Limiters and Discriminators

2—Foster-Seeley Discriminator; Designing for Minimum Distortion

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THE basic type of Foster-Seeley discriminator is shown in Fig. 1. A number of variants exist in practice, and these are discussed later. The audio output is the difference between the output voltages developed by the two diodes D1 and D2. At the centre frequency of the circuit, this output is zero, and the output swings above or below zero as the frequency of the applied signal shifts from its centre value. We shall assume initially that the diodes have 100 per cent rectification efficiency; the audio output is then equal to the difference of the peak values of the two r.f. voltages applied to the diodes.

Now let us analyse the r.f. side of the circuit. The transformation we shall employ is that shown in Fig. 2, the development of which was given in the first part of this article. The phase-difference transformer of Fig. 1 is then identical in performance with the arrangement of three tuned circuits shown. The "ideal" transformer T serves only to ensure that the current fed to its centre tap divides equally between the branches connected to its ends. We shall concentrate initially on the two tuned circuits connected between terminals 2, 3 and 2, 4. The parameters used are $x = 2Q_s df/f_o$ and $R_s/2$, the dynamic resistance of the tuned circuits. In the expression for x, df is the shift of the signal from its centre frequency f_o, whilst the value of Q_s is given by R_s/L_s. The centre frequency f_o is given by $f_o = 1/2\pi\sqrt{L_sC_s} = 1/2\pi\sqrt{L_pC_p}$. The special values of $x = \pm x_1$ give the displacement of the resonant frequencies of each of the two circuits from the centre frequency. In the circuit shown in Fig. 2, the resonant frequency of one tuned circuit is given by $\left(1+\frac{M}{2L_p}\right)^{-\frac{1}{2}} f_o$ and that of the other by $\left(1-\frac{M}{2L_p}\right)^{-\frac{1}{2}} f_o$. These values are equal approximately to $\left(1-\frac{M}{4L_p}\right) f_o$ and $\left(1+\frac{M}{4L_p}\right) f_o$ respectively, and hence $x_1 = 2Q_sM/4L_p = Q_sM/2L_p$. This can be rearranged into a more convenient form as follows.

$$x_1 = \frac{Q_sM}{2L_p}$$

$$= \frac{Q_sk\sqrt{L_sL_p}}{2L_p} \text{ where } k \text{ is the coupling coefficient}$$

$$= \frac{kQ_s}{2}\sqrt{\frac{L_s}{L_p}}$$

It is shown in the Appendix that if E_p and E_s are peak values voltages across the primary and secondary winding of the phase-difference transformer at resonance, then

$$x_1 = (E_s/2)/E_p$$

With a constant-current input I/2 to each of the two tuned circuits connected between terminals 2, 3 and 3, 4 the difference between the peak r.f. voltages between terminals 2, 3 and 2, 4 is given by

$$E = IR_s/4\,(a_1x + a_3x^3 + a_5x^5 \ . \ . \ .)$$

where

$$a_1 = 2x_1(1 + x_1^2)^{-3/2}$$
$$a_3 = x_1(2x_1^2 - 3)(1 + x_1^2)^{-7/2}$$
$$a_5 = \tfrac{1}{4}x_1\,(8x_1^4 - 40x_1^2 + 15)\,(1 + x_1^2)^{-11/2}$$

E is equal to the a.f. output provided that rectification efficiency is 100 per cent.

Except for the special case when the elements of the tuned circuit connected between terminals 1 and 2 become of infinite impedance, the input current, I, to the centre tap of the "ideal" transformer T is not constant. Its value can however be calculated. If the input current to terminals 1, 2 is I_{in}, then I is equal to I_{in} less the current flowing in the tuned circuit connected between terminals 1 and 2.

In order to simplify the treatment, we shall assume that the Q-values of the two circuits of the phase difference discriminator transformer are equal. Additionally, we shall employ the relationship $p = L_s/L_p$. The equivalent circuit can then be redrawn as shown in Fig. 3.

