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LB-939

THE TACITRON

A LOW-NOISE THYRATRON

CAPABLE OF CURRENT INTERRUPTION

BY GRID ACTION

RADIO CORPORATION OF AMERICA
RCA LABORATORIES DIVISION
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Approved

Stuart Wm Seeley

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Introduction

Two thyratron considerations are (1) spurious noise output and (2) speed and ease with which the anode current can be turned on and off. The spurious noise output of conventional thyratrons is such that they often cannot be operated close to sensitive communications equipment. In addition, while the anode current in conventional thyratrons can be established in a fraction of a microsecond, current interruption requires times of the order of 100 microseconds. During this time one is confronted with the additional disadvantage of having to remove the anode voltage.

The tacitron, a thyratron whose construction and operation are described and analyzed in this bulletin, is so quiet in operation that it can be operated in close proximity to sensitive communications equipment without noticeable interference. Furthermore, its anode current can be switched on or off in about one microsecond. With respect to arc drop, overall efficiency, tube construction, and other salient characteristics the tube is similar to conventional ones. The tacitron owes its unusual capabilities mainly to a grid design that insures tube operation in a unique discharge mode wherein ion generation takes place solely in the grid-anode region.

Besides being useful in applications in which conventional thyratrons are now used, this new tube offers the possibility of use in many new applications presently denied conventional thyratrons, either because of noise or recovery time considerations. This includes computers and all sorts of pulse circuits.

General Discussion

A thyratron is essentially a triode vacuum tube which has been filled with some gas such as xenon, argon, hydrogen or mercury. Electrons enroute from the cathode to the anode create ions which serve to neutralize the electron space charge. This makes possible the passage of large electron currents at low anode voltages, allowing the tube to operate with high efficiency. This important characteristic combined with simplicity, ruggedness, and high power sensitivity has made the thyratron a popular device for diverse switching applications.

The thyratron has, however, serious disadvantages. Two very important ones are: (1)

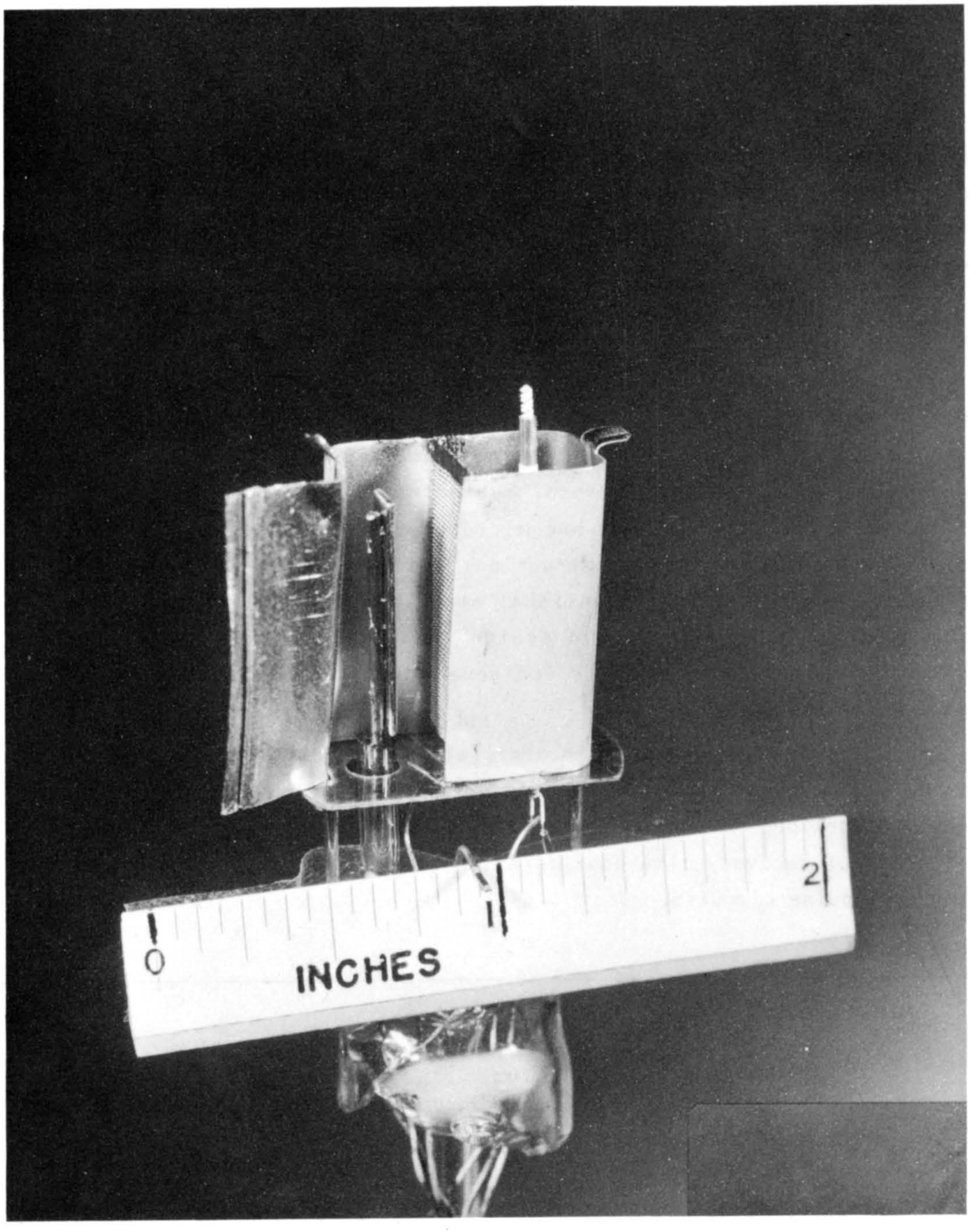

Fig. 1 - Experimental Tacitron. A section of the grid shield has been peeled back to expose the internal geometry.

e tube is often noisy,[1] and (2) once the anode current has started to flow it cannot be turned off by the grid. The latter difficulty has had to be surmounted by the troublesome expedient of first removing the anode voltage and then waiting for the tube to totally, or at least partly, deionize. This complicates the attendant circuitry and restricts the thyratron to relatively low frequency applications.

