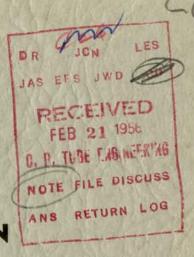
RB-32

SPECTRAL DISTRIBUTION
OF PHOTOCONDUCTIVITY





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Spectral Distribution of Photoconductivity

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A theoretical analysis of the shape of photoconductivity spectral distribution curves is presented, based upon the effects of surface and volume recombination of the charge carriers liberated by the light. Representative curves of photoconductivity vs. absorption are computed and compared with experimental observations. As an application of this analysis, experimental data for antimony sulfide are compared with a theoretical curve, and the difference is found to be resolvable into two bands representing non-photoconductive transitions.

Introduction

Photoconductive materials generally exhibit spectral distribution curves having a more or less sharp peak in the vicinity of the absorption edge. Characteristically, as one proceeds from the low energy (long wavelength) to the high energy (short wavelength) side of the absorption edge, the photoconductive response starts from zero, rises rapidly as the absorption edge is reached, goes through a maximum at some moderate value of absorption, falls again while the absorption coefficient is still rising, and usually appears to approach some asymptotic value greater than zero. This behavior is rather remarkable, since one would expect that each absorbed photon would generate a pair of charge carriers and hence that the photoconductive response should have risen to a maximum value, corresponding to absorption of substantially all of the incident radiation, with no decrease for higher values of absorption coefficient.

It has been customary to explain the peak in the photoconductive response by asserting that for high values of absorption coefficient the radiation is all absorbed in a comparatively thin layer near the surface and that the high density of carriers in this region leads to more rapid recombination and hence to a reduced equilibrium concentration. This explanation, however, is completely invalidated by the observation, found in substantially all cases, that the photocurrent in all spectral regions

is proportional to light intensity, at least for small intensities, while the peak in the spectral response curve remains. If the recombination of carriers were more rapid in the high absorption region because of the high carrier density, the photoconductive current would have to approach proportionality to the square root of the light intensity in this region.

In the case of very thin layers of photoconductors, as, for example, in the case of evaporated films, the peak of the spectral response curve is found to be greatly broadened and the drop in the high energy region is much less than is observed for thick specimens.

It is proposed that the spectral response curve may be explained by taking into account the recombination of carriers at the surface of the photoconductors. Thus, if the surface recombination rate is high, compared with that in the volume of the photoconductor, the equilibrium concentration of charge carriers will be less when these are generated close to the surface (high absorption region) than when they are distributed through the body of the material (lower absorption).

Analysis of the Problem

For simplicity, consider the photoconductor to have the form of a large sheet with thickness 1 (Fig. 1). Let it be illuminated on

one face by radiation having intensity I (photons/second). At each point through the specimen the density of charge carriers is n(x), and these charge carriers will diffuse in the + or - x direction because of the gradient of this density. It is assumed that the carrier pairs will undergo recombination in the volume of the material at a rate corresponding to a volume lifetime τ , and further, that they will recombine at the surface at a rate which may be represented by a surface recombination velocity S. The carrier pairs are being continuously generated at each point in the volume at a rate proportional to the intensity of the radiation at that point.

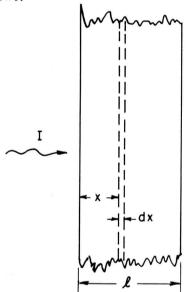


Fig. 1 - Schematic of ab sorbing crystal.

In a region at distance x from the surface on which radiation is incident, the rate of generation of carrier pairs is dn/dt = $Ae^{-\alpha x}$, where α is the absorption coefficient. To evaluate A, assuming that every photon absorbed creates a carrier pair, it is noted that for an infinitely thick specimen, the number of pairs generated would be equal to the number of photons incident.