The relationship between I and I_{in} is then given by

$$I = I_{in}\,\frac{1 + x_1^2}{\frac{1}{4} + x_1^2}\left[\frac{1 + 2x^2\frac{(1-x_1^2)}{(1+x_1^2)^2} + x^4\frac{1}{(1+x_1^2)^2}}{1 + 2x^2\frac{(1-n^2)}{(1+n^2)^2} + x^4\frac{1}{(1+n^2)^2}}\right]^{\frac{1}{2}}$$

where $n = kQ$ (see Appendix).

At first sight it would appear that the output

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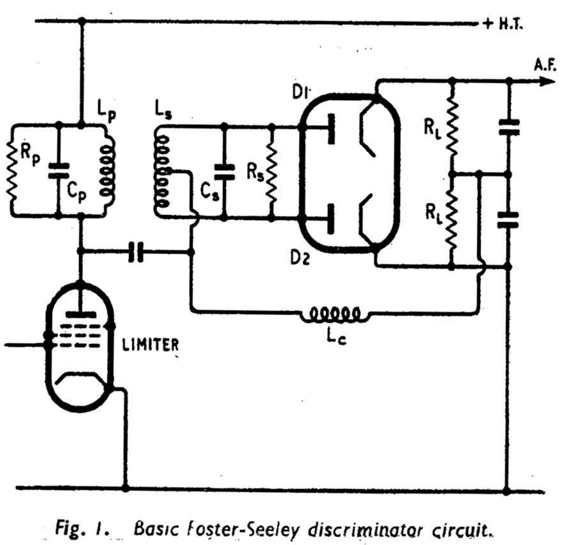

Fig. 1. Basic Foster-Seeley discriminator circuit.

for F.M. Receivers

In the first article of this series an equivalent circuit for the phase-difference transformer employed with the Foster-Seeley discriminator and the ratio detector was derived. This equivalent circuit enables these two forms of detector to be treated in the same manner as the Round-Travis circuit, already discussed, and in this part we shall discuss the Foster-Seeley circuit in greater detail.

voltage depends upon three variables, x_1 p and n. This is however, not true because

$$x_1 = \tfrac{1}{2}kQ\sqrt{\frac{L_s}{L_p}} = \tfrac{1}{2}n\sqrt{p}.$$

Thus there are only two independent variables.

From the equation for I, it will be apparent that I is independent of x only if $x_1 = n$. This is the special case referred to above, when the inductance L′ becomes infinite, capacitor C′ becomes zero, and R′ becomes infinite. In these conditions $\sqrt{p} = 2$, and hence $I = I_{in}$ as would be expected.

It was shown in Part I that the coefficient a_3 in the expression of the output voltage is zero when $x_1 = \sqrt{1.5}$. In the phase-difference transformer, the same conditions apply when $n = \sqrt{1.5}$, and $L_s = 4L_p$.

In the general case, when x_1 does not equal n, we can express I as a power series in x, as follows

$$I = I_{in}\frac{1 + x_1^2}{\frac{p}{4} + x_1^2}(b_o + b_2x^2 + b_4x^4 \ldots)$$

where

$$b_o = 1$$

$$b_2 = \frac{1 - x_1^2}{(1 + x_1^2)^2} - \frac{1 - n^2}{(1 + n^2)^2}$$

$$b_4 = \frac{2x_1^2}{(1+x_1^2)^4} + \frac{1-4n^2+n^4}{(1+n^2)^4} - \frac{1 - x_1^2}{(1+x_1^2)^2}\cdot\frac{1 - n^2}{(1+n^2)^2}$$

Inserting this value for I in the expression for E gives

$$E = \frac{I_{in}R_s}{4}\cdot\frac{1 + x_1^2}{\frac{p}{4} + x_1^2}(c_1x + c_3x^3 + c_5x^5 \ldots)$$

where

$$c_1 = (a_1b_o)$$

$$c_3 = (a_1b_2 + a_3b_o)$$

$$c_5 = (a_1b_4 + a_3b_2 + a_5b_o)$$

The distortion introduced is represented by the terms in x^3 and x^5. To minimize distortion, therefore, it is desirable that the coefficients of these terms should be as small as possible. The dominant term is the coefficient of x^3, and this can be made equal to zero by appropriate choice of parameters. For this condition, $a_1b_2 + a_3b_0 = 0$, and substituting values this gives

$$-x_1^2(2x_1^2 - 3)(1 + x_1^2)^{-7/2} = 2x_1(1 + x_1^2)^{-3/2} \times [(1 - x_1^2)(1 + x_1^2)^{-2} - (1 - n^2)(1 + n^2)^{-2}]$$

