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LB-968

SELECTIVITY AND TRANSIENT RESPONSE SYNTHESIS

RADIO CORPORATION OF AMERICA

RCA LABORATORIES DIVISION

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Selectivity and Transient Response Synthesis

Approved

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Introduction

Decade resistance and capacitance boxes are standard equipment in most development laboratories. These devices are useful because substitution measurements are usually quicker and easier to make than theoretical calculations.

This bulletin describes a filter circuit whose response can be adjusted to approximate a desired response. The versatility of this filter with respect to bandwidth and cutoff slopes is comparable to the universality of decade boxes. This filter may also be used to synthesize desired transient responses.

When substituted into, or added to a system, it enables the development engineer to obtain immediate results that would otherwise be available only after time-consuming, and often difficult, design calculations. Such a universal filter is, therefore, a most desirable piece of laboratory equipment.

The following sections will describe the theory[1], the design, and the uses of such a filter. Experimental results and applications are illustrated by means of oscillograms. The equipment has been found simple to operate and reliable in performance.

Theory of Operation

If a delay line, Fig. 1, is driven from its characteristic impedance and is open-circuited at the receiving end, it is shown in the Appendix, that when there are no losses, that

$$E_r = E_g e^{-j\theta} \qquad (1)$$

where E_r is the receiving end voltage.

E_g is the generator voltage.

θ is the length of line in radians or (the delay time)x(the radian frequency).

$$E_x = E_g \cos \frac{x\theta}{n} (e^{-j\theta}) \qquad (2)$$

$$E_n = E_g \cos \theta \, (e^{-j\theta}) \qquad (3)$$

where E_x is the voltage x units from the receiving end.

n is the total number of units in the line.

E_n is the voltage at the sending end.

Eqs. (1), (2), and (3) point up a startling fact. The voltages of these equations are always in phase, lagging the generator voltage by an angle θ which is proportional to frequency. This is physically possible because in

[1] M. S. Corrington and R. W. Sonnenfeldt, "Synthesis of Constant Time-Delay Networks", *Proceedings National Electronics Conference*, Vol. IX, pp. 50-63, Chicago, 1954.

the lossless line a wave traveling down the line will be reflected back into the line from the open receiving end and will then be absorbed at the terminated sending end of the line. Thus, the voltage at any point on the line is made up of the two waves, one traveling down the line, the other coming back, the time lapse between the two waves being proportional to the distance from the reflecting end. Since the voltage at any point on the line is in phase with the voltage at any other point on the line, it is possible to combine them algebraically, the result being the sum of the component voltages. Fig. 1 shows a delay line made up of an LC ladder with n sections total. Let us examine what the voltages are on this line. At the receiving end we have a voltage always equal to the generator voltage and lagging it by an angle

$$\theta = t_d \times \omega = 2\pi t_d \times f$$

where t_d is the total delay of the line.

One section back from the receiving end there is a voltage

$$E_1 = E_g \cos\frac{\theta}{n} \text{ at the same phase angle.}$$

Three sections back there is a voltage

$$E_3 = E_g \cos\frac{3\theta}{n}.$$

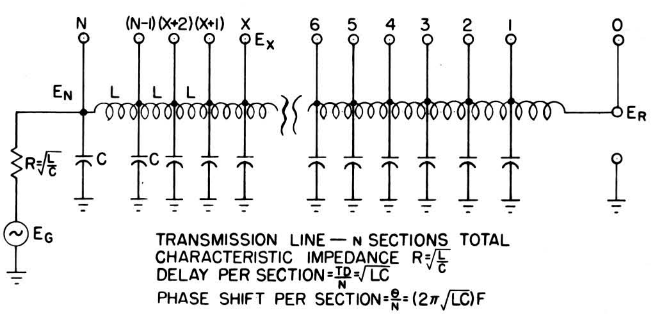


Fig. 1 - Lossless delay line.

Five sections back from the open end there is

$$E_5 = E_g \cos\frac{5\theta}{n}, \text{ and so on.}$$

Let us now examine what happens when these voltages are added together

$$E = E_0 + E_1 + E_3 + E_5 \dots\dots$$

$$= E_g \left(1 + \cos\frac{\theta}{n} + \cos\frac{3\theta}{n} + \frac{5\theta}{n} \dots\right). \quad (4)$$

This is recognized as a set of terms belonging to a Fourier series. Let us now assume that we are able to attenuate any of these voltages without phase-shift, and that in addition we could change the algebraic sign associated with each voltage at will. In that event we can proceed to construct a finite Fourier series with as many terms as we have connections to the line. For instance, we might construct the series:

$$E = E_g \left(1 + \cos y - 1/3 \cos 3y + 1/5 \cos 5y - \dots\right) \quad (5)$$

$$\text{where} \quad y = \frac{\theta}{n} = \frac{2\pi f t_d}{n} = \left(\frac{2\pi t_d}{n}\right) f$$

