	0.3—0.5	0.2—0.3	1.0
Distortionless power amplifier or modulator	0.3—0.5	0.2—0.4	2.0
Oscillator or power amplifier for radio-telegraphy	—1.0	0.55	1.0
Self-oscillator for radio-telephony	0.3	0.35	1.2
Power amplifier for radio-telephony, amplifying modulated wave	0—0.2	0.25	1.0
Ditto, amplifying continuous wave and subjected to modulation	0	0.3	1.5

If the above three quantities are decided, the following may be directly obtained from the diagram:

$$\frac{P}{E_sI_s}, \quad \frac{k\mathcal{E}_g}{E_s}, \quad \frac{\mathcal{E}_a}{E_s}, \quad \frac{I_a}{I_s}.$$

Moreover E_a/E_s can be roughly estimated according to the use of the tube and in relation to the value of $\mathcal{E}_a/E_s$, and from this the approximate values of E_aI_a/E_sI_s and η can be found.

High-Impedance or Low-Impedance Tube

For a given power output P, $\alpha=P/E_sI_s$ having been known, E_sI_s can be obtained and next comes an essential problem of splitting E_sI_s into the saturating voltage E_s and the saturation current I_s.

If one of any other operating conditions, such as anode voltage E_a

or anode circuit impedance R, is given at the same time, the problem is directly solved.

If this is not the case, especially in the design of standard types of tubes, most precise considerations must be taken for this from technical and economical points of view. This is the problem of selecting

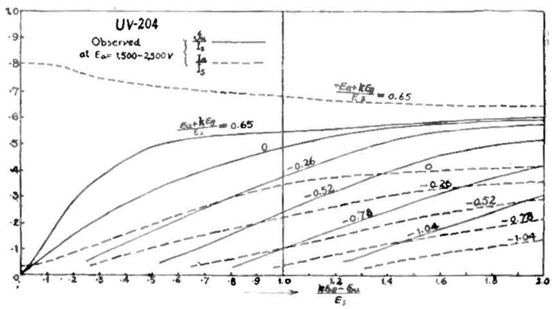

Fig. 30—Dynamic characteristics of UV204 tube obtained by experiment at radio frequency.

the type of the tube between a high-voltage low-emission type, and a low-voltage high-emission type. The general considerations on the two alternatives, high-impedance tube or low-impedance tube in other words, are as follows for a given power output:

TABLE XI

	High impedance tube	Low impedance tube	Remarks
Anode voltage	high	low	← Anode voltage must be within a practicable limit, from standpoint of insulations of the tube and other apparatus.
Saturation current	low	high	
Cathode power consumption	little	large	← Lower filament power is preferable as overall operating efficiency becomes higher and anode radiating area can also be reduced.
Circuit impedance	high	low	
Amplification constant	high	low	

Anode power conversion efficiency is not very different when it is considered on the diagram as it is determined from the working points, but in a higher voltage tube, anode voltage is more completely utilized and net efficiency will be a little higher than lower voltage tube.

This problem involves not only the economy of the tube itself but also that of apparatus with which the tube is to be used, such as output circuits, anode source of power, amplifiers preceding the tube under consideration, etc. The practicable limits in the values of E_a, R, cathode heating power, and relative amount of cathode heating power to anode loss, should also be taken into consideration.

Regarding the oscillatory power output of a triode, the following consideration is sometimes necessary. Output P, somewhat larger than the required value, has to be taken as the designing basis in order to take into account effect of the grid current, which was not done in the derivation of the dynamic characteristic diagram. The grid current affects the power output in two ways; first it reduces the anode current and hence the output, and secondly it absorbs some portion of the output in case of self-oscillators. An increase of 10 or 20 per cent of the required value will generally be sufficient, according as the tube is used as a power amplifier or a self-oscillator. More rigorous estimation of grid excitation loss will be given later.

Amplification Constant

From equation (15),

$$I_s = \frac{G}{(1+k)^{3/2}} E_s^{3/2} \quad \text{or} \quad (1+k)^3 = G^2 E_s^3 / I_s^2 = G^2 \bar{r}^3 I_s \tag{16}$$

and

$$P = \alpha E_s I_s$$

hence

$$(1+k)^3 = \left(\frac{G\alpha}{P}\right)^2 E_s^5. \tag{17}$$

G is not very much different with the type of tubes and is of the order as shown in the following table. This is because G is determined by the ratio of anode area to distances between electrodes, and in practical construction of the tube there are limits in the attainable anode area for a given length of the filament and for a given span of the filament the distances between electrodes must not be brought too close together in order to prevent an accidental short-circuit of the electrodes during operation.