This reduces to

$$\frac{1}{(1 + x_1^2)^2} = \frac{2(n^2 - 1)}{(1 + n^2)^2}$$

The graph of x_1 plotted against n is given in Fig. 4. This shows that if n is less than 1, c_3 cannot equal zero. It also shows that if n is between 1 and 1.2 approximately, there is little margin for error in adjustment of n, since x_1 is varying rapidly. The minimum value of x_1 occurs when $n = \sqrt{3}$; at this value $x_1 = 1$. Above $n = \sqrt{3}$, the slope of the curve is positive. This is of some importance, since x_1 is itself proportional to n. If n departs from its correct value, it is desirable that x_1 should change in the same sense to minimize the value of c_3.

The graph of x_1 plotted against n does not indicate any specific optimum value for n and x_1. However,

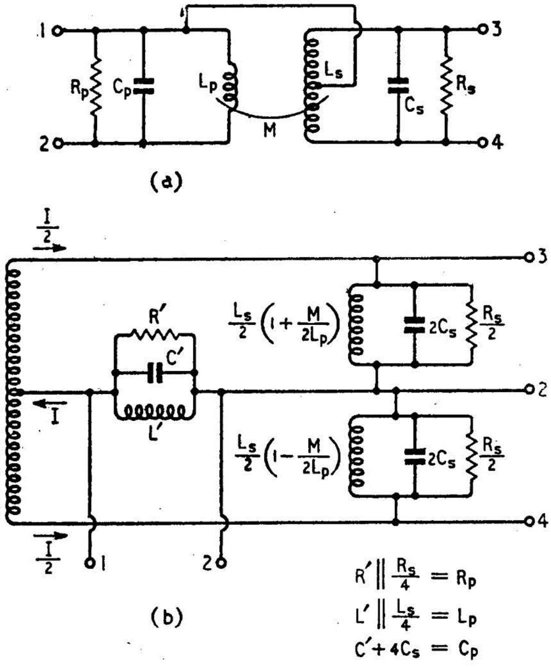

Fig. 2. (a) Phase-difference transformer, and (b) its equivalent circuit.

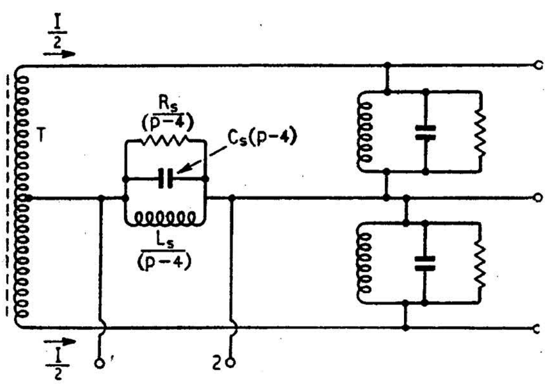

Fig. 3. Equivalent circuit of phase-difference transformer for $Q_s = Q_p$; $p = L_s/L_p$.

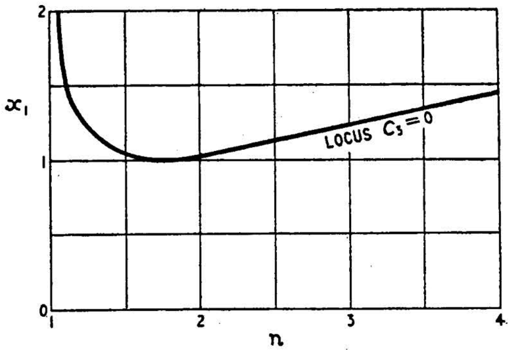

Fig. 4. Graph of x_1 against n for $c_3 = 0$.