This is recognized as a series expressing the square cosine function. Inspection of Eq. (5) shows that the variable is frequency, and thus we have produced a square wave selectivity curve, or more accurately an approximation depending on the number of terms in (5). It is obvious that any other cosine Fourier series can be approximated by the same method merely by choosing the proper taps on the line, the proper attenuation for each voltage picked off from the line, and the proper algebraic sign. As a result, then, this procedure enables us to synthesize to a very good approximation any selectivity curve that can be expressed by a Fourier cosine series. It is easily shown that physically realizable selectivity curves are even functions of frequency and are, therefore, expressible as Fourier cosine series provided that the selectivity meets the usual criteria for a function to be expressible as a Fourier series.[2] We have thus a very general method of synthesis producing selectivity at constant time delay since the phase angle is proportional to frequency. Fig. 2 shows schematically how this may be done. Attenuating resistors are used to feed two buses through SPDT switches,

[2] H. S. Carslaw, "Introduction to the Theory of Fourier's Series and Integrals", 3rd ed., Macmillan and Co. Ltd., London, 1930.

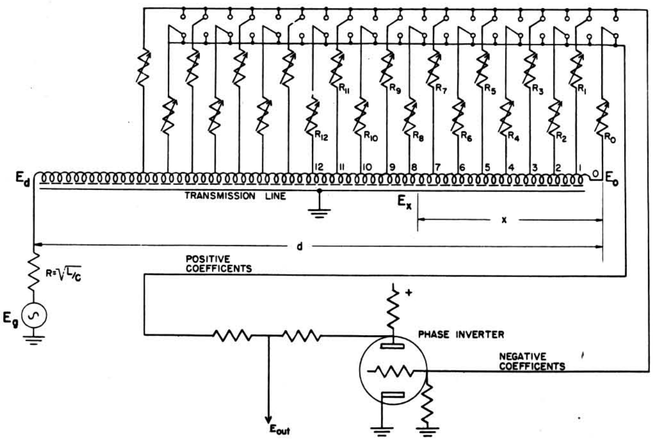


Fig. 2 - Delay line filter.

one bus for terms with positive signs, the other for terms with negative signs. The negative sign is obtained through phase inversion in a tube.

We shall now investigate the meaning of a square wave selectivity curve. Fig. 3a shows a square wave symmetrical about the frequency axis. Fig. 3b shows a wave to which has been added the steady value of 0.5, and Fig. 3c shows a square wave from which has been subtracted the steady value of 0.5. The meaning of Fig. 3a is that from 0 to a frequency f_0 we have a response of +0.5, between f_0 and $3f_0$ we have a response of -0.5, from $3f_0$ to $5f_0$ we have again a response of +0.5 and so on. Physically this means that wherever a minus sign is encountered there is a 180-degree phase shift. In the second case it will be noted that with 0.5 added to the curve we have between 0 and f_0 a response of unity, between f_0 and $3f_0$ a response of zero, between $3f_0$ and $5f_0$ a response of unity, and so on. Physically this means that there are alternating pass and stop bands, the first pass-band being between 0 and f_0, with the first stop-band being between f_0 and $3f_0$. This characteristic is therefore a low-pass filter if frequencies beyond $3f_0$ can be neglected. In the third case, with a DC value of 0.5 subtracted, there are alternating stop and pass-bands, the first stop-band occurring between 0 and f_0, the first pass-band being between f_0 and $3f_0$. This characteristic may be viewed either as high-pass or band-pass, depending on the frequency components considered. The steady-value term is, of course, the voltage from the receiving end of the line--see Eq. (1)--since amplitude response at this point is independent of frequency. We can therefore obtain any of the three conditions shown in Fig. 3 merely by adding the right amount of receiving end voltage to the other terms.

The length of line for the first tap is easily obtained. For instance, if we consider the selectivity curves of Fig. 3, we may write for Fig. 3a the Fourier series given in (5). From the curve and (5) we see that $y = 90^\circ$ when $f = f_0$ so that for a line of n sections with a total delay t_d we have the delay per section,

$$t_d/n = \frac{1}{4f_0} \qquad (6)$$

If it is assumed that $t_d/n = (LC)^{1/2}$, where L and C are inductance and capacity/unit length, (6) may be changed to

$$LC = 1/(16f_0^2) \qquad (7)$$

giving the LC product directly. L and C can be obtained from the relation above, and the relation

$$(L/C)^{1/2} = R \qquad (8)$$

where R is the characteristic impedance.