I =
$$\int_0^\infty Ae^{-\alpha \times} dx = A/\alpha$$
, i.e. $A = I_\alpha$

The carrier pairs leaving the region by diffusion will decrease the carrier density according to the equation dn/dt = -di/dx, where i = n dx/dt is the carrier current. The carrier pairs disappearing by recombination will be

represented by dn/dt = -n/t where τ is the volume recombination lifetime.

Combining, an equation of continuity is obtained:

$$dn(x)/dt = - di/dx + I\alpha e^{-\alpha x} - n/\tau$$
 (1)

For steady state conditions, dn/dt = 0, and

$$di/dx = I_{\alpha}e^{-\alpha \times} -n/\tau$$

For the diffusion carrier current, i = - D dn/dx, where D is the diffusion constant. Differentiating and substituting:

$$d^2n/dx^2 = n/D_{\tau} - I_{\alpha}/D_{e} - \alpha x$$

For convenience, let

$$1/D\tau = \beta^2$$
 and $I\alpha/D = \gamma$,

$$d^2n/dx^2 = \beta^2n - \gamma e^{-\alpha x}$$

This equation has the general solution:

$$n = Be^{-\beta X} + Ce^{\beta X} + \gamma/(\beta^2 - \alpha^2) e^{-\alpha X}$$
 (2)

Boundary condition are determined by the assumption that recombination occurs at each surface at a rate which may be represented by a recombination current $i_R = n_s S$, where n_s is the pair density at the surface and S is the surface recombination velocity. Thus, at the first surface: $i_{R_0} = -D \left(dn/dx \right)_{X=0} = -n_0 S$, and at second surface, $i_{R_1} = -D \left(dn/dx \right)_{X=1} = n_1 S$. Substituting these in Eq. (2) and solving for the constants B and C, gives

$$B = \frac{Y}{\beta^2 - \alpha^2}$$

$$\left\{\frac{(S-\alpha D)(D\beta-S)e^{-\alpha l}+(S+\alpha D)(D\beta+S)e^{+\beta l}}{(D\beta-S)^2e^{-\beta l}-(D\beta+S)^2e^{\beta l}}\right\}$$

$$C = \frac{\Upsilon}{\beta^2 - \alpha^2}$$

$$\left\{ \frac{(S - \alpha D)(D\beta + S)e^{-\alpha^{1}} + (S + \alpha D)(D\beta - S)e^{-\beta^{1}}}{(D\beta - S)^{2}e^{-\beta^{1}} - (D\beta + S)^{2}e\beta^{1}} \right\}$$

Assuming that the increase in conductivity caused by the radiation is proportional to N,

the total number of carrier pairs is

$$N = \int_{0}^{1} (x) dx = \frac{1}{\beta} \left\{ B(1 - e^{-\beta^{\dagger}}) + C(e^{\beta^{\dagger}} - 1) \right\}$$

$$+ \frac{Y}{\alpha(\beta^{2} - \alpha^{2})} (1 - e^{-\alpha^{\dagger}})$$

$$= \frac{Y}{\beta^{2} - \alpha^{2}}$$

$$\left\{\frac{1-\mathrm{e}^{-\alpha l}}{\alpha} - \frac{1}{\beta} \frac{\left[\mathrm{S}(1+\mathrm{e}^{-\alpha l}) + \alpha \mathrm{D}(1-\mathrm{e}^{-\alpha l})\right] \left[\mathrm{D}\beta \text{ sinh } \beta l + \mathrm{S}(\cosh \beta l - 1)\right]}{\left(\mathrm{D}^2\beta^2 + \mathrm{S}^2\right) \sinh \beta + 2\mathrm{D}\beta \mathrm{S} \cosh \beta}\right\}$$

$$= : \frac{\gamma}{\beta^{2} - \alpha^{2}} \left\{ \frac{1 - e^{-\alpha l}}{\alpha} - \frac{1}{\beta} \frac{\left[S(1 + e^{-\alpha l}) + \alpha D(1 - e^{-\alpha l}) \right]}{D\beta + S \coth (\beta l/2)} \right\}$$