Fundamental studies of externally-heated hot-cathode discharges[2,3,4] have illuminated e way to a tube design that overcomes these two difficulties. This design insures operation in a unique discharge mode wherein ion generation occurs solely in the grid-anode region. This mode is stable, noiseless,[1] and allows positive ion sheaths from a negative grid to span the grid holes and choke off tube current. In practice this mode of discharge is achieved by proper selection of the size of the grid openings, gas, gas pressure, and overall tube geometry. Aside from having a relatively opaque grid the tube is very similar to conventional ones with respect to geometry, efficiency, and operating conditions.

It is fairly well known that thyratron current can be interrupted by application of a reasonably small grid bias, providing that the tube current is very small, i.e., one per cent, or so, of the rated current. It has also been known[5] that there are certain conditions under which current cutoff with much larger currents can be achieved. These conditions have to do with gas, gas pressure and grid geometry. As will be seen, these conditions are included in a set of broader conditions that include overall tube geometry. In particular, it will be shown that the conditions for practical current cutoff are synonomous with those for noise-free operation.

In brief, current cutoff cannot be generally achieved in a conventional thyratron with any reasonable grid bias because the action of an increasing grid bias in expanding positive ion sheaths around the grid wires is largely cancelled by a feedback process that tends to decrease the size of the grid sheaths by causing an increase in the positive ion current to the grid wires. In the tube to be described this effect is minimized by restricting the site of ion generation to the grid-anode region.

Description of Typical Experimental Tacitron and Its Operating Characteristics

A photograph of a tube of the 100 milliampere size is shown in Fig. 1. In this photograph the top end mica has been removed and a portion of the shield electrode has been peeled back. In the shield compartment at the right of the photograph can be seen the 50-mil diameter oxide-coated cathode. Directly to the left of this is a grid mesh composed of 50 openings per linear inch. In this particular tube this mesh is electrically tied to the shield electrode. Facing this mesh, with a lead that comes from the stem through a glass insulator, is the anode.

A cross-section of the tube along with the basic operating circuit is shown in Fig. 2. The supply potential V and the resistor R limit the anode current to the value I. The grid is connected to a bias potential V_g.

The volt-ampere characteristic of the tube, as traced out by the usual 60-cycle sweep technique, is shown in the lower half of the photograph in Fig. 3. Here the anode potential is measured horizontally and the anode current vertically. The origin on the voltage axis is marked by a short vertical line. The scale can be judged from the fact that anode current occurs at about 13 volts and the maximum anode

[1] Throughout this paper the word "noise" will be taken to mean the coherent and incoherent anode voltage fluctuations, in the frequency range between several kilocycles and about 100 kilocycles, that are commonly found in hot cathode gas tubes. These fluctuations often produce interference at several hundred megacycles.

[2] L. Malter, E. O. Johnson, and W. M. Webster, "Studies of Externally Heated Hot-Cathode Arcs, I - Modes of the Discharge", *RCA REVIEW*, Vol. 12, pp. 415-435, September 1951.

[3] W. M. Webster, E. O. Johnson, and L. Malter, "Studies of Externally Heated Hot-Cathode Arcs, II - The Anode Glow Mode", *RCA REVIEW*, Vol. 13, pp. 163-182, June 1952.

[4] M. J. Druyvesteyn and F. M. Penning, "Electrical Discharges in Gases", *Rev. Mod. Physics*, Vol. 12, pp. 148-150, April 1940.

[5] . Fetz, "On the Control of a Mercury Arc by Means of a Grid in the Plasma", *Ann. der Phys.*, Vol. 37, pp. 1-40, January 1940.

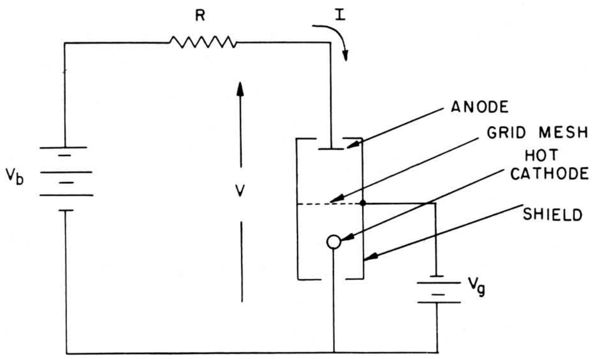

Fig. 2 – Cross-section of experimental Tacitron, together with test circuit.

current has a value of 400 milliamperes. It is seen that the anode voltage remains almost constant as the current increases smoothly up to a value close to the maximum. Beyond this value the discharge becomes unstable and gives rise to the voltage fluctuations evidenced by the broadening of the top portion of the characteristic. For comparison, the volt-ampere characteristic of a comparable commercial tube is shown in the upper portion of the same photograph along with the same current and voltage scales. It is seen that in the latter case the anode voltage fluctuations have a value of about 10 volts. It is just this sort of voltage fluctuation that gives rise to the troublesome noise and interference which commonly emanate from hot-cathode gas tubes. For example, when a tube having this noisy sort of volt-ampere characteristic is operated as a

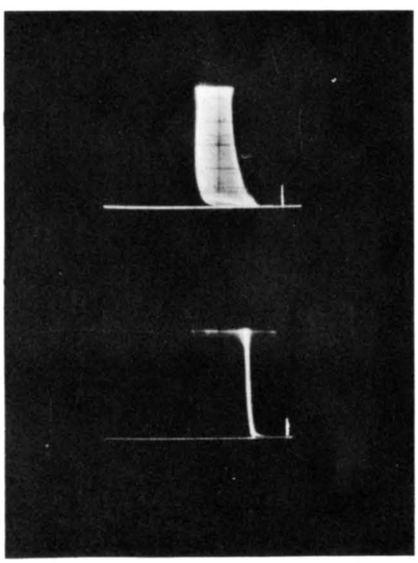

Fig. 3 – Oscilloscope trace showing volt-ampere characteristics of conventional thyratron (above) and Tacitron (below).

half-wave, 60-cycle rectifier near a television receiver, the picture is often badly disrupted. On the other hand, a tacitron, even when its anode lead is wrapped around the antenna terminals of the receiver, causes no noticeable interference.

