pls file
Tube design Handbook

## CALCULATION OF HEAT TRANSFER FOR WATER-COOLED, METAL-JACKETED, MERCURY POOL TUBES

The following is a general summary of the methods of evaluating the heat transfer characteristics of metal-jacketed mercury pool tubes which are cooled by forced convection of water. The usual method of cooling is by means of the circulation of water between the inner and outer jackets, the water entering from the bottom of the tube, leaving at the top. In order to render the problem amenable to analytic methods certain assumptions are made. Such assumptions as are necessary should be made in a "safe" direction; that is, in such a direction that the resulting calculations will lead to a cooling system design that will be adequate for normal tube operation.

Assumptions usually made are:

1. All of the power dissipated in the tube (equal to arc-drop volt-amperes) appears as heat in the coolant.

2. One third of the total power is dissipated by the anode or anode-grid assembly, the remaining two-thirds by the region of the tube below the grids.

3. By virtue of assumption (2) above, the tube can be regarded as two single stage heat exchangers in series.

4. Conduction through the arc-chamber wall is in a radial direction only.

Further assumptions which may be needed can be found in DF #64554, "Heat Transfer in Rectifier Tubes" E. F. Peterson.

To outline the method of solution of heat transfer problems, an example will be used. The tube to be investigated is the GL-6504, a large metal rectifier designed for use in a locomotive. The following information will be pertinent:

a. Water flow is 6 gpm.
b. Water enters the bottom of the tube at a temperature of 40°C
c. Twenty-two Kw are dissipated in the tube, 14 Kw below the anode-grid assembly, 8 Kw above.
d. The inner wall of the tube is .140" thick, of stainless steel of thermal conductivity K = .634 watts/in$^2$ - °C
e. Water flow is through an annular spiral of 2" X 1/4" cross section: We assume that all of the coolant flows through the spiral.

For the purposes of this example, we will concern ourselves only with the lower region of the tube. The same method can be then applied to the upper region, and the two sets of data super-posed where necessary to obtain the complete solution for the entire tube.

Determination - of the Type of Flow

In any fluid-cooled system, it is important that the nature of the flow of coolant be established. The rate at which heat will be absorbed by the coolant is dependent on the type of flow. The criterion by which the nature of the flow is determined is the "Reynold's number" for the particular flow rate and channel. It is defined as:

$$R = \frac{\rho V D}{\mu}$$

where $\rho$ = density of fluid

$V$ = velocity

$D$ = "equivalent diameter" = $\dfrac{4 \text{ X cross-section of flow}}{\text{wetted-perimeter}}$

$\mu$ = viscosity

The Reynold's number can also be determined from a nomograph in the Company's Design Data book on Heat Transfer. In order to render the example calculations complete, it will be calculated here.

For the case at hand, it will be assumed that flow takes place in a long rectangular channel of cross-section 2" x 1/4". In order to use a consistent set of units, it will be found to be convenient to use C.G.S. units. In this case then:

$$\rho = 1 \text{ gram/cm}^3$$

$$D = \frac{4 \times (2'' \times 1/4'') \times (2.54)^2}{2\ (2'' + 1/4'') \times (2.54)} = 1.13 \text{ cm}$$

$\mu$ = .6 Centi-poise - .006 poise (CGS units for viscosity) determined from tubes in the Heat transfer Design Data Book.

V = 118 cm/sec (calculated from the flow rate through this particular channel)

$$\text{so that } R = \frac{\rho V D}{\mu} = \frac{(1\ g/cc) \times (118\ cm/sec) \times (1.13\ cm)}{.6 \times 10^{-3}\ poise}$$

$$R = 2.3 \times 10^5$$

The value of R below which flow is laminar in straight ducts is ≈ $1.16 \times 10^3$ so that $R = 2.3 \times 10^5$ indicates the existence of turbulent flow. However, this value of R has been calculated for a straight duct and the duct under study is circular. A correction must be applied. Reference to the Design Data Book on Heat Transfer indicates the $R_{critical}$ for a 90° elbow of radius of curvature 4" and radius 1/4" is ≈ 7300 so that flow in such a bend will be turbulent for R greater than 7300. We can therefore safely assume that flow in the water spiral is turbulent and proceed on this basis.