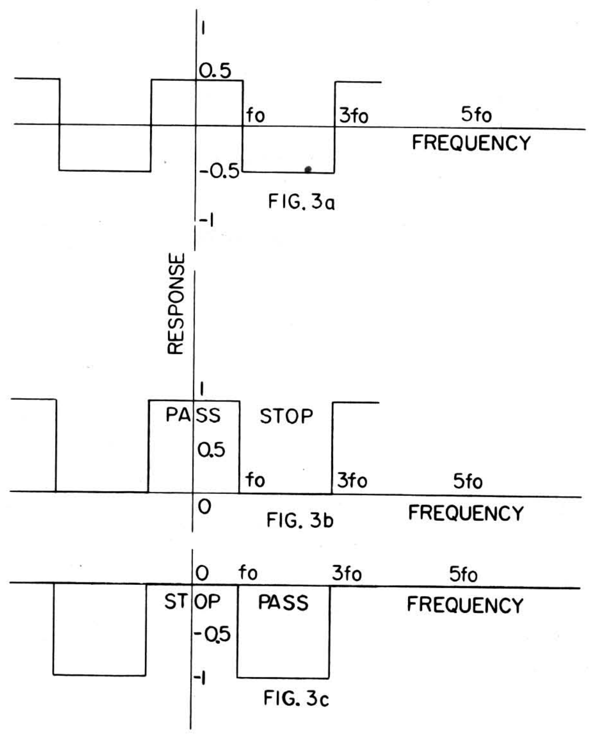
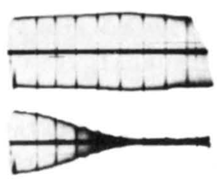
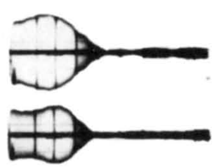


Fig. 3 - Square wave selectivity curves.

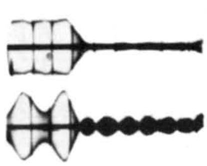
The other terms of the series can then be obtained from taps 3, 5, 7, and 9 and so on of a line of identical sections. If it is desired to obtain one-half the previous value of f_0, the first term would be picked off from tap 2 of the line and successive terms from taps 6, 10, 14, 18, and so on. For $f_0 = 1/3$ of the original value, one would use taps 3, 9, 15, 21, and so on. In this manner the cutoff frequency of the filter can be varied. The cutoff rate is controlled by the number of taps used. All of the filter characteristics made in this manner have the same constant time delay. Figs. 4a, b, c, d, show how a square wave selectivity curve is built up and Fig. 4e shows calculated responses for comparison.



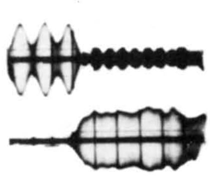
(4a) shows at the top the receiving end voltage when an FM sweep is applied to the sending end. At the bottom is shown the response when voltage from tap No. 4 of the line is added to the receiving end voltage. This is the fundamental component of the square wave selectivity curve to be synthesized. The vertical markers are at 500 kc intervals.



(4b) shows at the top the response when properly attenuated voltage from tap No.12 is subtracted from the response of Figure (4b), bottom. This is the third harmonic component of the selectivity curve. At the bottom of figure 4b is shown the response resulting when voltage from tap No. 20 is added. This is the fifth harmonic component.



(4c) shows at the top the response resulting from voltages from taps Nos. 4,12,20,28,36 and 44. This includes harmonics up to the 11th. It is seen how the approximation to the square wave becomes successively better. At the bottom of Figure (4c) is shown what happens when too much 7th harmonic is subtracted.



4d shows at the top the result of using too much 11th harmonic. At the bottom of Figure (4d) is shown what happens when the voltage from the receiving end is subtracted instead of added. The selectivity curve is now high-pass instead of low-pass. The irregularities in the response are due to imperfections in the transmission line at the higher frequencies.

Fig. 4 - Fourier synthesis of square wave selectivity.

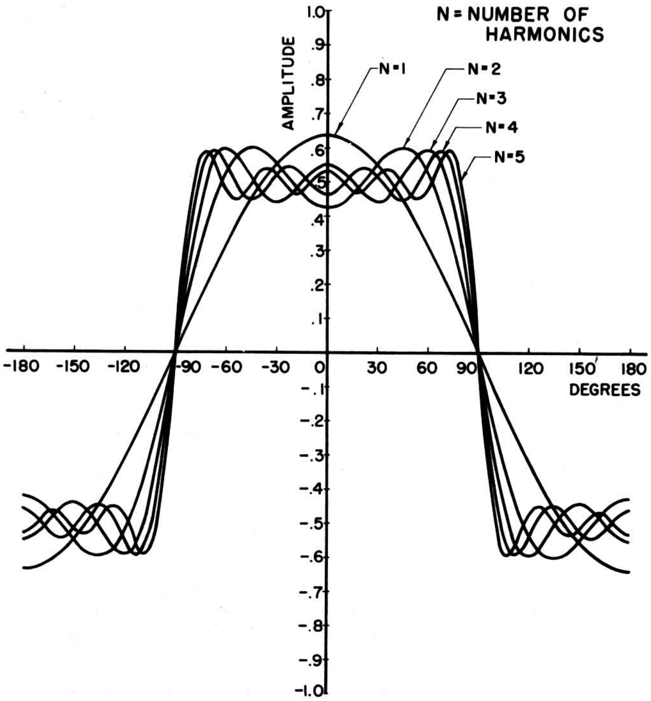


Fig. 4(e) - Fourier synthesis of square wave.