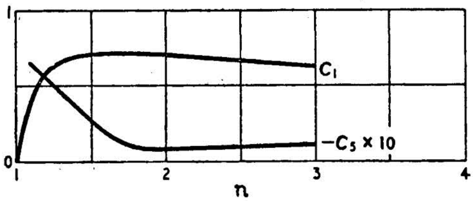

Fig. 5. Variation of c_1 and c_5 with n, for $c_3 = 0$.

this can be found by considering the coefficient of x^5 in the expansion. The value of this coefficient is plotted in the Fig. 5; at each value of n the value of x_1 is that which makes $c_3 = 0$. Additionally the graph shows the value of the coefficient of x; minimum distortion occurs when the ratio of the coefficient of x^5 to that of x is minimum. The optimum values would appear to be near $n = 2.0$. Consider $n = 2$; in this region the coefficient of x^5 is approximately -0.008. From Fig. 4, $x_1 = 1$ approximately. From $x_1 = \frac{1}{2} n\sqrt{p}$, $p = 1$, i.e. the two tuned circuits of the phase difference discriminator transformer are equal. From Fig. 5, the coefficient of x is 0.7 approximately.

The expression for the audio output voltage is then

$$E = 0.4\ I_{in}R_s\ (0.7x - 0.008x^5\ .\ .\ .\ )$$

The range of validity of this expression can be seen from Fig. 6. In this graph are two curves. Curve (a) is that obtained from the orthodox graphical solution for the Foster-Seeley response curve, which does not give any precise means of evaluating distortion; curve (b) is that derived from the expression above. It will be seen that the two curves are in very close agreement up to the value of x=1.4. This then is the limit of validity of the expansion above.

To evaluate the distortion present with the circuit constants chosen, consider an input signal $x_s \cos\omega_a t$. The a.f. output is then

$$E = 0.4\ I_{in}\ R_s \{0.7x_s \cos\omega_a t - 0.008\ (x_s \cos\omega_a t)^5\}$$

We can expand $\cos^5\omega_a t$ by means of the identity

$$\cos^5\theta = \frac{1}{16}\left\{\cos 5\,\theta + 5\cos 3\,\theta + 10\cos\theta\right\}$$

giving

$$E = 0.4\ I_{in}\ R_s \left\{ (0.7x_s - 0.005x_s^5)\cos\omega_a t - 0.0025\ x_s^5 \cos 3\omega_a t - 0.0005\ x_s^5 \cos 5\omega_a t \right\}$$

The reduction of the fundamental frequency component is negligible for the range of values of x_s of interest, i.e. $x_s < 1$. The percentage of third harmonic distortion is thus given by $\frac{0.0025 \times 100 \times x_s^4}{0.7}$. At $x_s = 1$, this is 0.35 per cent. By employing a smaller value of x_s a lower value of distortion is obtained, the distortion decreasing with x_s^4.

Consider a broadcast signal, with a deviation of 75 kc/s. If it is desired to operate the discriminator with 75 kc/s corresponding to $x_s = 1$, the parameters of the circuit are determined by $x = 2Q\ df/f_o$. With $df = 75$ kc/s at $x = 1$, and a centre frequency f_o of 10.7 Mc/s, the value of Q is 71. If we assume the two tuned circuits of the phase-difference transformer each employ a tuning capacitor of 50 pF, the dynamic resistance (R_s) is 22 kΩ. The input current I_{in}, is the peak value of the fundamental frequency component in the output of the preceding limiter stage; a typical value is 1 mA. The peak audio output is given by $0.28\ I_{in}R_s$, and in this example is 6.2 volts approximately.

It can be seen, from the expression for E, that the audio output is proportional to $I_{in}x$, where I_{in} is the input current, and x is a measure of the frequency