TABLE XII

Tubes		No. of tubes examined	G	
Anode voltage	Electrodes		Range	Average
6,000–10,000	cylinder	14	$0.08-0.4\times10^{-3}$	0.15×10^{-3}
200– 5,000	cylinder	8	$0.08-0.4$	0.17
"	plane	15	$0.15-0.9$	0.38
below 100	cylinder	10	$0.12-0.5$	0.18
"	plane	5	$0.28-0.5$	0.33

Total average: G for plane electrodes, 0.35×10^{-3}
G for cylindrical electrodes, 0.16×10^{-3}.

α and P having been known, and G being assumed, k is in definite relation to E_s as given by equation (17), and once E_s is determined k is directly obtainable.

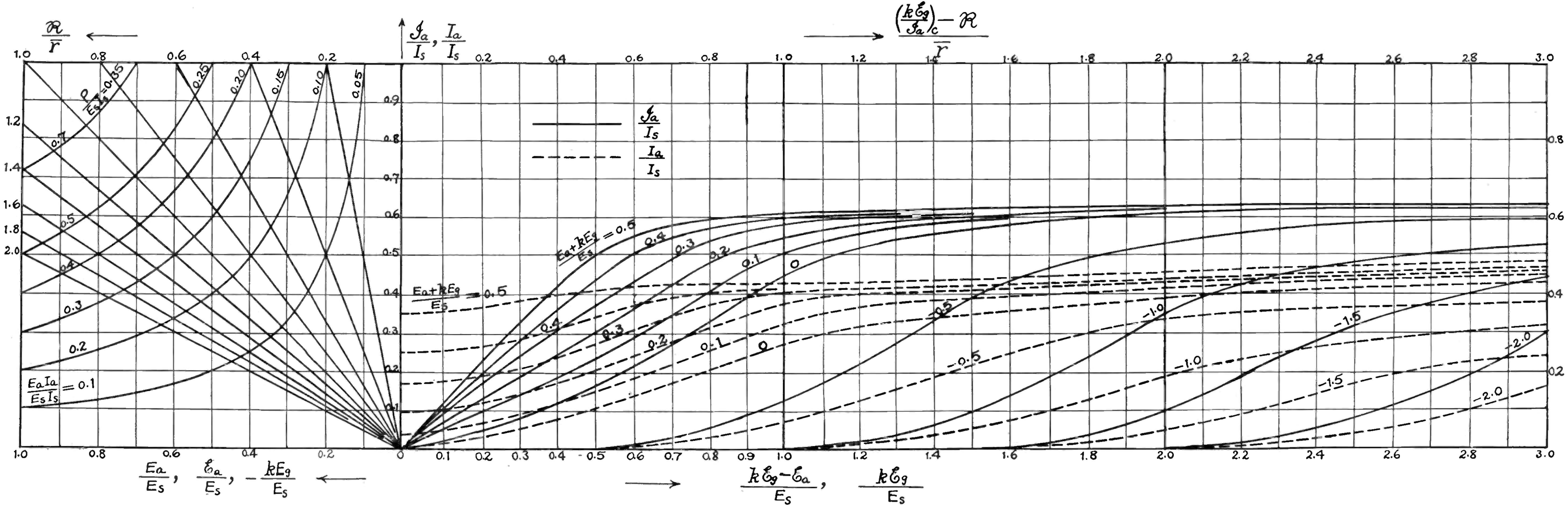

Fig. 31—Dynamic characteristic diagram.

In case of oscillators, self or separately excited, power lost in the excitation depends on the value of k. In a self-excited oscillator, this power is converted from the output of the tube and too much exciting power results in the lowering of the net output and overloading of the grid. In a separately-excited oscillator or power amplifier, the grid loss