Heat flow into Coolant

It is necessary that the rise in coolant temperature be calculated. There are 14 Kw of power to be absorbed by a flow rate of 10 gallons/min. in the first stage of heat exchange. Using the appropriate conversion factors these facts lead to the following equations:

$$Q = 14 \text{ Kw} \times (56.88 \text{ BTU}/_{\text{Kw-min}}) = 795 \text{ BTU/MIN}$$

$$6 \text{ gpm} \times (2.228 \times 10^{-3} \text{ ft}^3/\text{sec}) \times (60 \text{ sec/min}) \times (62.4 \text{ lbs/ft}) = 50.0 \text{ lbs/min.}$$

Since $Q = CM\Delta t$

and since $C = 1 \text{ BTU/lb} - {}^\circ\text{F}$,

$M = 50.0 \text{ lbs/min}$,

$Q = 795 \text{ BTU/min}$,

it follows that

$$\Delta t = 795\ {}^\circ\text{F} = 15.9{}^\circ\text{F}$$

It has been assumed that inlet water temperature was $40^\circ\text{C} = 104^\circ\text{F}$, therefore the the outlet temperature will be $T_{inlet} + \Delta t$,

$$\text{or} \quad T_{outlet} = 119.9^\circ\text{F} = 48.8^\circ\text{C}$$

The outlet temperature of water from the first stage is the inlet temperature of water in the second stage. The calculations for the second temperature increment are carried out in a manner identical with that shown above, except that only 8 Kw are to be absorbed by the coolant.

$$Q = 8 \text{ Kw} \times (56.88 \text{ BTU/Kw-min}) = 455 \text{ BTU/min}$$

again,

$$Q = cM\Delta t$$

$$\Delta t = \frac{455}{50}\ {}^\circ\text{F} = 9.1^\circ\text{F}$$

Total Rise in temperature of coolant $\cong 25^\circ\text{F} = 12.9^\circ\text{C}$

For an inlet temperature of $40^\circ\text{C}$ we can expect an outlet temperature of $52.9^\circ\text{C}$. A series of such calculations can be carried out for different flow rates and the resulting outlet temperatures can be plotted as a function of flow rate as is shown in Curve #1.

Wall Temperature

The wall temperatures at four points in the tube can now be calculated. Two of these are at the same point in the tube but will not be equal as a result of our having divided the tube into two descrete regions. However, it will be shown that this discontinuity is of no real importance.

The equation governing heat transfer through a wall is:

$$Q = \frac{KA(\Delta t)}{\ell}$$

where $Q$ = heat incident on an area A of a wall of thickness l having a thermal conductivity K, across which a temperature difference $\Delta t$ exists.

The temperature differential $\Delta t$ is unknown at the moment but we have assumed that all of the heat incident on the wall appears in the coolant. Thus all of the heat must be transferred through the wall.

It is useful at this stage to introduce the concept of heat flux, $\varphi$ or heat/unit area. For the lower region of the tube which is a cylinder of radius 4", height 5.875",

$$\varphi = \frac{Q}{A} = 94.7 \text{ watts/cm}^2$$

The equation solved for t now becomes

$$\Delta t = \frac{\varphi l}{k}$$

since $l = .140"$

and $k = .634 \text{ watts/in}^2 - {}^\circ C$

$\Delta t = 20.8^\circ C$ for the lower region of the tube and $9.2^\circ C$ for the upper region of the tube.

It should be noted that this differential is completely independent of coolant flow rate. Under the assumed conditions of power dissipation, the above temperature differential must exist across the wall regardless of the surface temperatures. The latter do depend on flow rate of coolant as will be shown.

Convection

The equation governing heat convection is

$$Q = h\,(\Delta t)$$

where h is the convection coefficient
Q = heat to be convected across a temperature differential $\Delta t$.

$\Delta t$ is the difference in temperature between the outer surface of the wall of the tube and the bulk fluid temperature. The latter can be taken as the inlet temperature $40^\circ C$.

The convection coefficient can be calculated but it is more readily found from tabulated data. Such a compilation has been made and appears in the Tube Department Files in "Basic Heat Transfer Data in Electron Tube Operation;" by Mrs. F. F. Buckland, AIEE Tech. Paper #51-227.

Reference to the data for the specific case at hand yields a value of 4.26 $\text{watts/in}^2 - {}^\circ c$ for h.

Using this value for h and the convection equation,

$$\Delta t = \frac{94.7 \text{ watts/in}^2}{h} = T_{wall} - T_{bulk\ fluid}$$

and since $T_{bulk\ Fluid} = 40^\circ C$ at the lower end of the tube,

$T_{wall} = 62.2^\circ C$ = temperature of outer surface of the arc-chamber wall.

It has been shown that $\Delta t_{wall} = 20.8^\circ C$

So that $T_{inner\ wall} = 62.2^\circ + 20.8^\circ C = 83.^\circ 0C$
which is the temperature of the inner surface of the arc-chamber wall.

Using the method outlined above, wall temperatures can be calculated for the cathode, anode, and mid-tube regions.

This information is generally enough to allow the engineer to design the proper cooling system for a given application. A graphical tabulation of data calculated using the above technique is shown in the Appendix for the GL-6504 Locomotive Ignitron.

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