Limiters and Discriminators for F.M. Receivers

By G. G. JOHNSTONE, B.Sc.*

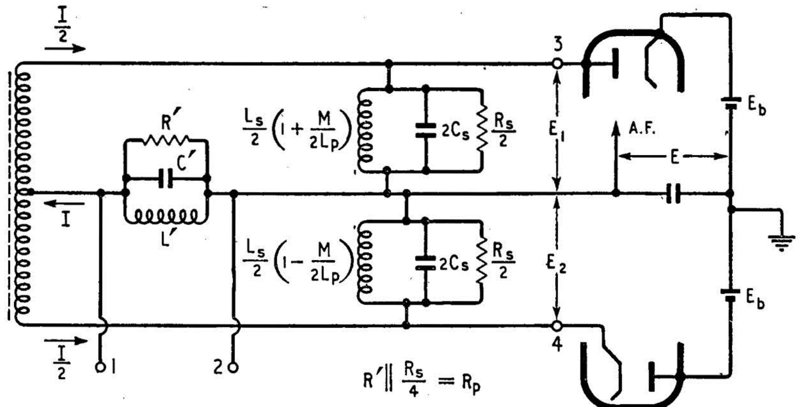
3—The Ratio Detector; Analysis of the "Idealized" Circuit

THE circuit of the conventional ratio detector circuit is essentially similar to that of a Foster-Seeley discriminator, and is shown in simplified form in Fig. 1. It differs from the Foster-Seeley circuit in that one of the diodes is reversed in sense, and in addition a capacitor of the order of 4–20 microfarads is connected in parallel with the load resistors R_L. Because of this capacitor, the voltage across the load resistors cannot change rapidly, and for the purpose of analysis, the two halves of the load circuit can be replaced by two batteries of appropriate voltage. This substitution is justified provided that the rate of variation of the carrier envelope is small compared with the time-constant of the load circuit. We shall return to this qualification later.

In part 1 of this series, an equivalent diagram for the phase-difference transformer shown in Fig. 1 was derived; this equivalent circuit is shown in Fig. 2. The analysis can be considerably simplified by assuming what we propose to show, namely, that the two tuned circuits connected between terminals 2, 3 and 2, 4 fed with equal currents provide a detector inherently insensitive to amplitude modulation. Thus we can ignore the tuned circuit connected between terminals 1, 2 initially, because its effect is to amplitude-modulate the current fed to the centre-tap of the "ideal" transformer T. The latter transformer ensures that the current divides equally between the branches connected to its ends.

We shall first consider two loss-free tuned circuits fed with equal currents with perfect diode detectors, i.e. rectification efficiency is 100 per cent. We shall use the circuit values appropriate to the phase-difference transformer, but the results are equally applicable to an arrangement of two tuned circuits, connected as shown in the equivalent circuit to be analysed (Fig. 3).

"Idealized" Ratio Detector.—Consider first the current flowing in the two diodes. Each diode conducts when the applied voltage reaches its peak value and a short pulse of current flows. It is a property of such a pulse that the d.c. component I_{dc} is equal to half the peak value of the fundamental frequency a.c. component I_{ac} in the equivalent frequency spectrum. These currents can be shown as loop currents flowing in the paths indicated in Fig. 4. There are currents at harmonic frequencies also, but we shall assume that the loop impedances are very low at these harmonic frequencies, and hence the voltages arising from these components can be ignored. There is only one path for the direct current in both diodes, and hence these currents must be equal. It then follows that the fundamental frequency a.c. components must also be equal in magnitude, although not necessarily in phase.