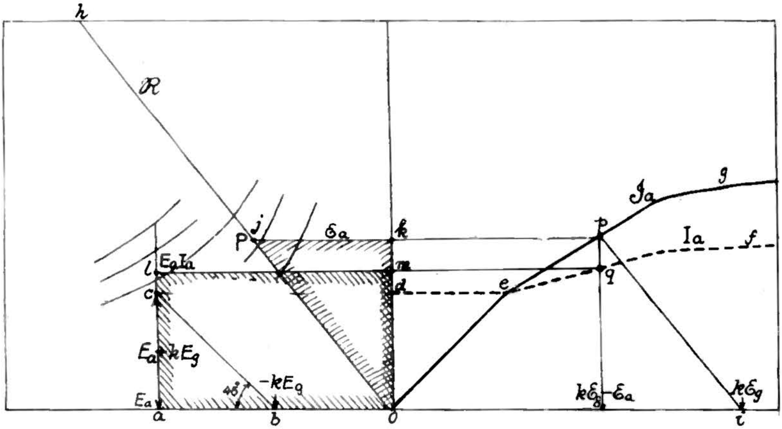

Fig. 32

is to be supplied from the preceding tube. In these cases the value of k may be estimated as follows; for the most favorable operation delivering maximum power at a given anode voltage, the minimum anode voltage $E_a - \mathcal{E}_a$ must be nearly equal to the maximum grid voltage $E_g + \mathcal{E}_g$,[11] E_g being negative, and this requires that

$$E_a - \mathcal{E}_a = E_g + \mathcal{E}_g$$

or

$$\frac{E_a}{E_s} - \frac{\mathcal{E}_a}{E_s} = \left(\frac{kE_g}{E_s} + \frac{k\mathcal{E}_g}{E_s}\right)\frac{1}{k}$$

hence

$$k = \frac{\left(\dfrac{k\mathcal{E}_g}{E_s}\right) + \left(\dfrac{E_a + kE_g}{E_s}\right) - \left(\dfrac{E_a}{E_s}\right)}{\left(\dfrac{E_a}{E_s}\right) - \left(\dfrac{\mathcal{E}_a}{E_s}\right)} \tag{18}$$

in which all quantities except E_a/E_s have accurately been known. Thus if k is given, E_a/E_s can be accurately determined and E_aI_a/E_sI_s and η are accordingly known.

[11] D. C. Prince, Proc. I. R. E., 11, 280; June, 1923.

For a rough estimation of the grid excitation loss in the above working condition, the grid current may be assumed to have a triangular wave form as shown in Fig. 36, and its peak value to reach

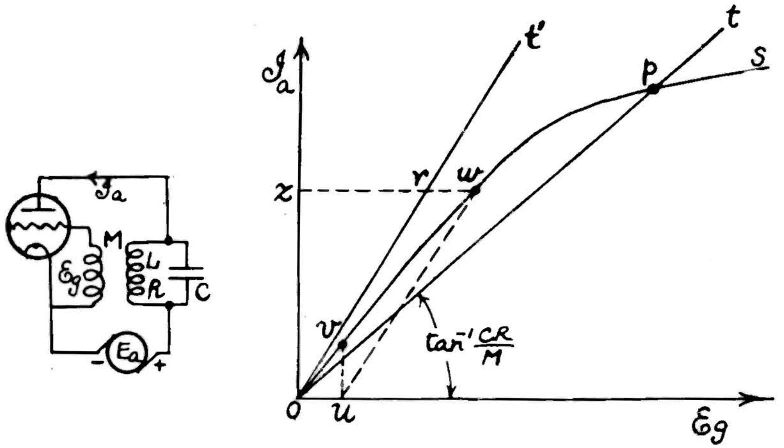

Fig. 33

$\frac{1}{2}I_s$. Of course this condition may not take place in actual case, but this will not be far from it. The results of calculations are as follows:

Grid current d-c component, $I_g = \mu I_s$.

Grid leak loss (or loss in charging the grid biasing battery), $W_l = -\mu E_g I_s$.

Grid excitation loss, $W_e = \nu \mathcal{E}_g I_s$.

Grid loss inside the tube or grid heating power, $W_g = (\nu \mathcal{E}_g + \mu E_g) I_s$.

Grid leak resistance required, $R_g = -E_g/I_g = -E_g/\mu I_s$.

in these relations E_g is a negative quantity, and μ and ν are the functions of $-E_g/\mathcal{E}_g$ or $-(kE_g/E_s)/(k\mathcal{E}_g/E_s)$ as given in Fig. 37.