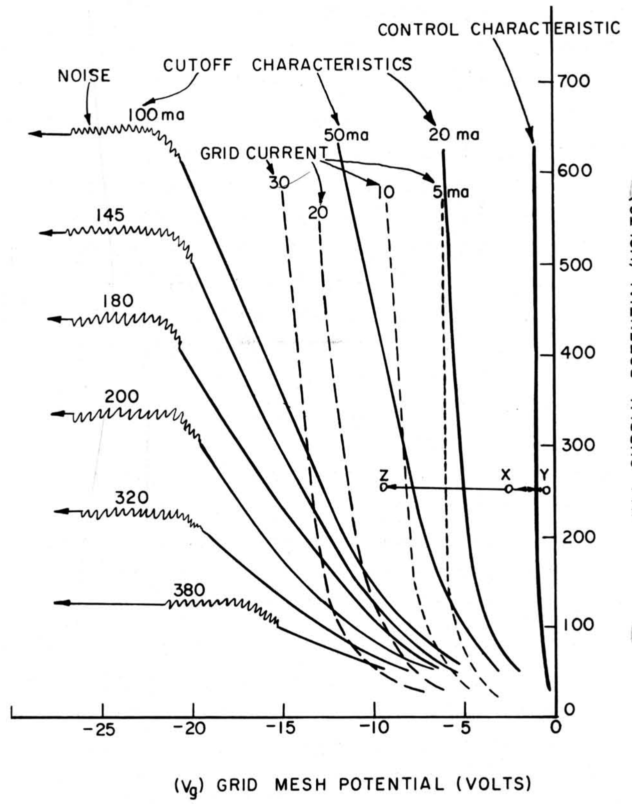

Fig. 4 – Firing and cut-off characteristics of experimental Tacitron of form shown in Fig. 1.

In Fig. 4 is plotted the combined firing and cutoff characteristics of a typical experimental tube such as the one shown in Fig. 1. The circuit from which these characteristics were obtained is the same as the one shown in Fig. 2. The solid curve on the extreme right represents the conventional control characteristic, or firing curve: the other solid curves to the left of this comprise the current cutoff characteristic. The dashed curves are the grid current characteristics which will be discussed later. To illustrate the use of the combined firing and cutoff curves, suppose that the unfired tube has grid and anode voltages represented by point x on Fig. 4. Now, with the supply voltage held constant, suppose that the bias is reduced from its value at point x to a new value indicated by point y. As with conventional tubes the tube will have fired when the control characteristic was crossed. If it is now wished to cut off a tube current of say, 50 milliamperes, it is necessary only to increase the grid bias until some point z, located to the left of the 50-milliampere curve, is reached. The current cutoff characteristic is independent of the rapidity of application of the grid bias.

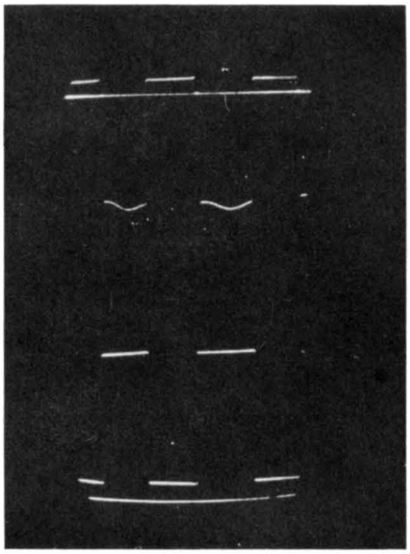

Fig. 5a – Grid and anode potential traces of Tacitron. Grid potential is alternated between minus 15 and minus 3 volts.

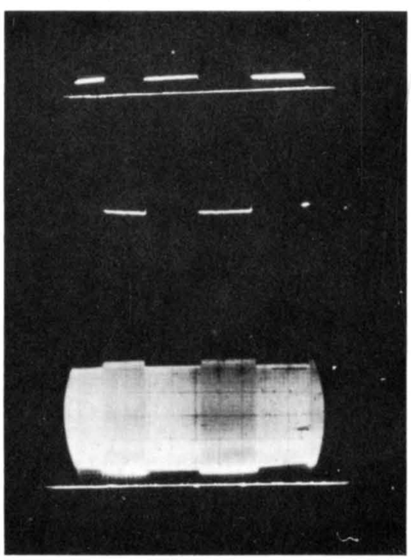

Fig. 5b – Same traces but for standard thyratron.

The wavy lines on the extremities of the cutoff characteristics indicate the presence of noise, such as that shown on the top of the volt-ampere characteristic of the lower curve in Fig. 3. As is apparent from the accompanying saturation in the cutoff characteristics, the presence of this noise is accompanied by an abrupt increase in the difficulty of achieving current cutoff. Practical operation of this tube is, therefore, limited to grid bias values which are smaller than about 15 volts.

The dashed lines in Fig. 4 are lines of constant grid current. These currents were measured when the tube was on the verge of cutoff. For example, when V_b = 300 volts and the anode current is 50 milliamperes, the grid bias voltage required for cutoff is 8.3 volts. The corresponding grid current is 10 milliamperes. The grid currents noted in the figure are reduced by a factor of three to five if the grid mesh is not connected to the shield electrode, and the latter is connected to the cathode.

The top photograph in Fig. 5 gives a more direct illustration of the cutoff action. The top half of this photograph is an oscilloscope picture of the grid voltage V_g in the circuit of Fig. 2 modified so that a switch operating at a 60-cycle rate, sets the grid bias at a 15-volt value* for half of a 60-cycle period, and then at a positive value of about 3 volts for the other half of the cycle. The unbroken horizontal line represents zero, or cathode potential. In the lower half of the same photograph is shown the simultaneous course of the tube's anode potential. The unbroken horizontal line again represents cathode potential. The anode potential is low when the grid is positive, and high when the grid is negative. In the first instance, the anode potential has a value of about 13 volts and the tube is passing a current of 100 milliamperes. In the second instance, the anode is at the 200-volt supply potential and the anode current is zero.