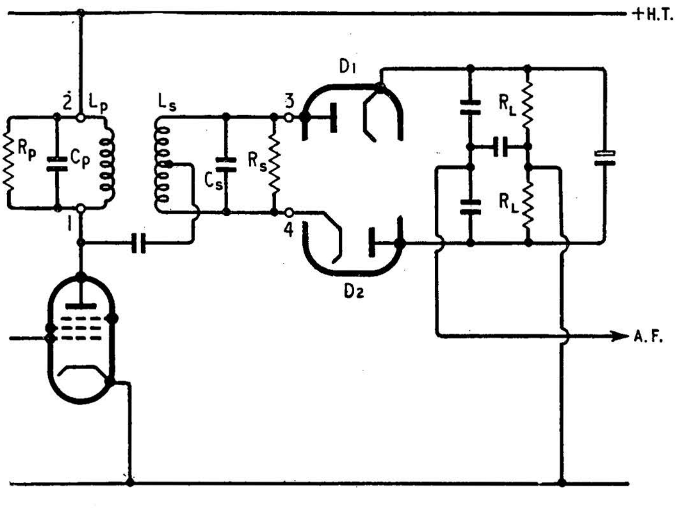


$R' \| \frac{R_s}{4} = R_p$

$L' \| \frac{L_s}{4} = L_p$

$C' + 4C_s = C_p$

Above: Fig. 2. Equivalent circuit of Fig. 1.



Below: Fig. 1. Simplified circuit diagram of ratio detector.

* B.B.C. Engineering Training Department.

The voltages across the tuned circuits are E_1 and E_2 respectively, and the fundamental frequency a.c. component in each diode is in phase with its applied voltage; this is again a property of the components of a short pulse of current. The tuned circuits are loss-free and the currents I_1 and I_2 fed to them are in quadrature with the applied voltages. Thus we may write $(I/2)^2 = I_{ac}^2 + I_1^2$ and $(I/2)^2 = I_{ac}^2 + I_2^2$ where I/2 is the input current to each half of the circuit. From these equations it follows that the magnitude of I_1 is equal to the magnitude of I_2. Thus a basic property of this circuit is that equal currents are fed to the two tuned circuits.

The magnitudes of the voltages E_1 and E_2 will be written as $|E_1|$ and $|E_2|$ and the magnitudes of the currents I_1 and I_2 as $|I_1|$ and $|I_2|$. If the magnitudes of the impedances of the tuned circuits are $|Z_1|$ and $|Z_2|$, then $|E_1| = |I_1| \cdot |Z_1|$ and $|E_2| = |I_2| \cdot |Z_2|$

We have assumed the diode rectification efficiency to be 100 per cent and hence $|E_1| = E_b + E$ and $|E_2| = E_b - E$, where E is the a.f. output voltage. We can thus write

$$E_b + E = |I_1| \cdot |Z_1|$$
$$E_b - E = |I_1| \cdot |Z_2|$$

Adding and substracting these expressions gives

$$2E_b = |I_1| (|Z_1| + |Z_2|)$$
$$2E = |I_1| (|Z_1| - |Z_2|)$$

and dividing gives

$$E = E_b \frac{|Z_1| - |Z_2|}{|Z_1| + |Z_2|}$$

This expression shows that the signal-frequency output voltage is independent of the input current I, i.e. that the circuit is not responsive to amplitude modulation.

To derive the relationship between the output voltage and signal frequency, it is convenient to replace the terms Z_1 and Z_2 by their corresponding admittances Y_1 and Y_2. Writing 1/Y for Z yields

$$E = E_b \frac{|Y_2| - |Y_1|}{|Y_1| + |Y_2|}$$

But

$$Y_1 = 2j\omega C_s + 2/j\omega L_s(1 + M/2L_p)$$

$$Y_2 = 2j\omega C_s + 2/j\omega L_s(1 - M/2L_p)$$

These expressions can be simplified by using Δf_1, and Δf_2 defined as

$$\Delta f_1 = f - f_1$$
$$\Delta f_2 = f - f_2$$

where f_1 and f_2 are the resonance frequencies of the two tuned circuits and f is the signal frequency.

In the neighbourhood of f_1 and f_2,

$$Y_1 = 4jC_s\Delta\omega_1 \quad \text{where } \Delta\omega_1 = 2\pi\Delta f_1$$
$$Y_2 = 4jC_s\Delta\omega_2 \quad \Delta\omega_2 = 2\pi\Delta f_2$$

These expressions can be given in terms of the half bandwidth $\Delta F = (f_1 - f_2)/2 = \Delta\Omega/2\pi$, and the separation $\Delta f = \Delta\omega/2\pi$ of the signal frequency from the centre frequency $f_o = (f_1 + f_2)/2$.