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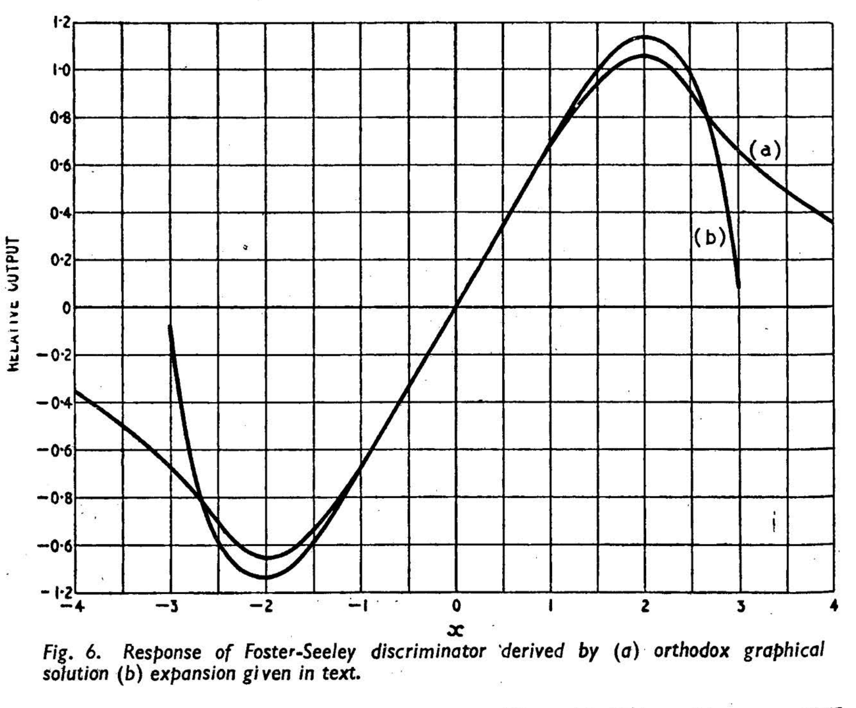


Fig. 6. Response of Foster-Seeley discriminator derived by (a) orthodox graphical solution (b) expansion given in text.

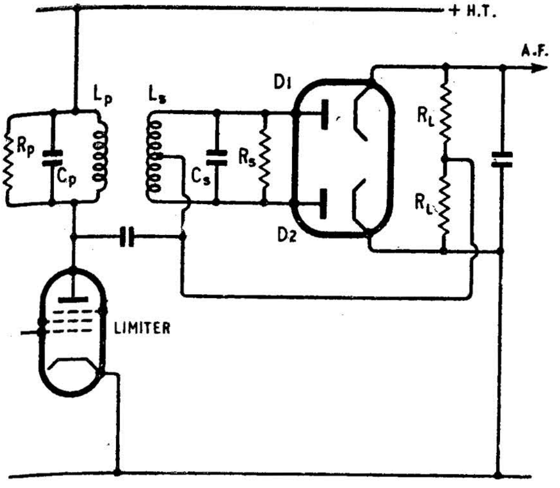


Fig. 7. Alternative form of Foster-Seeley discriminator.