No detailed studies have been made of the speed of cutoff other than noting, in cases such as described above, that the anode potential rises from the 13 to the 200-volt level in about half a microsecond. This time is tied up with circuit constants as well as with the dynamics of the ion sheaths which accomplish the current cutoff. There is no reason why the

*The wavy character of the 15-volt grid bias is caused by ripple in the grid voltage supply.

the propagation of these ion sheaths, and hence the actual cutoff cannot be made to proceed arbitrarily fast depending on how large and fast a grid bias one is willing to apply.

The photograph in the lower half of Fig. 5 is included to show what happens when cutoff of anode current is attempted with the thyratron which yielded the volt-ampere characteristic in the upper half of Fig. 3. Here the grid voltage scale is the same as in the preceding case and the amount of expansion of the anode voltage scale can be judged with reference to the 28-volt peak of the anode voltage fluctuations. The only effect that the application of bias has is the slight change in the fluctuation amplitude. In fact, this is all that happens even with a grid bias of 100 to 200 volts.

With respect to such things as heater power, peak inverse voltage, firing conditions, and life the tube does not appear to differ in any essential way from a comparable conventional tube. Tube life, as with other thyratrons, depends upon tube currents, the applied voltages, and the frequency of operation. These parameters are paramount in determining the severity of such deteriorating agents as sputtering and gas cleanup. A typical experimental tube of the sort previously described can be expected to have a life of about 800 hours when subjected to 420 cycles per second operation with the waveforms and also the current and voltage values associated with the top photograph in Fig. 5. Tube deterioration inevitably shows up as an increasing tube drop and a decreasing value of the peak noiseless current. Along with these symptoms there is a slow drift in the cutoff and control characteristics to lower grid bias values.

Whereas a tube of the 100 to 200 milliampere size has been described, experimental tubes of both larger and smaller sizes have also been successfully operated. For example, tubes have been constructed to handle an average current of about a quarter of an ampere. These tubes interrupt peak currents of about half an ampere in a circuit where the post-conduction anode potential was several kilovolts. The principles of operation thus appear to be applicable to tubes of a wide range of size.

Besides being useful in applications where conventional thyratrons are now used, this tube offers the possibility of use in many new applications which are presently denied conventional thyratrons, either because of noise or because of low speed of operation due to deionization time considerations. In particular, some possible new applications for the tube are magnetic core switching in computer circuits, fast-operating circuit breaking, and low impedance pulse work.

Theory

Noiseless Operation

Studies[2,3,4] of the discharge mechanism in hot-cathode gas diodes have defined four distinctly different modes in which the discharge may operate. Each of these is distinguished by the pattern of the observable glow and the interelectrode space potential. Generally, as the current in a hot-cathode gas diode is increased, the discharge will operate in one mode after the other in the idealized volt-ampere sequence illustrated in the left of Fig. 6. The basic circuit for obtaining this sequence is shown at the right of the same figure. This idealized sequence of modes is not usually observed in practice but, rather, has been pieced together from observations made on many tubes of different geometries, pressures, and noble gas fillings. Briefly, the different modes noted in the figure can be described as follows: (1) the "anode glow" mode, as the name implies, is visually characterized by a glow which exists only immediately adjacent to the anode surface; (2) the "ball of fire mode", as the name implies, is readily recognized by the presence of a sphere of luminosity, whose size is roughly inversely proportional to the gas pressure; (3) the "Langmuir" mode exists when the whole interelectrode space is suffused with luminosity except for a thin region immediately in front of the cathode surface; (4) the "temperature-limited" mode exists when the glow fills the entire interelectrode space and, as implied by the name, the cathode current is temperature limited. Since the last two modes are of no interest in this report, it may simply be noted that (1) the onset of the Langmuir mode occurs when the circuit current is roughly one-half of the free field emission of the hot cathode and

(2) in both the Langmuir and temperature-limited modes the space potential close to the cathode surface is approximately equal to the tube drop.

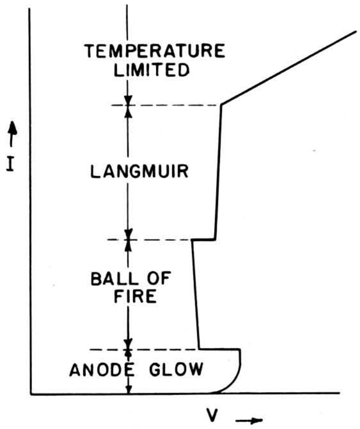

Fig. 6a - Volt-ampere characteristic of indirectly-heated hot-cathode-arc diode.

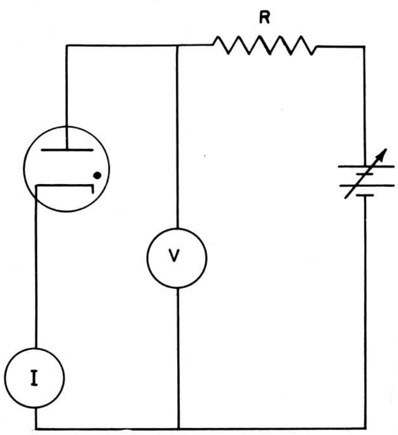

Fig. 6b - Circuit used in obtaining experimental characteristic of Fig. 6a.

It seems quite certain that commercial hot-cathode gas tubes normally operate in some combination of the first three modes, particularly the ball of fire and Langmuir, since the anode glow mode is usually not found when the tube current is greater than several per cent of the rated current. This is unfortunate because the anode glow mode is the only one that can be relied upon for noiseless operation: the other modes are almost inevitably noisy and so unstable that the discharge usually jumps erratically from one to the other. Thus, the problem in getting noiseless hot-cathode gas tube operation becomes one of finding how to make the tube remain in the anode glow mode up to appreciable currents. Before discussing the practical solutions to this problem it is expedient to treat the anode glow mode in somewhat more detail.