Then

$$\Delta f = \Delta f_1 - \Delta F = \Delta f_2 + \Delta F$$

Thus

$$Y_1 = 4jC_s(\Delta\omega + \Delta\Omega)$$

$$Y_2 = 4jC_s(\Delta\omega - \Delta\Omega)$$

The values of $|Y_1|$ and $|Y_2|$ are plotted in Fig. 5, together with $|Y_1| + |Y_2|$. The resultant characteristic of $E_b(|Y_2| - |Y_1|)/(|Y_1| + |Y_2|)$ is plotted in Fig. 6. This shows that the idealized ratio detector has perfect linearity over the range between the resonance frequencies of the circuits, the output in this range being given by $E = -E_b\Delta f/\Delta F$. In practice, of course it is not possible to realize loss-free tuned circuits and diodes with 100 per cent efficiency. An approach to this condition might be made by applying regeneration to the tuned circuit, but this has not been done to the author's knowledge.

The a.m. rejection properties of a ratio detector must be considered in two parts (a) the degree of

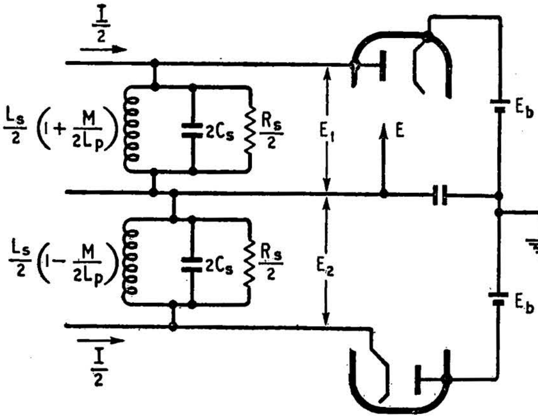


Fig. 3. Simplified circuit for initial analysis.

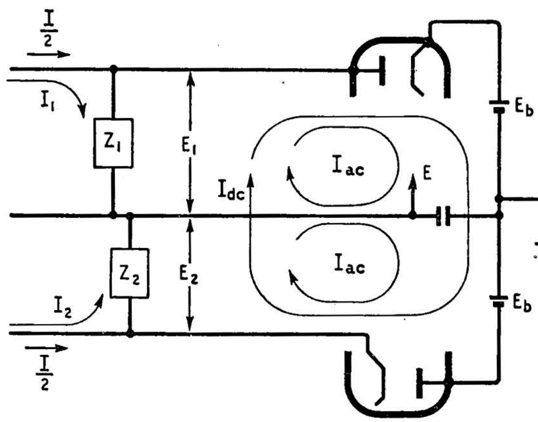


Fig. 4. Showing current paths.

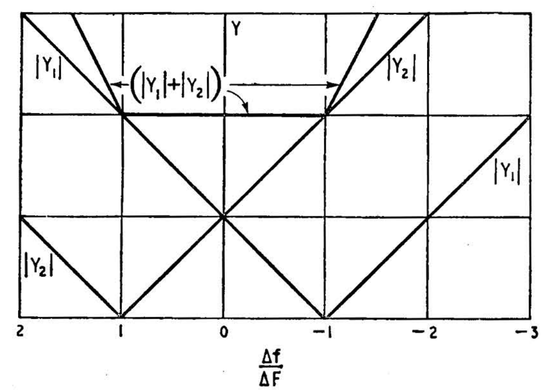


Fig. 5. $|Y_1|$, $|Y_2|$, and $(|Y_1| + |Y_2|)$ plotted against $\Delta f/\Delta F$.

a.m. rejection, and (b) the working range of signal amplitude over which this rejection is maintained. In the idealized ratio detector considered above, the output was shown to be independent of I, the r.f. input current; thus the detector has no response to a.m. However, it was implicit in the analysis that the input current input was sufficient to maintain current in the diodes at all times, i.e. that the peak voltage across each tuned circuit never fell below that of the battery. The condition where the diodes are cut-off can only occur when the amplitude of the input signal is decreasing, i.e. there is "downward" amplitude modulation. There is no corresponding limit if the signal amplitude increases.