It will be noted that this method gives rise to multiple responses, as shown in Fig. 3. This may not be desirable, and only a single pass-band may be wanted. There are several ways of obtaining this. The delay line filter may be cascaded with a conventional filter which has linear phase and unity response in the pass-band of the line filter. There is no restriction upon the phase and amplitude response of this second filter in the stop-bands of the line filter, and it may therefore be provided with attenuation characteristics suitable for suppression of unwanted response regions without affecting the phase linearity in the pass-band. Another method is to adjust the Fourier series generated so as to increase the relative widths of the stop-bands so that undesired multiple responses fall outside the bandwidth of the system with which the line filter is to be associated. This is done by "lowering the duty cycle" of the selectivity square wave by selection of the proper Fourier series terms for a pulse rather than a square wave.

A later section will describe in detail the circuitry employed to obtain these results. To summarize, this method utilizes a transmission line, open-circuited at the receiving end, driven from its characteristic impedance and having constant delay over the basic range in which it is to operate as a universal filter. Voltages are picked off from this line, and after suitable attenuation and polarity inversion, where needed, these voltages are added up in accordance with the harmonic analysis schedule of the desired selectivity curve.[3]

As has been pointed out, all filter characteristics obtained in this manner will have the same time delay characteristics. Frequently it is desirable to have filters whose phase characteristics are other than linear. Such a need arises when filters with prescribed transient responses must be designed. It is known that amplitude and phase characteristics of a filter are completely determined by the transient response.[4]

It will now be shown how transient responses may be synthesized from a transmission line terminated at the receiving end in its characteristic impedance. For an ideal line, having no losses, it is found that

$$E_x = 1/2\ E_g\ e^{-j(n-x)\theta} \qquad (9)$$

where E_g is the generator voltage.

E_x is the voltage x units from the receiving end.

n is the total number of units in the line.

and θ is the phase delay/unit of line = $t_d/n\ (2\pi f)$ where t_d is the total delay of the line.

Eq. (9) shows that the line merely introduces a delay, without a selectivity characteristic when terminated. If a unit step function is applied to the line, this waveform will be propagated down the line without distortion. There will be no reflection. If the line is provided with taps, it is possible to combine voltages from these taps in much the same manner as before. Fig. 5 shows the voltage at the receiving end, one line unit back from the receiving end, 3 line units back from the receiving end, and 10 units back from the receiving end. The next figure shows how a number

[3] R. P. G. Denman, "36 and 72 Ordinate Schedules for General Harmonic Analysis", *Electronics*, Vol. 15, pp. 44-47, September 1942.

[4] W. L. Sullivan, "Analysis of Systems with Known Transmission Frequency Characteristics by Fourier Integrals", *Electronic Engineering*, Vol. 61, pp. 248-256, May 1942.

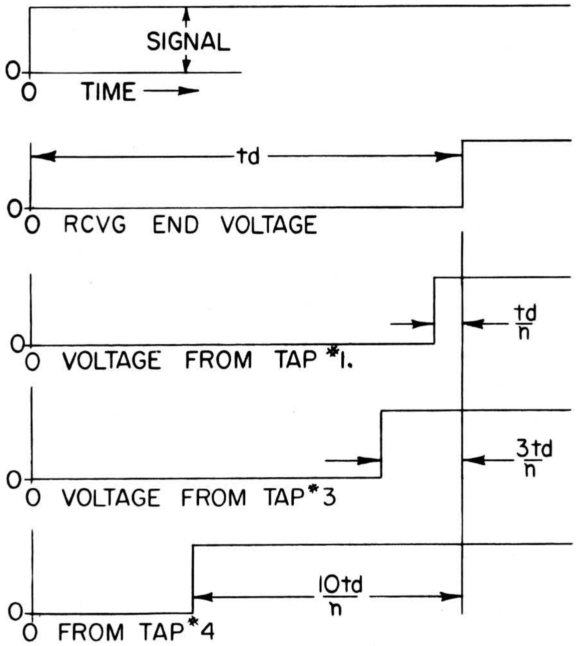


Fig. 5 - Time domain step responses.

of such voltages may be combined for the transient response of Fig. 6. It is to be noted that each of the component voltages has been attenuated by a proper factor and some have been inverted in polarity to obtain the composite response. Fig. 7 is a schematic diagram for this arrangement. Any response within the limits imposed by the total number of taps available, and the delay between taps, may be synthesized in this manner. An analysis would show that this mode of operation is equivalent to the simultaneous synthesis of amplitude and phase characteristics uniquely determined by the nature of the transient. This amplitude and phase characteristic would operate on any other waveform applied to the filter.