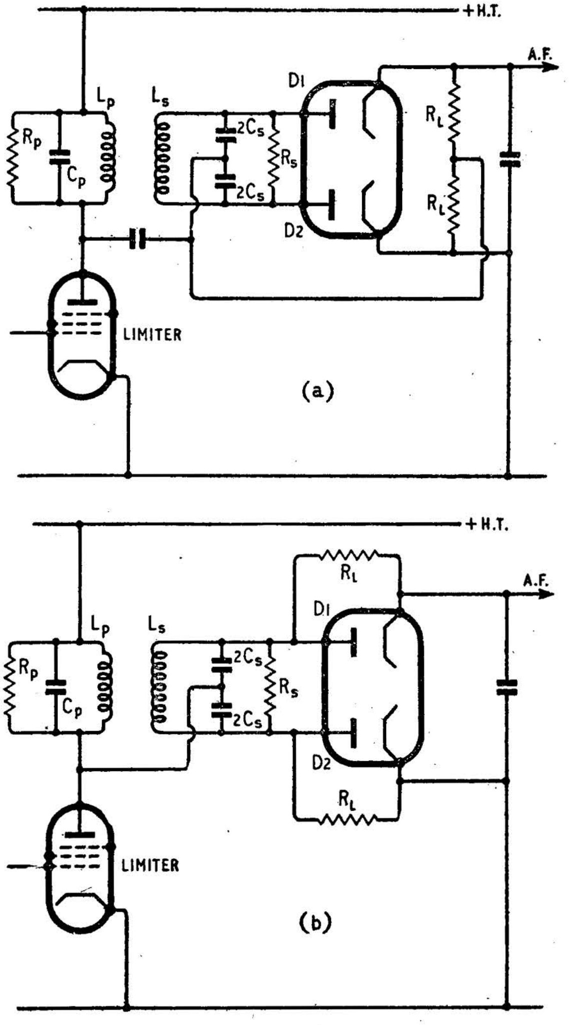
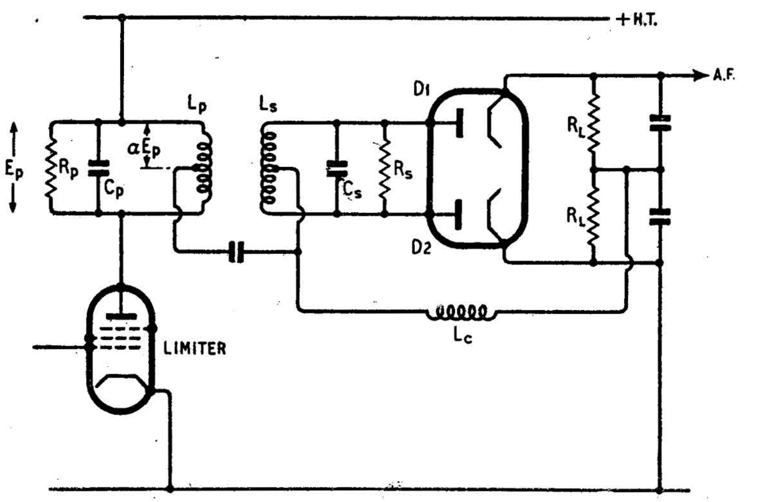


Fig. 8. Further alternative forms of Foster-Seeley circuit, with the secondary circuit centre taps produced by the capacitors.

Fig. 9. Foster-Seeley discriminator with tapped primary circuit.



shift. If there is amplitude modulation present, the magnitude of I_{in} varies, but if $x = 0$, i.e. the signal is at the centre frequency there is no output due to a.m. For any other value of x, i.e. if the signal is mistuned or frequency-modulated, there is an output due to the amplitude modulation. Because the output is proportional to $I_{in}x$, the a.m. and f.m. signals are multiplied together and there is complete cross-modulation. Thus a Foster-Seeley discriminator must be preceded by a limiter stage.

To complete the survey of the Foster-Seeley circuit, there are a number of practical points to be considered. The first of these concerns the diode load resistors. These should not be too large, as otherwise "diagonal clipping" can occur. Briefly this happens if the input to one diode falls rapidly. If the time constant of the load circuit is too great, the cathode potential cannot fall sufficiently quickly, and the diode may be cut off. For this reason, the diode load resistors are usually limited to 100 kΩ, and the shunt capacitors to 50 pF.

This relatively low value of diode load in turn means that the damping imposed in the tuned circuits cannot be neglected. The input resistance of each diode at r.f. is $R_L/2\eta$, where R_L is its load resistance and η is the rectification efficiency. In the equivalent circuit of Fig. 3, the relationship between the resistances of the two tuned circuits connected between terminals 2, 3 and 2, 4 and that connected between terminals 1, 2 then differs from that postulated. The condition can, however, be re-established if an additional resistor equal to $R_L/(p-4)$ is connected between terminals 1 and 2, i.e. across the primary winding of the original circuit. However, the values of p employed in practice are often less than 4, implying that a negative resistance is required. This obviously cannot be realised in practice. It suggests, however, that the Q values can be equalized if the secondary Q value without the diodes connected is lower than the primary Q value. Given equal initial Q values, a resistor can be connected in parallel with the secondary circuit to achieve this result. Its value can be calculated if it is remembered that the damping imposed on the primary circuit is $R_L/4\eta$, whilst that imposed on the secondary circuit is R_L/η. In the example considered above, with equal primary and secondary circuit, the resistance would be $R_L/3\eta$.

It is common practice to omit the r.f. choke L_c shown in Fig. 1, giving the circuit of Fig. 7. In this case it must be remembered that there is then additional damping equal to $R_L/2$ imposed on the primary circuit, since the primary circuit can "see" the two resistors R_L in parallel.