Replacing the values of β and γ :

$$N = \frac{\tau I}{1 - \alpha^{2} D \tau} \left\{ (1 - e^{-\alpha l}) - \frac{[\alpha S \tau (1 + e^{-\alpha l}) + \alpha^{2} D \tau (1 - e^{-\alpha l})]}{1 + S \sqrt{\tau / D} \coth l / 2 \sqrt{D \tau}} \right\} (3)$$

It is convenient to make use of the following dimensionless parameters:

$$\frac{1}{\sqrt{D\tau}} \equiv \lambda \ (\sim \text{ thickness of photoconductor})$$

$$\alpha \sqrt{D\tau} \equiv \zeta \ (\sim \text{ absorption coefficient})$$

 $S\sqrt{\frac{\tau}{D}} = \xi$ (~ ratio of surface to volume recombination rates)

Hence,

$$\alpha 1 \equiv \lambda \zeta$$
.

Then

$$N = \frac{\tau I}{1 - \zeta^2} \left\{ (1 - e^{-\lambda \zeta}) \right\}$$

$$=\frac{\left[\zeta \xi \left(1 + e^{-\lambda \zeta}\right) + \zeta^{2} \left(1 - e^{-\lambda \zeta}\right)\right]}{1 + \xi \coth \lambda/2}$$
 (4)

Letting Z = $\lambda \zeta$ = αl , and expressing the photoconductivity in the form P = N/I $_{\tau}$,

$$P = \frac{N}{I_{\tau}} = \frac{1}{\lambda^{2} - Z^{2}} \left\{ \lambda^{2} (1 - e^{-Z}) - \frac{[\xi \lambda Z (1 + e^{-Z}) + Z^{2} (1 - e^{-Z})]}{1 + \xi \coth \lambda / 2} \right\}$$

$$= \frac{1 - e^{-Z}}{1 + \xi \coth \lambda/2}$$

$$\left\{ \frac{\lambda^2 + \xi \lambda^2 \coth \lambda/2 - \xi \lambda Z \coth Z/2 - Z^2}{\lambda^2 - Z^2} \right\}$$

$$= \frac{1 - e^{-Z}}{1 + \xi \coth \lambda/Z}$$

$$\left\{ 1 + \frac{\xi \lambda [\lambda \coth \lambda/2 - Z \coth Z/2]}{\lambda^2 - Z^2} \right\}$$
 (5)

It is of interest to examine several limiting

1. Small absorption $a \rightarrow 0$, $\therefore Z \rightarrow 0$

Hence
$$1 - e^{-Z} \cdot z$$
 $Z \cdot coth \cdot Z/2 - 2$

Then $P \rightarrow 1 + \xi \cdot coth \cdot \lambda/2$

$$\left\{ 1 + \frac{\xi \lambda (\lambda \cdot coth \cdot \lambda/2 - 2)}{\lambda^2} \right\}$$

$$\rightarrow 0 \text{ as } Z \rightarrow 0$$

This result is essentially trivial: if light is not absorbed, then there can be no conductivity.

2. Large absorption

$$a \rightarrow \infty$$
 .: $Z \rightarrow \infty$, $e^{-\lambda Z} \rightarrow 0$, coth $Z/2 \rightarrow 1$

Then
$$P \rightarrow \frac{1}{1 + \xi \coth \lambda/2}$$

$$\left\{1 + \frac{\mathcal{E}\lambda(Z - \lambda \coth \lambda/2)}{Z^2}\right\}$$

$$\rightarrow \frac{1}{1 + \mathcal{E} \coth \lambda/2}$$

The photoconductive response approaches a constant value.