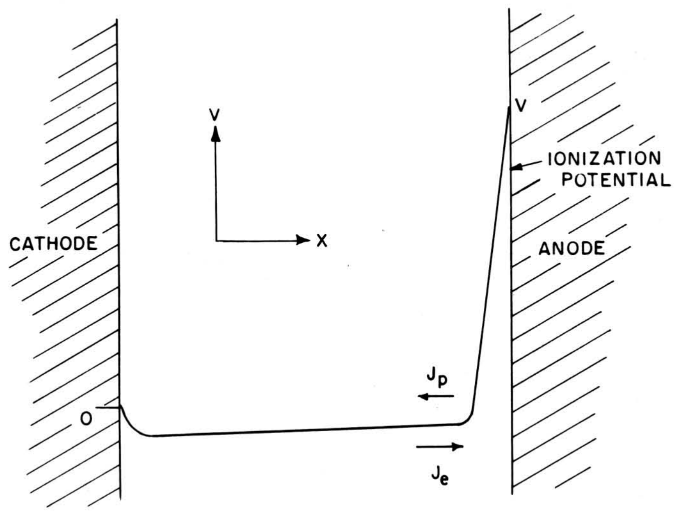

Fig. 7 - Potential distribution between hot cathode and anode of diode gas discharge operating in anode glow mode.

In Fig. 7 is shown the potential distribution between the cathode and anode of a tube operating in the anode glow mode. The main volume of the interelectrode space contains a plasma, and is characterized by very small electric fields. Nearly all of the applied voltage appears across the sheath, or high-field region, which separates the plasma from the anode. Only here do electrons attain sufficient energy to ionize and excite gas atoms, the latter phenomenon producing the characteristic glow pattern from which the name of the mode was derived. It is to be noted in the figure that the anode potential V is somewhat greater than the ionization potential. The ions generated in the sheath fall back into the plasma and sustain it against diffusion losses[6] of the ions to the end micas and other insulating surfaces where surface recombination takes place. The ions are trapped between the cathode and anode by the positive potential barriers at these electrodes. However, certain portions[3] of an oxide cathode surface do not have the potential barrier shown in the figure and so can provide a surface for recombination.

[6]Under the conditions of these experiments volume recombination losses are negligible.

As will be pointed out, this factor plays an important role in the operation of the tube, particularly at higher anode currents. Electrons from the hot cathode enter the plasma against the retarding potential indicated in the left of the figure. This potential is necessary if the current demanded by the circuit is less than the emission capability of the cathode. The cathode electrons diffuse through the plasma to the edge of the sheath where they are "pulled" toward the anode. Since the cathode electrons enter the plasma against a retarding potential they arrive in the plasma with a temperature corresponding to that of the cathode. With no significant energy sources or sinks in the plasma the plasma electrons remain in thermal equilibrium with the cathode. Consequently, it is not unexpected that this plasma is stable and free from the oscillations that are often generated[7] when a stream of energetic electrons is fired into a plasma as is the case with both the ball of fire and Langmuir modes. A detailed analysis[3] of the anode glow mode shows that it has the electrical characteristics of a non-linear positive resistance with no obvious tendency for instability.

From both analysis and experiment it appears that there are four conditions that must be satisfied if the anode glow mode is to exist. The first two of these set upper and lower gas pressure limits. These limits are very wide and include the usual pressures used with hot-cathode gas tubes. The third limit requires that the saturated cathode emission current be at least twice the circuit current. This, too, is easily satisfied in practice. The fourth condition is really the crucial one. It states that the ratio of ion to electron current densities at the anode edge of the plasma must always be less than either of two quantities described by the relations

$$\frac{J_p}{J_e} < \frac{\mu_p}{\mu_e} F \qquad (1)$$

$$\frac{J_p}{J_e} < S \sqrt{\frac{m}{M}} \qquad (2)$$

[7] See for example, G. Wehner, "Plasma Oscillator", *J.A.P.*, Vol. 21, pp. 62-63, January 1950.

Here, J_p is the ion current density flowing out of the electron sheath at the anode, J_e is the electron current density flowing into this sheath from the plasma, μ_p is the ion mobility in the plasma, and μ_e is the electron mobility in the plasma, F is dimensionless factor involving plasma geometry and the mean free paths of the plasma electrons, m and M are the electron and ion masses, respectively, and S is a factor involving anode sheath geometry. The first relation arises from a lack of capacity of the plasma in the vicinity of the anode to carry the required electron current: the second arises because of space-charge considerations within the anode sheath. If either of these relations is violated the electron sheath will move away from the anode toward the cathode, and the discharge will shift to the ball of fire mode. In that case a plasma, which is a virtual anode, bridges the gap between the new location of the sheath and the anode surface. For infinite plane-parallel structures $F = 1$, for a cylindrical case with an inner hot cathode surrounded by an anode $F < 1$, and for the reverse cylindrical case $F > 1$: the same can be said for the factor S. The mobilities are functions of gas pressure, but the ratio is not, having an essentially constant value for a given gas. Practically all of the electron current is carried through the tube by electrons, so that J_e is equal to the circuit current divided by the anode area. Thus, since the mobility and mass ratios are fixed, the only way to insure satisfying both relations (1) and (2)* is to make F and S as large as possible and the current ratio as small as possible.

To minimize the current ratio the ion current density, J_p and hence the ion losses in the tube, should be minimized. This can be realized in a "trapped" structure wherein negative electrodes, micas, and any other surfaces that can accept an ion current are eliminated. A practical realization of this goal consists of the cylindrical diode shown in Fig. 8. Here, because of the anode end plates, the ion losses are cut to zero[6] except for the stray losses to the oxide cathode mentioned earlier. An experimental tube of this

*Since the right-hand members of these relations are roughly equal, it is not possible to say which relation is the more critical.

sort passed well over an ampere of anode glow current. With the end plates made several volts negative, to insure ion losses at these electrodes, the maximum anode glow current was only 10 milliamperes. This approach to noiseless operation is not particularly practical because of the sensitivity to cathode quality. A slight deterioration in cathode uniformity, as inevitably occurs during life, causes a large relative increase in ion loss and so causes a significant reduction in the maximum noiseless current.