The maximum degree of "downward" amplitude modulation that the detector can handle can be calculated readily. As stated above, the limit occurs when the current through the diodes falls to zero; under these conditions, all the input current (I/2) flows into the tuned circuit. Near the centre frequency, the impedance of each tuned circuit is approximately equal in magnitude to $1/4C_s\Delta\Omega$, and the voltage across each tuned circuit is equal in magnitude to E_b. The resultant current is of magnitude $E_b\, 4C_s\Delta\Omega$. The fundamental frequency a.c. component in the diodes is $2E_b/R_L$ (i.e. twice the d.c. component). These two components are in quadrature and hence

$$(I/2)^2 = E_b^2\,(16\,C_s^2\Delta\Omega^2 + 4/R_L^2)$$

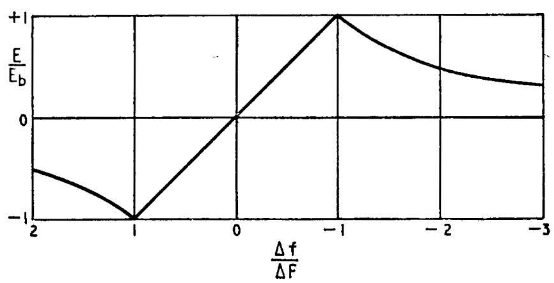


Fig. 6. Audio output plotted against frequency for "idealized" ratio detector.

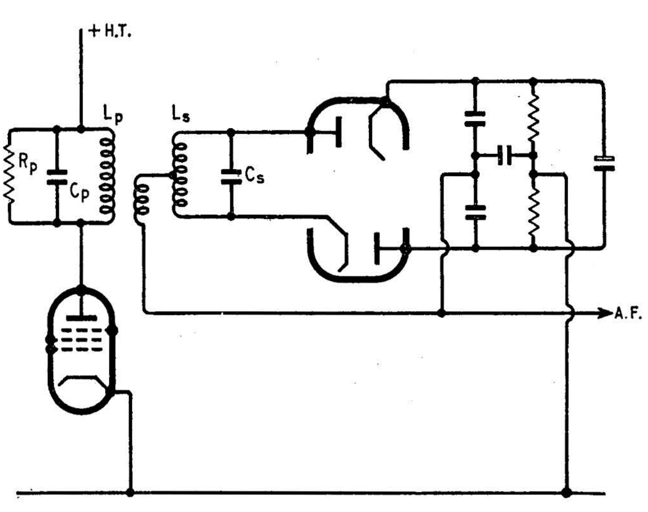


Fig. 7. Ratio detector with tertiary winding closely coupled to primary winding.

When the diode current is cut-off, the input current I′ is given by

$$(I'/2) = E_b(4C_s\Delta\Omega)$$

Thus the maximum amplitude modulation depth that the detector can handle is given by

$$m_{max} = 1 - I'/I$$

$$= 1 - \sqrt{\frac{16\,C_s^2\Delta\Omega^2}{16\,C_s^2\Delta\Omega^2 + 4/R_L^2}}$$

$$= 1 - \sqrt{\frac{4\,R_L^2C_s^2\Delta\Omega^2}{1 + 4R_L^2C_s^2\Delta\Omega^2}}$$