3. Surface recombination negligible compared with volume recombination.

i.e.,
$$S << D/\tau$$
 or $\xi << 1$.
Then $P \rightarrow (1 - e^{-Z})$

$$\left\{1 + \frac{\xi \lambda (\lambda \coth \lambda/2 - Z \coth Z/2)}{\lambda^2 - Z^2}\right\}$$

$$\rightarrow (1 - e^{-Z})$$

Here P increases monotonically from 0 for small α to 1 for large α . There is no peak in the photoconductivity curve.

4. Surface recombination large compared with volume recombination. $\xi >> 1$.

Then P = $\frac{1 - e^{-Z}}{5 \cdot \coth \lambda/2}$

$$\begin{cases} 1 + \frac{\xi \lambda(\lambda \coth \lambda/2 - Z \coth Z/2)}{\lambda^2 - Z^2} \end{cases}$$

This approaches 0 as $\alpha \rightarrow 0$ and approaches 1 for large α . For intermediate values ξ coth $\lambda/2$ of α , there is a peak which may be approximated as follows:

For λ and Z greater than about 3, coth $\lambda/2$ and coth Z/2 both approach unity.

Then P =
$$\frac{(1 - e^{-Z})}{g}$$

$$\left\{1 + \frac{g \lambda(\lambda - Z)}{\lambda^2 - Z^2}\right\}$$

$$= \frac{1 - e^{-Z}}{g} \left\{1 + \frac{g\lambda}{\lambda + Z}\right\}$$

This quantity has a maximum given by:

$$\frac{dP}{dZ} = 0 = \frac{d}{dZ} \left\{ 1 - e^{-Z} + \frac{\xi \lambda (1 - e^{-Z})}{\lambda + Z} \right\}$$

$$e^{-Z} + \frac{\xi \lambda (\lambda + Z) e^{-Z} - \xi \lambda (1 - e^{-Z})}{(\lambda + Z)^2} = 0$$

which reduces to

$$e^{Z} = 1 + \lambda + Z$$
 for $(\lambda + Z)^{2} < \langle \xi \lambda \rangle$.

For
$$\lambda = 3$$
 $Z_{max} = 1.75$
10 2.61
100 4.7
1000 6.9

For this case, there will be a peak in the photoconductivity response corresponding to small values of Z .

5. Small sample thickness. $\lambda \rightarrow 0$.

Then
$$P = \frac{1 - e^{-Z}}{2 \xi/\lambda} + \frac{\xi \lambda (2 - Z \coth Z/2)}{-Z^2}$$

$$\sim \lambda/2\xi (1 - e^{-Z})$$

As in the case of small surface recombination velocity, there is a monotonic increase in P from small to large α , but the magnitude of the photoresponse is also small and proportional to the sample thickness.

For the general case, the equation for photoconductivity response was evaluated numerically. For convenience in computation, Eq. (5) may be modified to give

$$P = \frac{N}{I\tau} = \frac{2}{(1 + \coth Z/2)(1 + \xi \coth \lambda/2)}$$

$$\left\{1 + \frac{\xi(\lambda/2)[\lambda/2 \coth \lambda/2 - Z/2 \coth Z/2)]}{(\lambda/2)^2 - (Z/2)^2}\right\}$$

The spectral response curves have been computed over a wide range of parameters. Specifically, the ranges considered are:

Thickness of Photoconductor:
$$l = 10^{-3}$$
 cm. to 10^{-1} cm. Diffusion Constant: $D = 10^{-1}$ cm. $^2/sec$.

rusion Constant:
$$D = 10^{-6} \text{ cm.}^{-6}/\text{sec.}$$

to 10 cm²/sec.

Volume Recombination Time: τ = 10^{-11} sec. to 10^{-5} sec. Surface Recombination $S = 10^2$ cm/sec. to Velocity: 10^4 cm/sec.