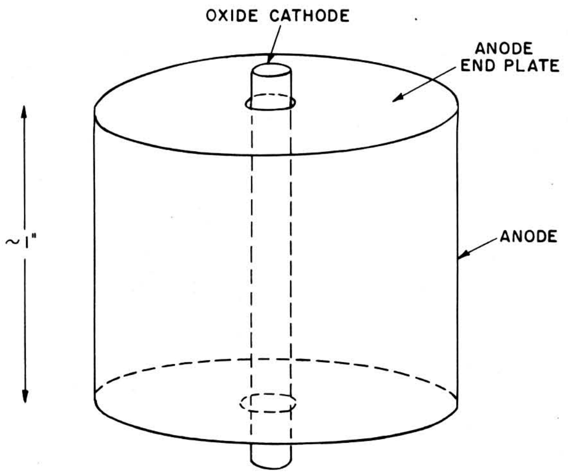

Fig. 8 – Experimental gas diode with metal end plates connected to the anode to achieve effective ion trapping. This insures operation in the anode glow mode up to high current values.

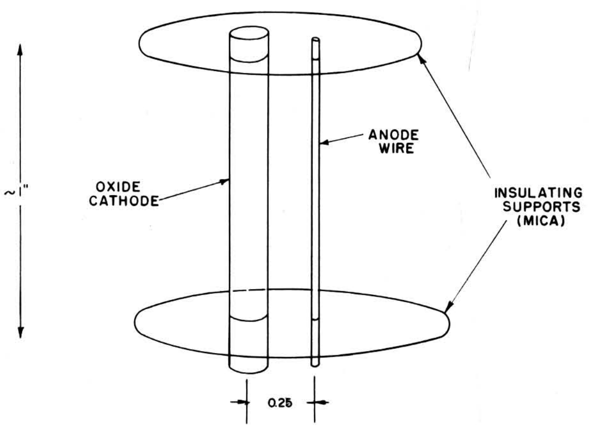

Fig. 9 – Experimental gas diode with wire anode to effect anode-glow operation up to high currents.

Another approach to the construction of noiseless gas diodes is the use of a geometry which makes the factors F and S large. This is most practically accomplished by a tube of the sort shown in Fig. 9, where a wire of about 20 mils diameter replaces the usual cylindrical anode which surrounds the cathode. The use of the fine wire anode is, mathematically speaking, an approach to the case where a cylindrical cathode surrounds a coaxial cylindrical anode. Noiseless anode glow currents of an ampere or so are readily obtained with the structure shown in the figure. Visually, the anode glow mode in these tubes is manifested by a glow which closely hugs the anode wire.

Now the more important problem of achieving noiseless operation with a three-element gas tube, such as the thyratron, will be considered. This is a more complicated problem than with the diode. Studies of thyratron discharges show, however, the existence of modes comparable to the anode glow and ball of fire modes in the diode. Further, the mode which compares to the anode glow mode is noise-free

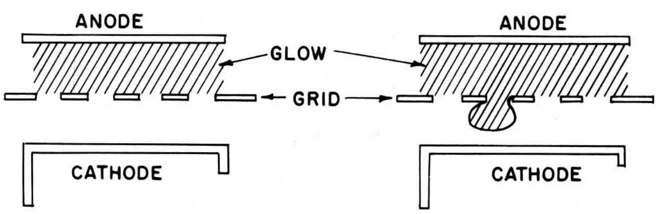

Fig. 10a – Gas discharge glow pattern in case of triode operating in analogue to anode-glow mode.

Fig. 10b – Gas discharge glow pattern in case of triode operating in analogue to ball-of-fire mode.

and allows the grid to interrupt the tube current: the mode which can be compared to the ball of fire mode is noisy and very difficult to cut off by grid action. In the anode glow mode analogue of the thyratron, ionization and excitation take place solely in the region between the grid and anode. Visually, this is evidenced by the uniform glow which fills the grid-anode region and the absence of glow in the cathode-grid region. This glow pattern is exemplified by the sketch shown in the left of Fig. 10. In the right of the same figure is shown the ball of fire analogue, wherein a ball of luminosity protrudes through a grid opening into the cathode space. Severe fluctuations of the anode potential always attend the appearance of this luminous protuberance.

The potential diagram in Fig. 11 corresponds to the anode glow, or quiet mode which will hereafter be referred to as the "tacitron" mode to avoid confusion with the anode glow mode in diodes. There are two regions of plasma in this discharge. The one in the cathode region has a potential slightly negative with respect to that of the cathode; the one in the anode region has a potential about equal to that of the anode. The conditions in the cathode-grid plasma are identical to those in the anode glow plasma of the diode. The ions which supply this plasma are generated in the grid-anode region and fall through the double sheath* which exists in the plane of the grid wires.

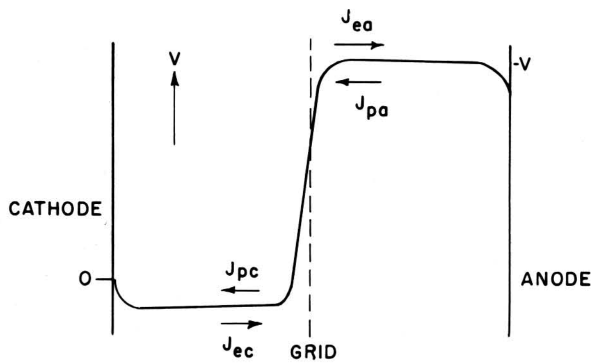

Fig. 11 - Potential distribution in triode operating in analogue to anode-glow mode.