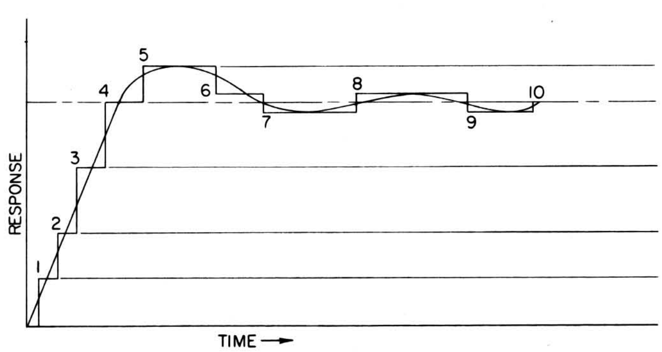


Fig. 6 - Step approximation of transient response.

It is not as easy to calculate by analytical methods the amplitude and phase characteristics resulting, as was the case with the open-ended line. But fortunately this is rarely necessary, since this method is intended for synthesis procedures in the time domain while the former serves for synthesis in the frequency domain. The synthesis in the time domain is straight-forward, requiring only step approximations to a desired transient response. Figures in a later section will show how well such approximations can be made.

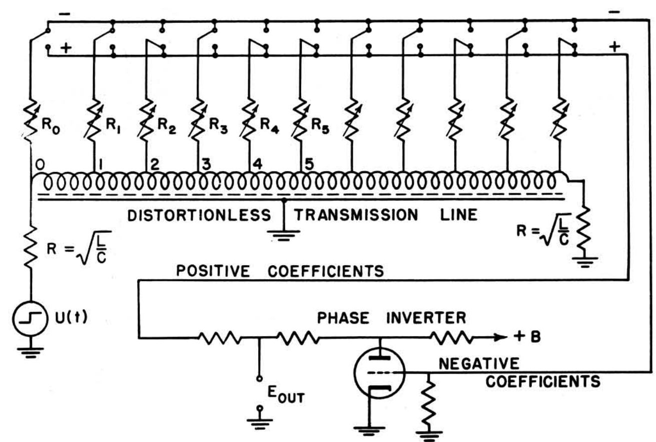


Fig. 7 - Delay line transient synthesis filter.

Equipment Design

Fig. 8 shows a circuit diagram of the universal filter. It is provided with a specially designed delay line of 50 sections, having a characteristic impedance of 185 ohms, with good amplitude and phase response to about 5 Mc. Each section is connected to a pin jack. Ten cathode followers, each provided with a jack, serve to pick off voltages from the line without loading it. Each of the cathode followers is provided with an attenuation control, and the output from each of these attenuators can be switched by means of a single-pole, double-throw switch onto either of two buses to select the algebraic sign. The switches have a center position, in which they are open.

One bus is used to feed the cathode of the amplifier V_{12}, while the other bus feeds the