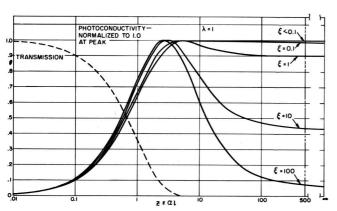


Fig. 2 - Photoconductivity-Absorption Curve. $\lambda = 1, \xi = 0.01 \text{ to } 100$

Corresponding to various possible combinations of these values, the dimensionless parameters have the following ranges:

$$\lambda = 10^{-1} \text{ to } 10^{5}$$

$$\xi = 10^{-4} \text{ to } 10^{2}$$

In order to exhibit the forms of the computed photoconductivity curves, each curve was normalized to a value of 1.000 at its peak.

Figs. 2 to 7 show the manner in which the photoconductivity curves vary with recombination rate (\mathcal{E}) for fixed sample thickness (λ) and the variation with thickness for given re-

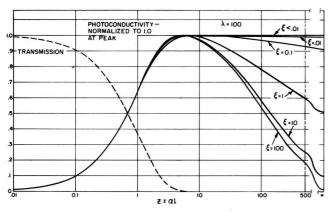


Fig. 3 - Photoconductivity-Absorption Curve. $\lambda = 100$, $\xi = 0.01$ to 100

combination rate. The abscissae of the curves are $Z = \zeta \lambda = \alpha l$ and the curves, representing photoconductive response as a function of absorption coefficient, are equivalent to spectral curves of photoconductivity. They differ

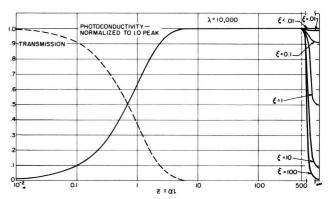


Fig. 4 - Photoconductivity-Absorption Curve. $\lambda = 10,000, \ \xi = 0.01 \ to \ 100$

in shape from such curves, as normally presented, with either wavelength or photon energy as abscissa, in being greatly stretched out beyond the absorption edge. This stretching in the horizontal direction occurs because after the absorption edge is reached, the absorption coefficient changes very rapidly for small change in wavelength or photon energy. For conveniet reference, the optical transmission is also plotted. (T = $e^{-\alpha l}$).

Characteristically, each curve approaches an asymptotic value of photoconductive response for large values of absorption coefficient, and perhaps the most conspicuous distinguishing feature of a curve is the ratio of this asymptotic response to the peak response. Table I gives the values of these ratios for the ranges

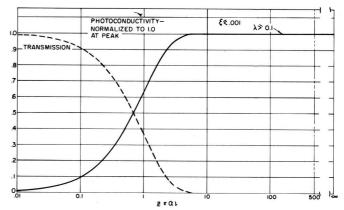


Fig. 5 - Photoconductivity-Absorption Curve. $\xi < 0.001, \ \lambda \ge 0.1$

Table I

$\frac{P_{\infty}}{P_{\Lambda}} = \frac{Photoconductivity at High Absorption Coefficient}{Peak Photoconductivity}$									
£ =		.0001	.001	.01	• 1	1	10	100	
λ =	0.1	1.0000	1.0000	1.0000	.9993	.9921	.8967	.4278	
	1	1.0000	1.0000	.9993	.9922	.8981	.4317	.0684	
	10	1.0000	.9996	.9951	.9447	.6073	.1261	.0141	
	100	•9999	.9991	.9910	.9167	.5159	.0957	.0105	
	1000	•9999	.9990	.9903	.9100	.5025	.0917	.0100	
	10,000	.9999	.9990	.9901	.9092	. 5003	.0910	.0099	
-	100,000	.9999	.9990	.9901	.9091	.5001	.0909	.0099	

of λ and ξ used herein. It is notable that, for small values of λ and ξ (small thickness, small surface recombination), this ratio is practically unity, and no peak exists. For large values of λ and/or large values of ξ , there is a peak, which becomes more pronounced very rapidly in

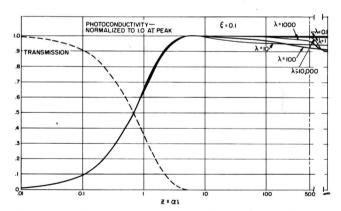


Fig. 6 - Photoconductivity-Absorption Curve. ξ = 0.1, λ = 0.1 to 10,000

the case of increasing ξ and more slowly for increasing λ , with the peak shape becoming constant for large values of λ . (Fig. 8) shows a contour map of the asymptote-to-peak ratio as a function of ξ and λ .