If the grid loss inside the tube be allowed up to $1/b$ of the anode loss, $1/b\,(E_aI_a - P) \geqq (\nu \mathcal{E}_g + \mu E_g) I_s$ or

$$\frac{1}{b}\left(\frac{E_aI_a}{E_sI_s} - \frac{P}{E_sI_s}\right) \geqq \left(\nu\frac{k\mathcal{E}_g}{E_s} + \mu\frac{kE_g}{E_s}\right)\frac{1}{k}$$

hence

$$k \geqq b\frac{\nu\left(\dfrac{k\mathcal{E}_g}{E_s}\right) + \mu\left(\dfrac{kE_g}{E_s}\right)}{\left(\dfrac{E_aI_a}{E_sI_s}\right) - \left(\dfrac{P}{E_sI_s}\right)}. \tag{19}$$

If, on the other hand, the grid excitation loss is to be limited within $1/d$ of the power output,

$$\frac{P}{d} \geqq \nu \mathcal{E}_g I_s \quad \text{or} \quad k \geqq d\nu\left(\frac{k\mathcal{E}_g}{E_s}\right)\Big/\left(\frac{P}{E_s I_s}\right). \tag{20}$$

All the quantities in the parentheses have been known and the lower limit of k can be determined from these relations. The exact value of E_a/E_s must be derived by equation (18), after the value of k is determined.

It is advisable to calculate the required quantities for several values of k or E_s, and to select among them a most preferable one.

Dimensions of the Electrodes

The main factor determining the anode construction is the anode loss W_a which may be taken as $(E_a I_a - P)$. But in the case of amplifiers or modulators for radiotelephony, the dead anode loss corre-

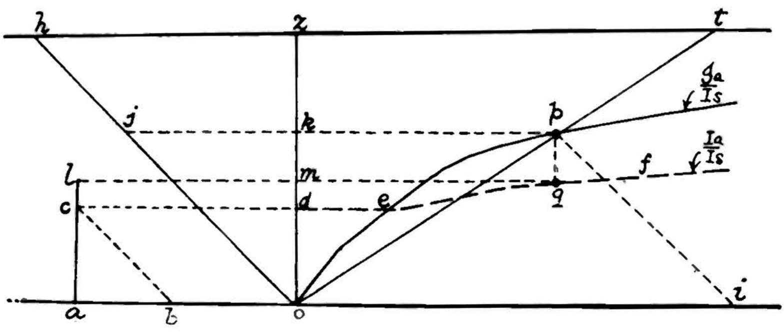

Fig. 34

sponding to the working point on the static characteristics, i.e., $GE_a\{(E_a+kE_g)/(1+k)\}^{3/2}$ must be taken as the anode loss, as this amount of loss actually takes place when speech is not being transmitted.

The allowable limit of anode loss is dependent on the material of the anode and its cooling condition. The following data giving the maximum limit of the loss continuously applicable to the anode may be taken in the ordinary construction.

TABLE XIII

Materials	Allowable loss (watt/cm²)
Nickel	$w=3$
Molybdenum	5
Tungsten	8
Copper, water-cooled	20

The filament heating power W_f must also be considered in estimating the anode cooling area, as some portion of it will have to dissipate through the anode. Suppose q part of the filament power is radiated through the anode, then

$$A \geqq \frac{W_a + qW_f}{w} \tag{21}$$

q depends on the degree at which the filament is hidden in the anode, and in ordinary cases in which the anode is open on both ends, q may be taken as about 0.5. The writer deems it reasonable to take the effective area A explained in Part I, instead of the actual area, in the above expression.

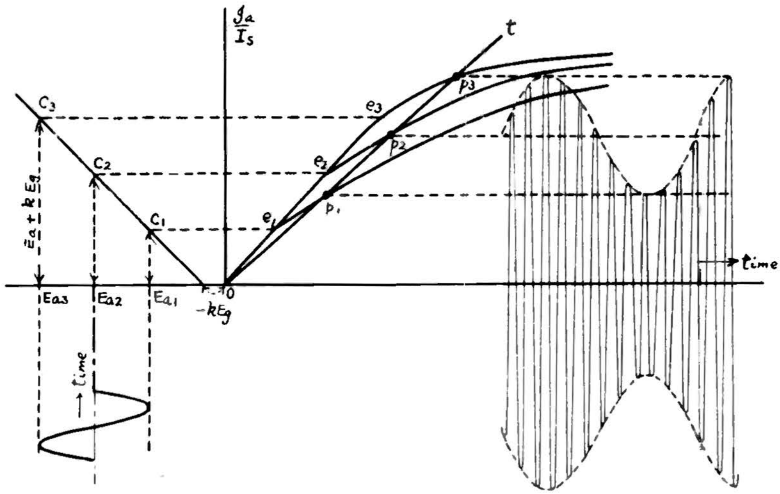

Fig. 35

In high-voltage tubes grid voltage also becomes high with respect to the cathode, and appreciable distances must be kept between grid and cathode in order to avoid severe electrostatic attraction. On the other hand, in low-voltage tubes, x_g can be reduced, and if the cathode is made of W-shaped filament arranged in a plane, A may be greatly increased without difficulty in construction. Thus for anode voltages less than say 4,000 v, plane anode is preferable, while for higher voltages cylindrical anode is advantageously applied.