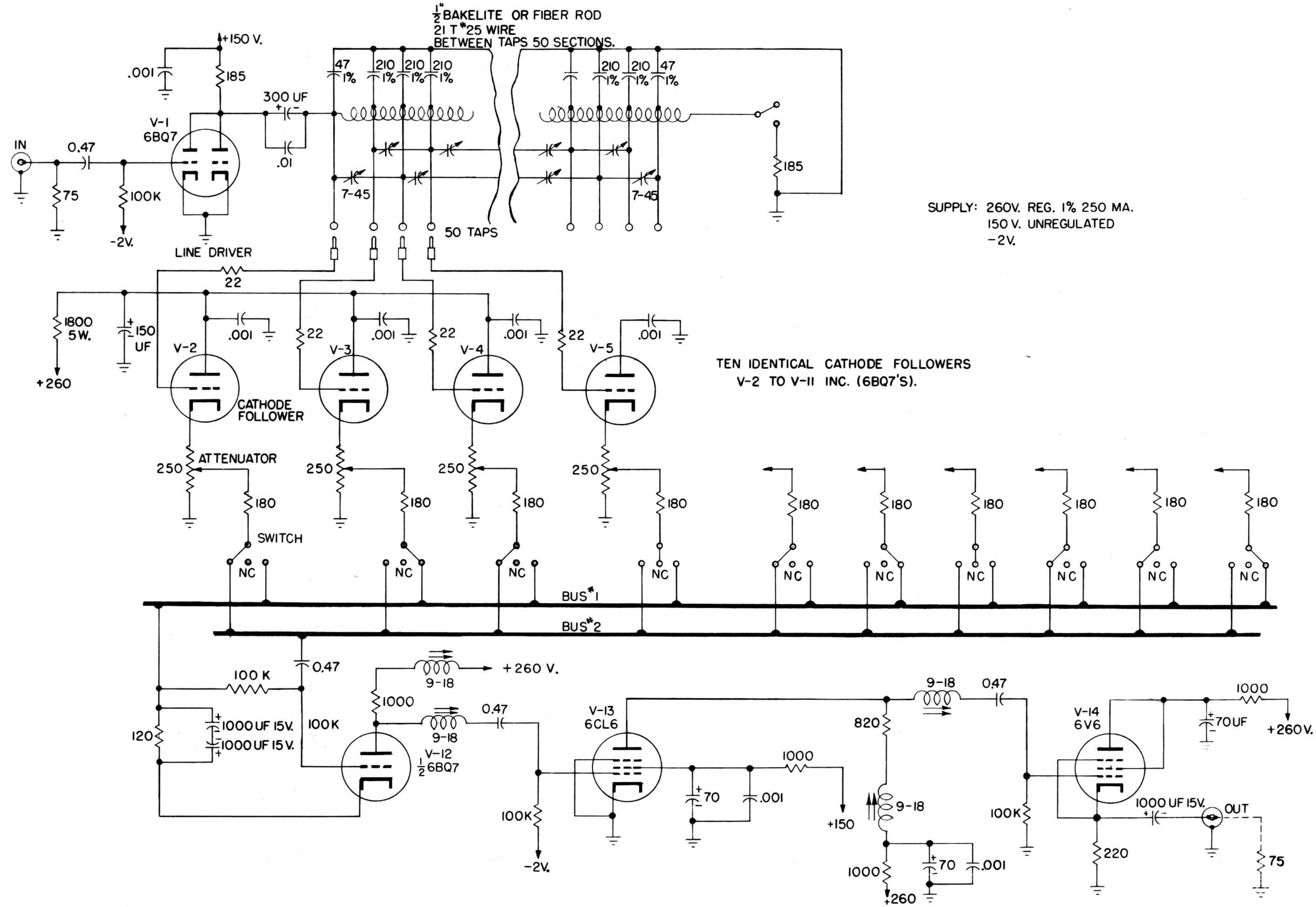


Fig. 8 - Schematic of universal filter.

grid of this tube, bias being obtained across a by-passed resistor in the cathode circuit. At the plate of V_{12} the cathode voltage will appear amplified in the same phase, while the grid voltage will appear with a 180-degree phase inversion, or with its algebraic sign changed. V_{13} and V_{14} are a wideband amplifier circuit, with cutoff at about 11 Mc, and unity amplitude response and essentially linear phase up to approximately 6.5 Mc. Output is low-impedance to feed 75-ohm lines.

The tapped line is provided with a fixed termination at the sending end, and a switch and a termination at the receiving end. The receiving-end termination can therefore be either 185 ohms or open so that the line can be used either for synthesis in the frequency, or in the time domain, depending on the switch position.

It is seen that any ten voltages can be picked off from the line and can be combined in any manner desirable, so that either ten terms of a Fourier series can be obtained, or any ten-step approximation to a transient response function can be made.

In the design of this equipment, care must be taken to keep chassis currents to a minimum, to keep the power supply impedance low, and to decouple individual stages. Difficulty was experienced with a cadmium plated chassis, and in the final design a solid brass chassis was used. Parasitic suppressors were included wherever practical. When these precautions are observed, the filter circuit will be stable, free from spurious responses and re-settable.

The transmission line was wound on ½-inch bakelite rod, carefully checked for uniformity. The winding is 21 turns of No. 25 enamel wire between taps, and every effort was made to obtain a smooth, continuous winding. The shunt capacitors are 210 μμf, 1 per cent, with low temperature coefficient, and the bridging capacitors are adjustable 7-45 μμf. They are adjusted for best response at the receiving end of the line and should all be adjusted to approximately the same value--about 28 μμf. The line should be kept well away from the chassis, to minimize losses, and should be securely mounted. Figs. 9 and 10 are photographs showing constructional details. It should be noted that additional lines may be provided to extend the usefulness of this equipment. Voltages can be picked off from any other line in the same manner by the cathode followers, but the line impedance must be kept reasonably low to minimize loading effects.