Comparison with Experimental Results

Detailed comparison of the predictions of this theory with experiment is rather diffi-

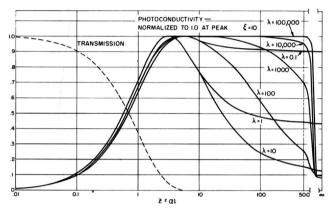


Fig. 7 - Photoconductivity-Absorption Curve. $\xi = 10$, $\lambda = 0.1$ to 100,000

cult, in view of the meager data on absorption on the short wavelength side of the absorption edge. The general shapes of observed photoconductivity spectral distribution curves are in reasonable agreement with those predicted by this theory, when due account is taken of the non-uniform variation of absorption coefficient through the spectrum.

One general observation that has been made is that photoconductivity curves obtained for thick specimens (of the order of magnitude of 1mm or greater) invariably show a peaked form, with a tendency to approach an asymptotic value in the high absorption region. Thin films of the same materials (thickness in the micron range), on the other hand, show either no peak or a very broad peak with a small drop in the high absorption region. These characteristics are in good agreement with this theory.

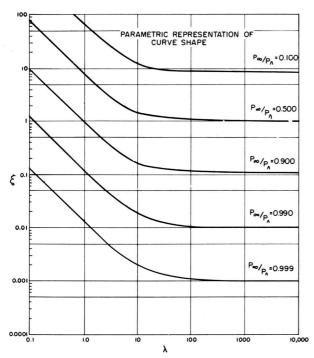


Fig. 8 - Contour Representation of Ratio of Asymptotic to Peak Values of Photoconductivity as Function of ξ and λ .

On the low absorption (long wavelength) side of the peak, experimental curves generally show less photoconductive response, corresponding to a given absorption coefficient, than would be expected from this theory. The situation is exemplified by the experimental curve for a crystal of antimony sulfide(Sb,S3), shown in Fig. 9, together with the theoretical curve (for $\lambda = 1000$, $\xi = 1$) which seemed most nearly to match the location of the peak and the ratio of peak height to asymptotic value at large α . The disparity apparently arises from the assumption that every photon absorbed creates a pair of charge carriers -- i.e., the assumption that only band-to-band transitions occur. However, it is substantially never true that photoconductors are of such purity and perfection that no other type of transition can occur. Absorption in the long wavelength tail may arise from a variety of electronic transitions, not all of which are of such nature that they give rise to photoconductivity.

This divergence between experimental data and theoretical predictions may be utilized to obtain information concerning electronic transitions, occurring in the long wavelength tail, which do not give rise to photoconductivity.

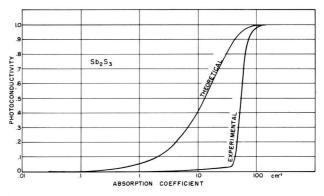


Fig. 9 - Photoconductivity-Absorption Curves for Antimony Sulfide.

Such an analysis was carried through for the Sb.S. data. By subtracting the absorption values of the theoretical curves from the corresponding experimental values, a curve for "excess" absorption is obtained. Fig. 10 shows the tail of the experimental absorption curve and the corresponding curve of excess absorption. This excess absorption curve was then resolved into absorption bands, assumed to have Gaussian form, and these bands are also shown in Fig. 10. No great accuracy is claimed for the second absorption band, since the absorption data in the neighborhood of hv = 1.6 e.v., corresponding to very low light transmission by the samples available, have considerable uncertainty.