The order of x_a in the existing tubes is as follows:

TABLE XIV

E_a below 100 V	X_a = 0.2–0.5
E_a 100–1,000	0.3–1.0
E_a 1,000–10,000	0.6–3.5

The actual anode surface area should be made as effective as possible in order to reduce the mass of the anode material and hence to minimize the absorbed gas evolving in the evacuation process, as well as from an economical point of view. The plate should be as thin as mechanically allowable, but not so thin as to avoid free conduction heat, otherwise local overheating or melting may occur at accidental overload of the anode.

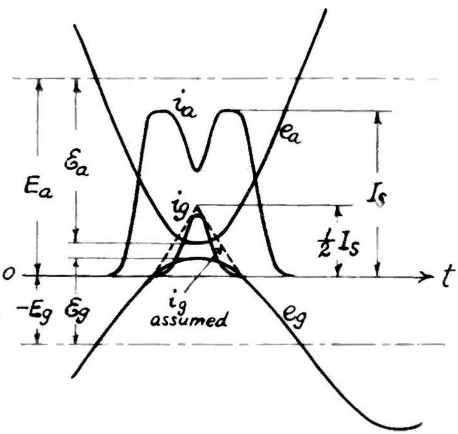

Fig. 36

The anode area A and anode-to-cathode distance x_a or z_a must be in conformity with each other to satisfy the expression,

$$G = 2.33 \times 10^{-6}\frac{A}{z_a z_g}.$$

The grid construction is determined from the amplification constant. When x_a is given and other dimensions p and d_g remain unvaried, k is maximum at $x_g \doteqdot 0.4\,x_a$, and it is advisable to take this in order to reduce the mass of the grid as well as to minimize the effect of non-uniformity of x_g on the value of k. Thus

$$\frac{k}{\log\frac{x_a}{x_g}} = \frac{k}{0.4} = \frac{cL_g}{\log\coth\pi a}.$$

The table in Part I is to be referred to for the calculation, and ranges of actual values of L_g and πa in existing tubes are shown in Fig. 39.

L_g and πa having been determined, the next problem is to determine

p and r_g. This means a decision as to whether to use thick wires at broad pitch or to use thin wires closely arranged.

It is preferable to take πa small in order to reduce the mass of the grid and to facilitate evacuation. This is effected by thin grid wires at comparatively narrow pitch, but in extreme cases mechanical rigidity of the grid will be lost and deformation or break down will occur during

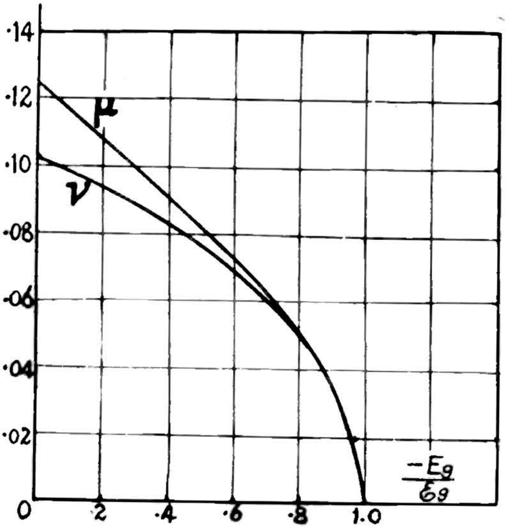

Fig. 37

operation by the heating of the grid. Fig. 38 shows actual values of p and r_g in existing tubes, from which the general tendency can be observed. Materials best suited for grid are molybdenum and nickel, but the latter has a remarkable effect of secondary emission and often causes the "blocking" of oscillators.

Design of the Cathode

Design of a tungsten cathode to give the required emission is fully described in the original paper, applying a chart worked out by N. Kato[12] based on the data given by Forsythe and Worthing.[13] There have been presented various other methods on the design of cathode, and no explanation is made here.