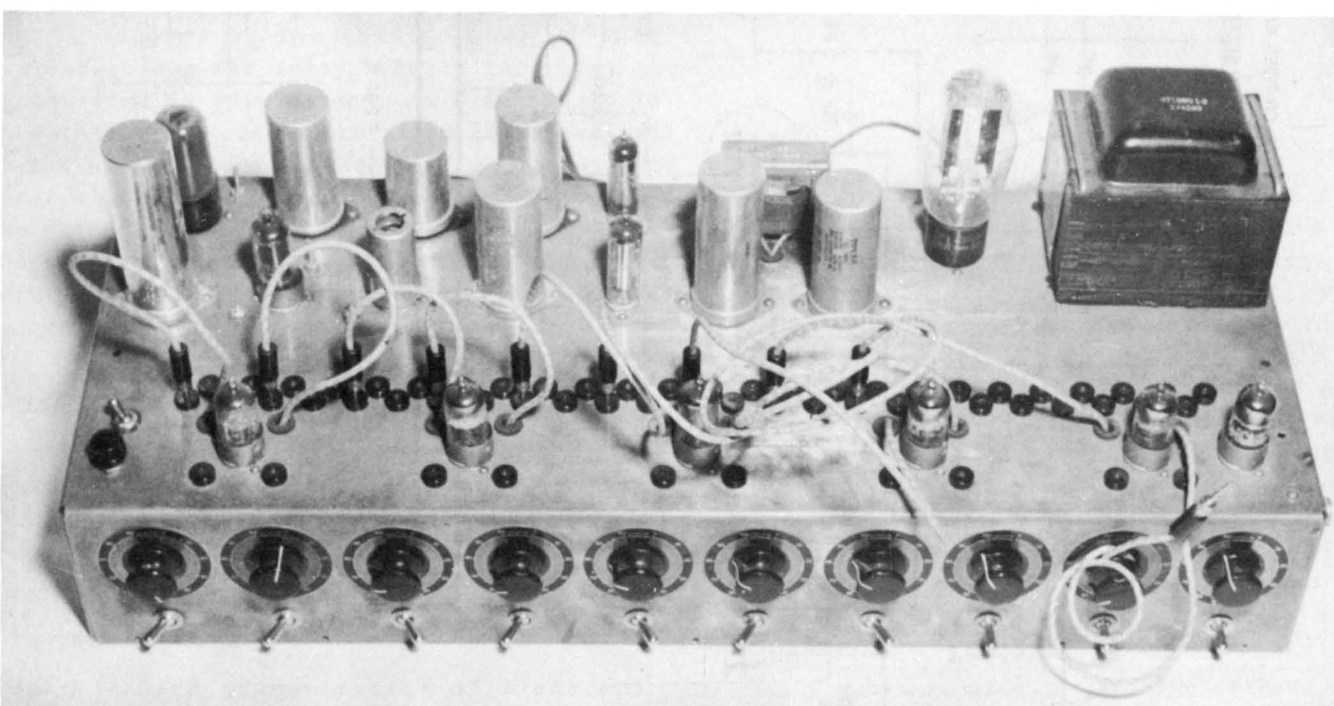


Fig. 9 - Top-view of universal filter.

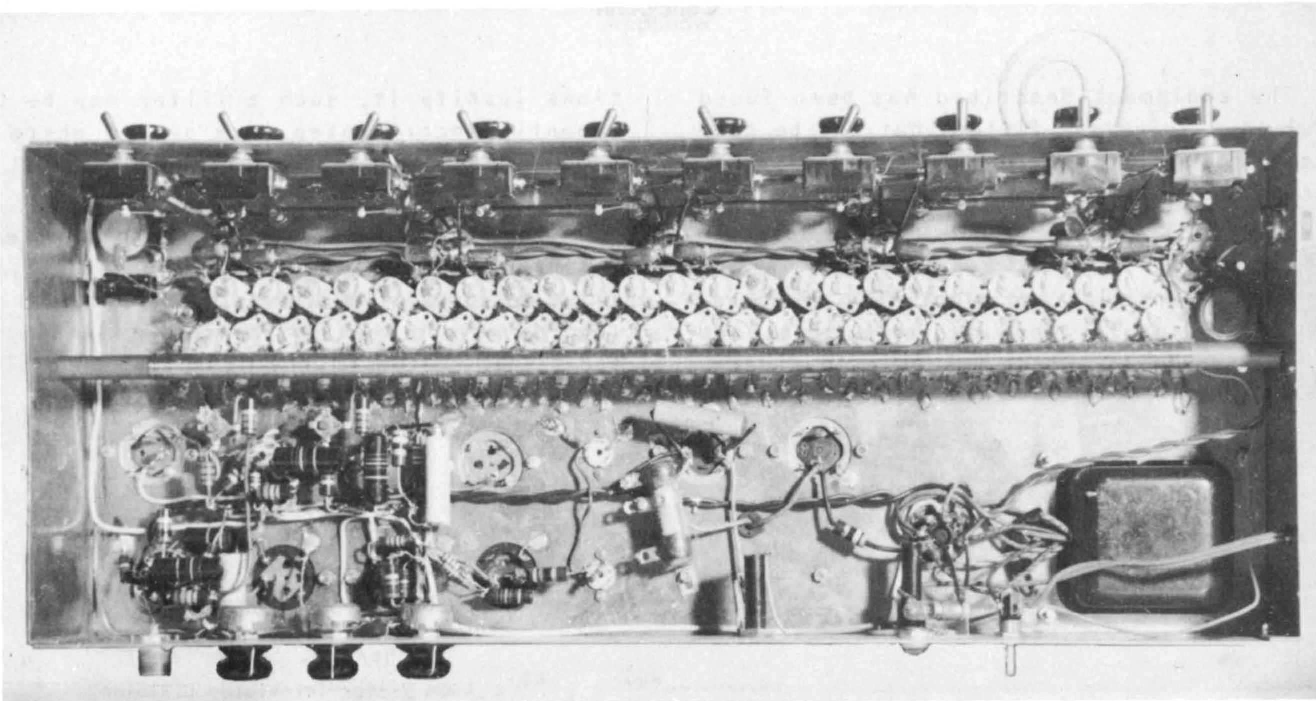


Fig. 10 - Bottom-view of universal filter.