An alternative form of Foster-Seeley circuit is that shown in Figs. 8(a) and 8(b). In the arrangement, the secondary centre-tap is provided by the capacitors. Because there is no longer d.c. continuity for the diodes of Fig. 8(b), the diodes are shunt-fed. The damping imposed on the secondary circuit is the R_L/η in parallel with $2R_L$, whilst that on the primary is $R_L/4\eta$ in parallel with $R_L/2$. In one other variant the primary as well as the secondary is tapped, as shown in Fig. 9. If the ratio of voltage tapped off to the total primary voltage is a, then this circuit can be analysed by replacing the primary circuit by a single un-tapped circuit, with parameters as follows:

$$L'_p = L_p a^2 \qquad E_p' = aE_p$$
$$C'_p = C_p/a^2 \qquad M' = Ma$$
$$R'_p = R_p a^2 \qquad k' = k$$
$$I' = I/a \qquad Q' = Q$$

This circuit has one particular advantage. If $a = \frac{1}{2}$, i.e. the primary circuit is centre-tapped, the damping applied to primary and secondary circuits by the diode loads is automatically equalized. In such a circuit, the value of p in the équivalent circuit is equal to 4, i.e. the analysis simplifies to the case when the tuned circuit in parallel with the input terminals vanishes. For this condition of operation, the optimum value of $n = \sqrt{1.5}$.

APPENDIX

The equivalent diagram for two circuits coupled by mutual inductance is shown below. The circuit equations are

$$E_s = jX_{cs}i_s$$
$$E_p = jX_{cp}(i-i_p)$$
$$0 = Z_p i_p - j\omega M i_s - jX_{cp}i$$
$$0 = Z_s i_s - j\omega M i_p$$

where
$$X_{cs} = 1/\omega C_s$$
$$X_{cp} = 1/\omega C_p$$
$$Z_p = jL_p\omega + \frac{1}{j\omega C_p} + r_p$$
$$Z_s = jL_s\omega + \frac{1}{j\omega C_s} + r_s$$

In the region near the resonants frequency (f_o) of the two circuits, $i_p > i$. Then

$$E_s = -jX_{cp}X_{cs}\frac{\omega M}{Z_pZ_s + \omega^2M^2}i$$
$$E_p = X_{cp}^2\frac{Z_s}{Z_pZ_s + \omega^2M^2}i$$

Near resonance $\omega L_s - 1/\omega C_s$ is approximately equal to $2L_s\delta\omega$, where $\delta\omega$ is the departure from $\omega_o(=2\pi f_o)$. Similarly, $\omega L_p - 1/\omega C_s = 2L_p\delta\omega$. This gives

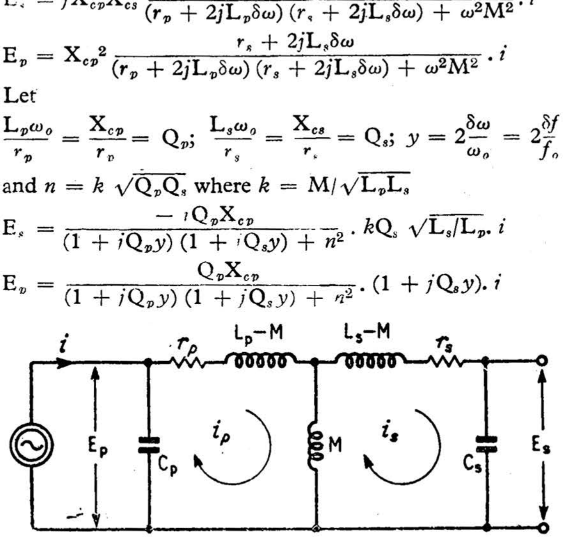
$$E_s = jX_{cp}X_{cs}\frac{\omega M}{(r_p + 2jL_p\delta\omega)(r_s + 2jL_s\delta\omega) + \omega^2M^2}\cdot i$$
$$E_p = X_{cp}^2\frac{r_s + 2jL_s\delta\omega}{(r_p + 2jL_p\delta\omega)(r_s + 2jL_s\delta\omega) + \omega^2M^2}\cdot i$$