For a close approximation of the reasonable life of a tube, which in

[12] N. Kato, "Graphs for the Design of Bright-emitting Tungsten Filament," Circulars of the Electrotechnical Laboratory, No. 50; February, 1928. (Written in Japanese.)

[13] W. E. Forsythe and A. G. Worthing, *Astrophysical J.*, 61, 146, 1925.

ideal case is the life of the cathode, an economical condition must be dealt with in such a way that the total running cost of a tube is a

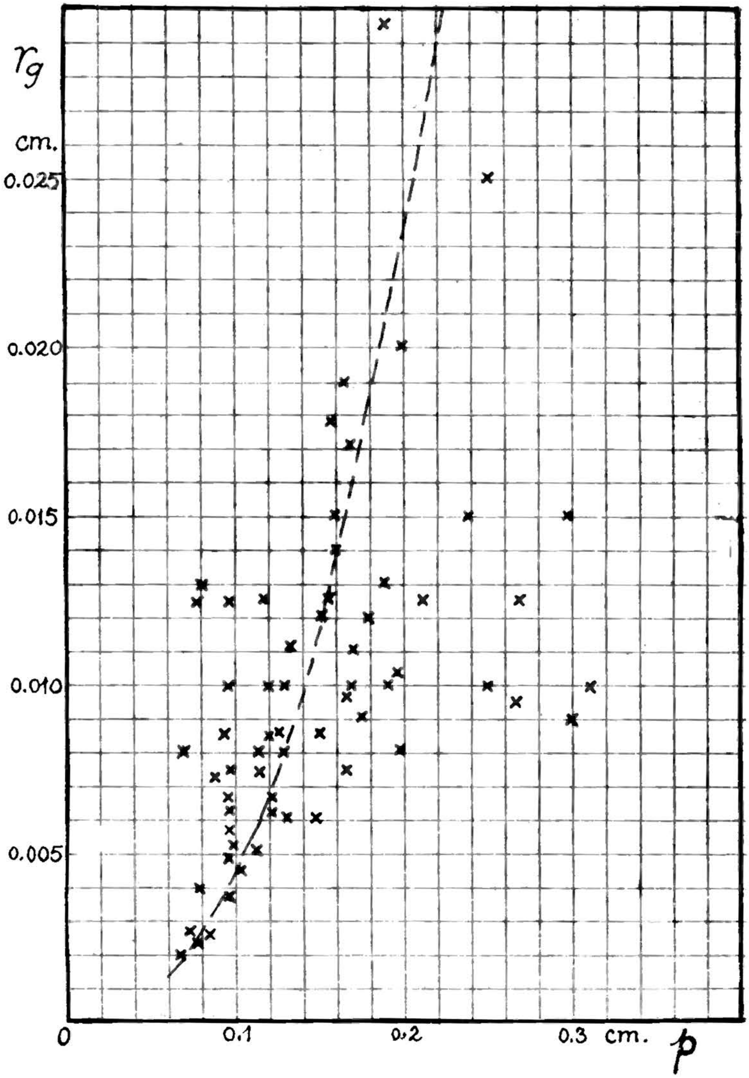

Fig. 38

minimum for a given power output. This condition is given by the following relations: total cost per watt output per hour,

$$T = \frac{Ct}{PL} + \left(\frac{1}{\eta} = \frac{W_f}{p}\right) C_g$$

in which C_t = cost of the tube, and C_p = cost of electric power per watt-hour. On the other hand, for the filament of a certain diameter the life L depends on its working temperature, which in turn is determined by the filament efficiency I_s/W_f at which it is operated, thus