Experimental Results

The equipment described has been found most useful for the rapid synthesis of filter characteristics in the range from 0 to 4 Mc. Fig. 11 shows a measured selectivity curve with its associated phase curve. Fig. 12 shows several square wave responses synthesized on the equipment.

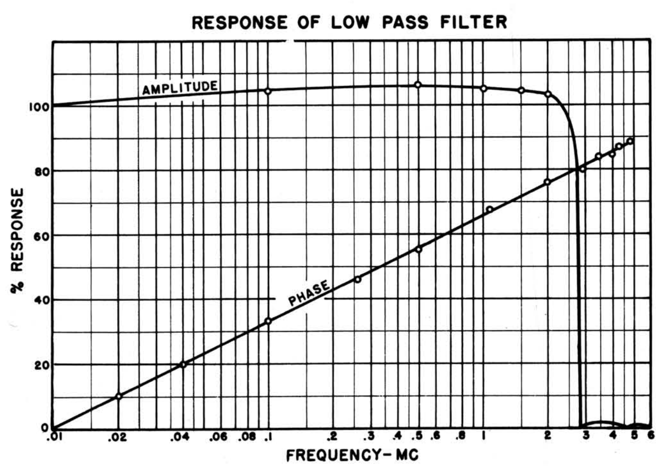


Fig. 11 - Measured amplitude and phase curve.

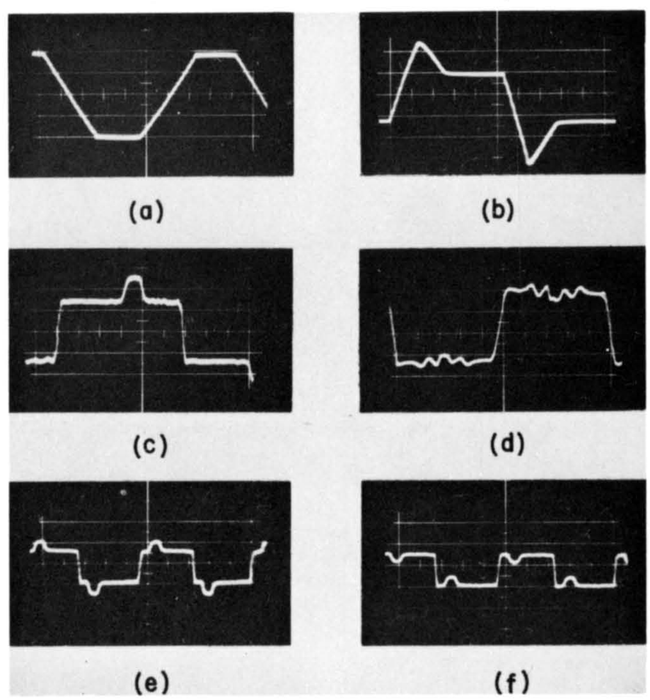


Fig. 12 - Synthesis of transients.

Conclusion

The equipment described has been found useful as a universal filter that can be connected into experimental circuits or systems. If properly used, it can lead to improved and more rapid design procedures, particularly in television applications.

Its universality and simplicity of operation make it useful when system evaluations must be undertaken. Where economic considerations justify it, such a filter may be permanently incorporated in a system where its performance advantages outweigh its cost and circuit complexity.

The range of such a filter may be greatly extended by additional delay lines of appropriate design. It is stable in operation so that records may be kept of particular settings to facilitate re-checks under identical conditions.

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Appendix

In a lossless transmission line let E_x be the voltage at a point x units from the receiving end, I_x the current at point x, L the inductance per unit length, C the capacitance per unit length, and let ω be the applied frequency. Then

$$\frac{dE_x}{dx} = i\omega L I_x \tag{1}$$

and

$$\frac{dI_x}{dx} = i\omega C E_x \tag{2}$$

since there are no losses. These equations can be solved to give

$$I_x = A_1 e^{i\theta x} + A_2 e^{-i\theta x} \tag{3}$$

$$E_x = A_1 (L/C)^{\frac{1}{2}} e^{i\theta x} - A_2 (L/C)^{\frac{1}{2}} e^{-i\theta x} \tag{4}$$

where A_1 and A_2 are constants and $\theta = \omega(LC)^{\frac{1}{2}}$.

Let the generator voltage be E_g with internal impedance $R = (L/C)^{\frac{1}{2}}$. Then for a open-ended line

$$I_o = 0 = A_1 + A_2$$

so

$$A_1 = -A_2. \tag{5}$$

The voltage at the sending end is

$$E_d = 2A_1 (L/C)^{\frac{1}{2}} \cos \theta d = E_g - I_d (L/C)^{\frac{1}{2}} \tag{6}$$

where I_d is the sending-end current.

From (3) $\quad I_d = 2A_1 i \sin \theta d.$

Substituting in (6)

$$A_1 = 1/2 E_g (L/C)^{-\frac{1}{2}} e^{-i\theta d}. \tag{7}$$

This gives

$$E_x = E_g \cos \theta x \, e^{-i\theta d}. \tag{8}$$