This can be simplified by rearranging the terms, and introducing Q_w, the working Q-value of the tuned circuits, i.e. the Q-value measured under quiescent conditions with the diode circuit damping present. The damping due to the presence of the diode load resistor is equivalent to a resistance of $R_L/2$ in parallel with each circuit, and thus the working Q-value may be defined as

$$Q_w = \frac{R_L}{2}\cdot\frac{1}{(L_s\omega_o/2)} = R_L\omega_oC_s$$

Then

$$m_{max} = 1 - \sqrt{\frac{(2Q_w\Delta F/f_o)^2}{1 + (2\,Q_w\Delta F/f_o)^2}}$$

If m is large, the expression simplifies to

$$m_{max} = 1 - 2\,Q_w(\Delta F/f_o)$$

From the expressions derived above, the conditions that the detector should have good "downward" a.m. rejection properties are apparent. These are that Q_w and ΔF should be small, and f_o should be large. In general the half-bandwidth ΔF cannot be decreased indefinitely, or distortion results. Similarly f_o is fixed by other considerations. Hence Q_w is the only independent variable. If, for example, $\Delta F = 75$ kc/s, and $f_o = 10.7$ Mc/s and the detector is required to handle a.m. to a modulation depth of 0.9, then

$$0.9 = 1 - 2Q_w(75/10700)$$

$$Q_w = 7.2 \text{ approximately.}$$

In terms of circuit values, this requires that the load resistor be 1,150 ohms approximately, if the tuning capacitance of the phase-difference transformer is 50 pF.

The above calculations show clearly that the conditions for good "downward" modulation handling capabilities require a narrow bandwidth; thus if a wide-band detector is required, it must be preceded by a limiter stage, since its inherent ability to deal with a.m. is seriously impaired by its wide bandwidth. This situation cannot be remedied by reducing Q_w, since in a practical circuit there is a limit to this imposed by considerations of diode efficiency.

To complete the investigation of the "idealized" ratio detector we will deduce the sensitivity of the circuit. It was shown above that the a.f. output voltage is proportional to E_b and, other parameters being fixed, it is desirable that E_b should be as large as possible. The usual way of achieving this object is to employ a tertiary winding closely coupled to the primary circuit as shown in Fig. 7. This steps up the input current and reduces the impedance level. The use of a tapped primary circuit

(*continued on page 127*)

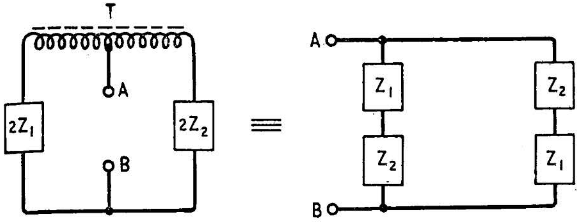


Fig. 8. Equivalent circuit to determine impedance measured between terminals A and B.

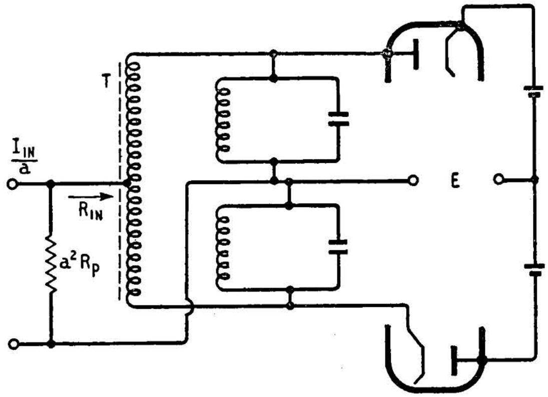


Fig. 9. Circuit for determining division of input current I_{in}/a.

was discussed in part 2 of this article, where it was shown that the primary circuit proper must be replaced before the analysis is commenced by another tuned circuit, whose parameters are given below. The fraction a represents the fraction of the primary circuit voltage tapped off by the tertiary winding. $L_p' = a^2L_p$; $C_p' = C_p/a^2$; $R_p' = a^2R_p$; $I' = I/a$; $M' = aM$.

To determine the value of I we must calculate the input impedance presented at the centre tap of the transformer T in the equivalent diagram of Fig. 2. This can best be done by utilizing one of the intermediate stages in the derivation of the equivalent circuit, which was given in part 1; this is shown in Fig. 8. At the centre frequency, the input impedance R_{in} is purely resistive and can be shown to be

$$R_{in} = (R_L/4)/\{1 + (2Q_w\Delta F/f_o)^2\}$$

We shall assume that the signal is close to the centre frequency. This is the resonance frequency of the circuit connected between terminals 1 and 2 and hence the reactive component of its impedance can be ignored. Since $R_s/4$ is infinite, the equivalent circuit reduces to that of Fig. 9.