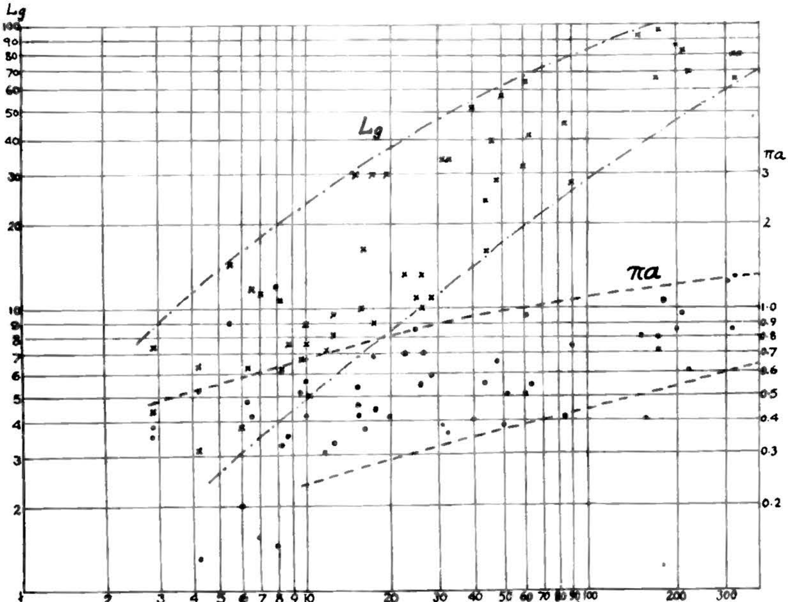

Fig. 39—Ranges of $Lg(x)$ and $\pi a(\cdot)$ in existing tubes.

$$L = \phi \left(\frac{W_f}{I_s}\right)^{\sigma}.$$

Minimum of the operating cost T occurs at a life

$$L_{\text{opt.}} = \phi^{1/1+\sigma} \left(\frac{\sigma C_t}{C_g I_s}\right)^{\sigma/1+\sigma}.$$

For tungsten filament, the two constants σ and ϕ are of the following orders,[14]

$$\sigma \doteqdot 2.6 \quad \frac{d_f \quad 0.005 \quad 0.020 \quad 0.050}{\phi \quad 0.00034 \quad 0.0026 \quad 0.033}.$$

[14] H. Simon, *Telefunken Z.*, Fig. 44 on p. 39; October, 1927.

It is preferable to design the filament to give a slightly higher value of emission than desired in order to give allowance for the discrepancies which might take place, as the emission is quite a variable quantity. For tungsten filaments it is sometimes necessary to take a slightly longer life than calculated, as a portion of the designed life will be lost during the evacuating process by evaporation.

Conclusion

In the above discussions the fundamental principles involved in the writer's system of designing is explained. Many of the factors that have to be dealt with from the practical point of view in the manufacturing are lacking, and further development is necessary in this respect, but it is hoped that this paper will be of some use to manufacturers.

The writer wishes to acknowledge his indebtedness to E. Yokoyama, under whose direction the present work has been carried out, and also to N. Kato, I. Miura, and S. Maeda for their assistance.

List of Symbols

A effective anode area (cm^2)
a ratio of grid conductor area to grid surface area
c conversion factor of k for plane-electrode tubes
d_f filament diameter (cm)
d_g grid wire diameter (cm)
e_a anode voltage (volt)
e_f filament terminal voltage (volt)
e_g grid voltage (volt)
e_g' equivalent grid voltage (volt)
E_S saturating voltage (volt)
E_a steady anode voltage (volt)
$\mathcal{E}_a$ alternating anode voltage, max. value (volt)
$\mathcal{E}_g$ alternating grid voltage, max. value (volt)
g mutual conductance (mho)
G perveance
i electron current (ampere)
i_a anode current, instantaneous value (ampere)
i_g grid current (ampere)
i_f filament current (ampere)
I_a anode mean or direct current (ampere)
I_g grid mean or direct current (ampere)
I_S saturation current (ampere)
$\mathbf{I}_a$ anode alternating current, fundamental component max. value (ampere)
k amplification constant
L life of a cathode (hour)
L_g ratio of total length of effective grid wires to axial length of the grid.
m number of points of support of the filament
n number of grid wires parallel to filament

p pitch of adjacent grid wires between centers (cm)
P power output of a triode (watt)
q portion of the filament power radiated through the anode
r anode resistance (ohm)
r mean anode resistance (ohm)
R a-c impedance (resistive) of the anode circuit (ohm)
r_f filament radius (cm)
r_g grid wire radius (cm)
s number of supports of grid wires which lie in the stream of electrons
t thickness of grid support (cm)
w permissible anode loss per unit area (watt/cm^2)
W_a anode loss (watt)
W_f filament heating power (watt)
W_g grid internal loss (watt)
W_e grid excitation loss (watt)
W_l grid leak loss (watt)
x_a distance of anode to cathode axis (cm)
x_g distance of grid to cathode axis (cm)
z_a mean shortest distance of anode to filament (cm)
z_g mean shortest distance of grid to filament (cm)
η anode power conversion efficiency