Let

$$\frac{L_p\omega_o}{r_p} = \frac{X_{cp}}{r_p} = Q_p;\quad \frac{L_s\omega_o}{r_s} = \frac{X_{cs}}{r_s} = Q_s;\quad y = 2\frac{\delta\omega}{\omega_o} = 2\frac{\delta f}{f_o}$$

and $n = k\sqrt{Q_pQ_s}$ where $k = M/\sqrt{L_pL_s}$

$$E_s = \frac{-jQ_pX_{cp}}{(1 + jQ_py)(1 + jQ_sy) + n^2}\cdot kQ_s\sqrt{L_s/L_p}\cdot i$$
$$E_p = \frac{Q_pX_{cp}}{(1 + jQ_py)(1 + jQ_sy) + n^2}\cdot(1 + jQ_sy)\cdot i$$



whence $$E_s = \frac{-jkQ_s\sqrt{L_s/L_p}}{1 + jQ_sy}E_p$$

let $p = L_s/L_p$, and $y = 0$, i.e., the circuits are at the resonant frequency. Then if $|E_s|$ is the magnitude of E_s, and $|E_p|$ is the magnitude of E_p,

$$\tfrac{1}{2}kQ_s\sqrt{p} = (|E_s|/2)/|E_p|$$

Finally if $Q_p = Q_s$, $Q_pX_{cp} = R_p$, and $Qy = \frac{2Q\delta\omega}{\omega_o} = x$

$$E_p = \frac{R_p(1 + jx)}{(1 + jx)^2 + n^2}i$$

In the equivalent circuit discussed in the text, this voltage is that applied to the tuned circuit connected between terminals 1 and 2 when $i = I_{in}$. In the text $p = L_s/L_p = R_s/R_p = C_p/C_s$. Thus the tuned circuit between terminals 1 and 2 consists of three branches in parallel; (a) inductor $L_s/(p-4)$; (b) capacitor $(p-4)C_s$; (c) resistor $R_s/(p-4)$ (see fig. 3). Then the current I′ flowing into the three elements is given by

$$I' = E_pY$$

where
$$Y = (p-4)/j\omega L_s + j\omega C_s(p-4) + (p-4)/R_s$$
$$= (1/j\omega L_s + j\omega C_s + 1/R_s)(p-4)$$
$$= \frac{p-4}{R_s}(1 + jx)$$

Thus
$$I' = \frac{R_p(1 + jx)}{(1 + jx)^2 + n^2}\cdot\frac{p-4}{R_s}(1 + jx)\,I_{in}.$$
$$= \frac{p-4}{p}\,\frac{(1 + jx)^2}{(1 + jx)^2 + n^2}I_{in}$$

But the current I fed to the centre-tap of transformer T is equal to $I_{in} - I'$. Hence

$$I = I_{in}\left\{1 - \frac{p-4}{p}\cdot\frac{(1 + jx)^2}{(1 + jx)^2 + n^2}\right\}$$
$$= I_{in}\frac{pn^2 + 4(1 + jx)^2}{pn^2 + p(1 + jx)^2}$$
$$= I_{in}\frac{4}{p}\,\frac{x_1^2 + (1 + jx)^2}{n^2 + (1 + jx)^2}$$
$$= I_{in}\frac{1 + x_1^2}{\frac{p}{4} + x_1^2}\cdot\frac{1 - x^2/(1 + x_1^2) + 2jx/(1 + x_1^2)}{1 - x^2/(1 + n^2) + 2jx/(1 + n^2)}$$

If $|I|$ is the magnitude of I and $|I_{in}|$ that of I_{in} then

$$|I| = |I_{in}|\,\frac{1 + x_1^2}{\frac{p}{4} + x_1^2}\left\{\frac{1 + 2x^2(1 - x_1^2)/(1 + x_1^2)^2 + x^4/(1 + x_1^2)^2}{1 + 2x^2(1 - n^2)/(1 + n^2)^2 + x^4/(1 + n^2)^2}\right\}^{\frac{1}{2}}$$