In an ordinary thyratron, the grid generally has one or a few large apertures and the discharge will operate in the tacitron mode for currents which are only a few per cent of the rated current. If, however, the grid is made to have a large number of small holes, whose size is properly related to the gas, gas pressure, and overall tube geometry, noise-free conduction in the tacitron mode can be obtained up to much larger currents and, in addition, the grid cutoff action at substantial values of plate current is possible.

Although the details of the discharge phenomena in the three element tubes are difficult to deal with, the overall picture is fairly well understood and capable of treatment in terms of the diode operation. As with the diode the ratio of ion to electron current density is of cardinal importance and, as implied in Fig. 11, these densities have different values on either side of the grid. These differences arise because the grid intercepts or blocks some of the charged particles flowing from one plasma to the other. If ϕ_p is defined as the effective transparency of the grid to ions and ϕ_e as the effective transparency to electrons, then

$$J_{pc} = \phi_p J_{pa} \tag{3}$$

and

$$J_{ea} = \phi_e J_{ec} \tag{4}$$

Here, J_{pc} is the ion current density entering the cathode plasma, J_{ec} is the electron current density leaving this plasma, J_{ea} is the electron current density entering the anode plasma, and J_{pa} is the ion current density leaving the anode plasma. On the cathode side of the grid conditions are identical with those in the diode case so that for tacitron operation it is necessary that:

$$\frac{J_{pc}}{J_{ec}} < \frac{\mu_p}{\mu_e} F_c \tag{5}$$

On the anode side of the grid, the plasma boundary conditions are inverted and, therefore, the inverse of (5) must hold:

$$\frac{J_{pa}}{J_{ea}} > \frac{\mu_p}{\mu_e} F_a \tag{6}$$

The quantities F_c and F_a correspond to the F in Eq. (1), being written with respect to the appropriate plasma. Both of these quantities are approximately unity and, to a first approximation, we have assumed that the mobility ratios are the same in both plasmas. Eqs. (3) and (4) combine to give

$$\frac{J_{pc}}{J_{ec}} = \phi_p \phi_e \frac{J_{pa}}{J_{ea}} \tag{7}$$

Thus, if ϕ_p and ϕ_e are less than unity, the inequalities (5) and (6), which require that

$$\frac{J_{pc}}{J_{ec}} < \frac{J_{pa}}{J_{ea}} \tag{8}$$

may be satisfied. In this way the grid acts as a "matching" device between the two plasmas. An

*A sheath containing a counter flow of electrons and ions is often known as a double sheath.

exact computation of ϕ_p and ϕ_e requires an intimate knowledge of the electric field in the grid openings and has not been accomplished. There is little doubt, however, that the transparencies are less than the optical transparency, which is obviously less than unity for any physically realizable grid.

Both J_p and J_e will vary across the grid openings and will have maximum values in the center. These maximum values, which might be comparable to the current densities present in the absence of a grid, would not satisfy both (5) and (6). These densities are, however, averaged out shortly after the particles enter the plasmas. In the analysis, the critical inequalities apply to ions and electrons flowing according to the laws of diffusion and not to particles injected at high velocity into a plasma. Thus, they do not apply to the ion and electron currents flowing through the grid until the individual particles have been scattered by collisions. By the time the injected particles have lost their incident velocity, they are spread out over a larger area than the grid holes and the maximum values of J_{pc} and J_{ea} are reduced. The degree to which this averaging action takes place depends on the ratio of the grid hole dimension to the mean free paths of the injected ions and electrons. The implications are that tacitron operation will fail if the largest grid hole dimension is greater than the mean free path of the charged particles passing through these holes. Since the mean free paths are inversely related to gas pressure, an upper limit is set on the gas pressure for a given size grid aperture. This critical limit is reasonable from the standpoint that a plasma finds it easier to exist if the channel or hole through which it passes has a lateral dimension larger than the mean free path of the plasma particles. Fetz[5], who has considered essentially the same question, attributes the onset of the ball of fire analogue to an unstable condition which arises in the sheath at the grid hole. This instability is presumably due to the ion current, arising from generation in the region near the grid hole, being too large to satisfy the mass ratio condition expressed in Eq. (2). The theory and experiments of Fetz show an inverse relation between gas pressure and grid hole size and, furthermore, predict magnitudes which are of the same order as those computed below from a simple relation between grid hole size and mean free paths. Thus, while the failure of the tacitron mode with increasing pressure can be interpreted in various ways[8], all of these give approximately the same answer.

Using the notion of equivalence between mean free paths and grid hole size, the maximum pressure p, in millimeters of mercury, that can be tolerated by a tube with a multi-holed grid is given by

$$p = \frac{G^*}{d} \tag{9}$$

where d is the major dimension of a grid hole, and G is taken as the electron mean free path at a pressure of one millimeter of mercury. Since Eq. (9) is only a rough criterion there is no real reason to use either the electron or ion mean free path in preference to the other. For convenience the values of G for the various noble gases are tabulated below in Table I.

Table I

GAS	G*	W
Helium	0.12	1.0
Neon	.083	1.8
Argon	.014	13
Krypton	.011	-
Xenon	.007	25

The general relationship† that exists between tube drop, gas pressure, gas, size of grid openings, and tube geometry for experimental structures is shown in Fig. 12. In order to keep the curve general for all the noble gases the ratio of tube drop V to the ionization potential V_i is used as ordinate. The tube parameter quantity pK is used for the abscissa. The quantity p is the gas pressure in milli-

[8] Another reason, which may be important in certain cases, stems from the high pressure restriction noted earlier. Briefly, this restriction applies because Eqs. (1) or (2) are violated by the excessive ion current needed to supply the larger aggregate of plasma particles which can collect at the higher pressures.

*Landolt-Bornstein, ZAHLENWERTE UND FUNKTIONEN AUS PHYSIK UND CHEMIE, J. Springer, Berlin, 1950.

†For the case of zero grid bias.

meters of mercury and the K is given by the semi-empirical relation

$$K = W \sqrt{ab}\phi \qquad (10)$$

where W is a gas constant whose empirically determined value is given in Table I, a is the average grid-anode distance in centimeters, b is the average cathode-grid distance in centimeters, and ϕ is the effective or empirical, mesh transparency whose values for different commonly used meshes are tabulated below in Table II.