The proportion of the input current I_{in} fed to the centre-tap of the transformer T is thus given by

$$I = \frac{I_{in}}{a}\cdot\frac{a^2R_p}{R_{in} + a^2R_p} = I_{in}\,\frac{aR_p}{R_{in} + a^2R_p}$$

This reaches a maximum value when

$$a^2 = \frac{R_{in}}{R_p} = \frac{1}{R_p}\,\frac{R_L}{4\,\{1+(2Q_w\Delta F/f_o)^2\}}$$

At this value of a

$$I = I_{in}/2a$$

If we assume that $(2Q_w\Delta F/f_o)$ is small, as required for good "downward" a.m. handling capacity, the expression for a simplifies to

$$a^2 = R_L/4R_P$$

We have shown earlier that the output voltage $E = -E_b\Delta f/\Delta F$. The value of E_b can be determined simply when the value of a is chosen for power matching as described above, when $I = I_{in}/2a$. The power delivered to the centre-tap of the transformer T is $P = \frac{1}{2}(I_{in}/2a)^2R_{in} = \frac{1}{2}(I_{in}/2a)^2a^2R_p = \frac{1}{8}\,I_{in}{}^2R_p$.

The network has no losses, and hence this power is delivered entirely to the load circuit. This power is given by

$$P = 2\,E_b{}^2/R_L$$

whence

$$E_b = \tfrac{1}{4}\,I_{in}\sqrt{R_pR_L}$$

Thus for maximum output for a given value of Δf, R_p and R_L should be large, and ΔF small. But R_L is directly proportional to Q_w, and it was shown earlier that for good "downward" a.m. handling capacity, Q_w should be small. This is diametrically opposed to the condition for maximum sensitivity as shown by the expression above, which requires R_L large. The expression shows also that a wide-band discriminator can only be secured at the expense of sensitivity, and as shown earlier, at the expense also of the "downward" a.m. handling capacity.

(*To be continued*)

New "Wireless World" Books

Foundations of Wireless by M. G. Scroggie, B.Sc., M.I.E.E. This well-known book on the basic theory of radio transmission and reception has now reached its sixth edition. An introductory section on the use of algebraic symbols, graphs and circuit diagrams clarifies the small amount of mathematics that is included. The fundamental laws of electricity and the theory of valves are discussed and no previous technical knowledge on the part of the reader is assumed. Subjects which are treated include television, radar and transmission lines. There is a new chapter on semi-conductors, which, with other additions, keeps this book right up-to-date. The book has 349 pages and costs 12s 6d (postage 1s).

Television Receiving Equipment by W. T. Cocking, M.I.E.E. The advent of Band III and other new developments in television have necessitated almost complete rewriting, and the addition of 169 pages of new material to this, the fourth, edition of a book which has become one of the classics of the subject. Topics discussed in fuller detail in this edition include magnetic deflection (greatly expanded) and the increasing problems of interference. The comprehensive treatment covers both theoretical and practical aspects of the subject. Modifications necessary to deal with systems other than the British are also discussed. The book has 454 pages and costs 30s (postage 1s 6d).

Television Engineering, Vol. III by S. W. Amos and D. C. Birkinshaw. This volume in the series of B.B.C. Engineering Training Manuals covers the properties, generation and application of the various waveforms used in television (sinusoidal, rectangular, sawtooth and parabolic). The treatment is mainly devoted to the generation of these waveforms, and being more fundamental and descriptive, is less mathematical than the two previous volumes. Vol. IV (in preparation) dealing with the various circuit techniques, will complete the series. The book has 226 pages and costs 30s (postage 11d).

These books are issued for *Wireless World* by our Publishers, Iliffe and Sons Ltd., Dorset House, Stamford Street, London, S.E.1; publication date, March 7th.