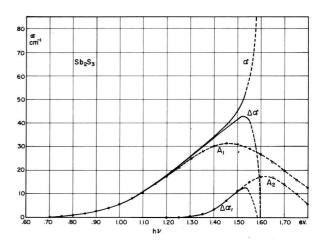


Fig. 10 - Low-Energy Tail of Absorption Curve for Antimony Sulfide α - Experimental Absorption Curve; $\Delta\alpha$ - Difference between Experimental and Theoretical Curves; A_1 - First Absorption Band; $\Delta\alpha$ - Difference between $\Delta\alpha$ and A_1 Curves; A_2 - Second Absorption Band.

Applying Smakula's formula to the absorption bands, values are obtained for densities of impurities responsible for these two bands.

The calculated values are about 8 x 10^{16} cm⁻³ and 3 x 10^{16} cm⁻³, respectively. These results are plausible.

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Appendix

The technique of resolving the absorption curve into absorption bands having Gaussian form may be of interest. The absorption band has the equation

$$\alpha = Ae^{-k(h\nu - h\nu_0)^2}$$
 or $\ln \alpha = \ln A - k(h\nu - h\nu_0)^2$

For convenience of computation, it is preferable to use logarithms to the base 10: $\log \alpha = \log A - K(h\nu - h\nu_0)^2$.

The problem is then that of determining the constants A, K, h ν_0 , such that this curve will fit as well as possible to the experimental curve. A number of experimental points $\alpha_n(h\nu_n)$ are selected. The assumption is made that the first of these, $\alpha_1(h\nu_1)$, lies on the Gaussian, and the fit of the others is checked.

Writing

$$\log \alpha_n = \log A - K(h\nu_n - h\nu_o)^2$$

and

$$\log \alpha_1 = \log A - K(h\nu_1 - h\nu_0)^2$$

and subtracting:

$$\log \alpha_{n} - \log \alpha_{1} = -K (h\nu_{n} - h\nu_{0})^{2} - (h\nu_{1} - h\nu_{0})^{2}$$
$$= -K (h\nu_{n} - h\nu_{1}) (h\nu_{n} + h\nu_{1} - 2h\nu_{0})$$

Define

$$z_{n,1} = \frac{\log \alpha_n - \log \alpha_1}{h\nu_n - h\nu_1} = -K(h\nu_n + h\nu_1 - 2h\nu_0)$$

If, now, z_n is plotted on a linear scale against ($h\nu_n$ + $h\nu_1$), a straight line should be obtained. Its zero intercept will be 2 $h\nu_0$ and its slope will be -K.

The computation of z_n is repeated with each of the selected points in turn being assumed to lie on the curve and a plot will be obtained similar to that shown in Fig. 11. It is seen that a straight line can be found which fits well to most of the points. Those points departing widely from the line are associated with experimental points which do not fall on the curve sought. From the line so determined, mean values of K and h ν_0 are found. These are then substituted in the equation log A = log α_n + K(h ν_n - h ν_0) for each of the experimental points and the mean of the values of log A is found.

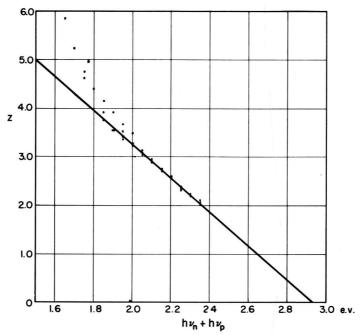


Fig. 11 - Part of $z = \frac{\log \alpha_n - \log \alpha_1}{h\nu_n - h\nu_1}$ us $h\nu_n + h\nu_1$.

Having determined a first absorption band in this fashion, it is plotted together with the experimental absorption curve and subtracted, point-by-point, from the experimental curve to give a residual which may then be analyzed in the same manner for further absorption bands.