Table II

Mesh Size (holes per inch)	Wire Size (mils)	Transparency Optical	Effective (ϕ)
16 x 16	12	0.66	0.66
24 x 24	8.5	.635	.52
40 x 40	6.5	.55	.25
50 x 50	7	.44	.10

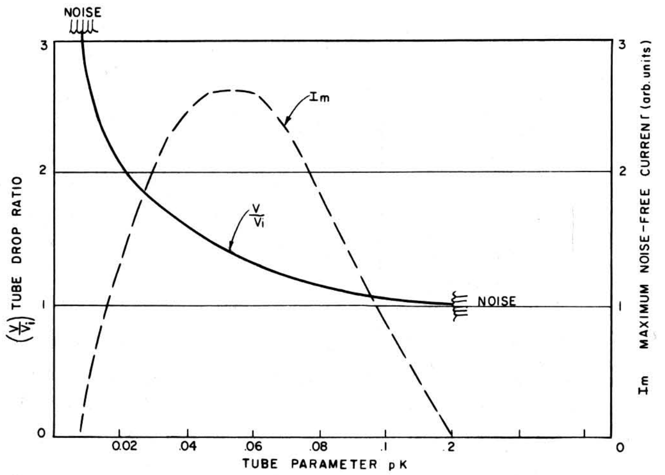

Fig. 12 - Variation of tube drop and maximum noise free current with tube parameters and gas pressure.

Eq. (10) is based on the analysis given in the appendix and has been found to predict the the voltage ratio to within about 25 per cent for the noble gases listed in Table I over the following range of geometric variables: a, between 0.1 and 3 centimeters; b, between 0.1 and 1.5 centimeters; ϕ, over the range noted in Table II. The type of structure used in compiling these results is shown in Fig. 13. No unusual precautions were taken in the construction of these tubes other than making sure that the top and bottom mica supports fitted tightly against the ends of the grid mesh cylinders.

The onset of noise at the low pressure end of the curve, where operation is in the transition range between gas and vacuum tube operation, probably originates from the low pressure restriction on the anode glow mode mentioned earlier. This end of the curve is of little technical interest because the high tube drop causes such a severe combination of sputtering, gas cleanup, and cathode bombardment that tube life is short. Operation at the $V/V_i = 2$ point where $pK = 0.02$, is advantageous for plasmatron operation.[9]

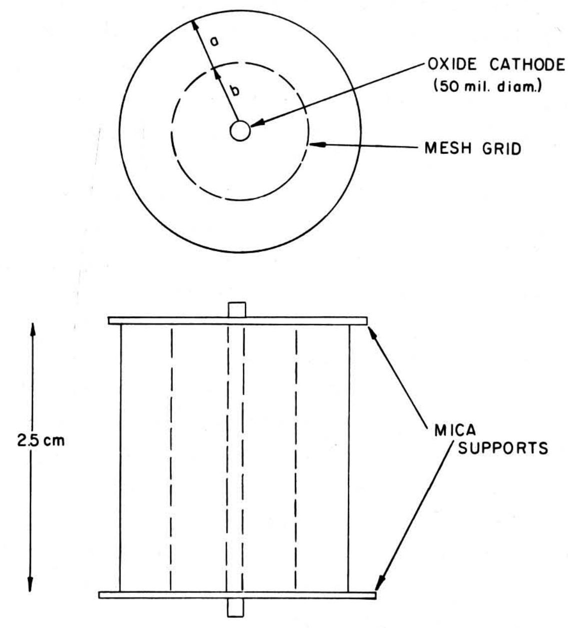

Fig. 13 - Experimental structure used in arriving at results of Fig. 12.

The onset of noise at the high pressure end of the curve is more reliably predicted by Eq. (9) than by the value of pK indicated on the abscissa.

The dotted curve in Fig. 12 is a rough indication of the behavior of the maximum noiseless current as a function of the variable pK. This current is necessarily low near the high pressure end of the curve, because here the ratio of ion to electron current density is very close[8] to the critical value given by either Eq. (1) or (2). Thus a small increase

[9]E. O. Johnson and W. M. Webster, "The Plasmatron, A Continuously Controllable Gas-Discharge Developmental Tube", *Proc. I.R.E.*, Vol. 40, pp. 645-659, June 1952.

in anode current increases the ion losses at the cathode sufficiently for the critical value to be reached. As the value of pK is reduced the value of this current ratio is decreased. Hence, larger anode currents can be tolerated before increased ion losses, due to the cathode shortcomings noted earlier, cause the critical ratio to be reached. The maximum current should start to decrease at a low value of pK because of the low pressure restriction mentioned earlier and, also, because of the additional ion losses at the cathode caused by the excessive ion mean free paths at low pressures. Here, ions falling through the drop at the grid make so few collisions with neutral atoms in the grid-cathode region that these ions retain, on the average, sufficient energy to overcome the small potential barrier at the cathode.

In the design of a tube for either noiseless rectification or grid cutoff operation the best compromise between low tube drop, maximum noiseless current, and reserve pressure occurs at a value of pK of about 0.07.

Whereas the curve in Fig. 12 is empirically based on the cylindrical geometry shown in Fig. 13, it can be used as an effective guide for the design of structures of other geometries. For example, the tube shown in Fig. 1 has the following parameter values: ϕ = 0.3, a = 0.5 cm, and b = 0.5 cm. Since the gas is xenon at a pressure of about 0.05 millimeters of mercury, the value of pK is 0.133 and the curve in Fig. 13 predicts a voltage ratio of about unity, a value that compares well with the actual observed value of approximately unity. In general, anything done to a tube structure to increase ion loss, or to make ion generation more difficult, will inevitably tend to increase the tube voltage drop.

As for mechanical construction, tubes with the noiseless feature are no more difficult to fabricate than other gas tubes. Care has to be taken, however, to insure that there are no holes or spaces along the edges of the mesh grid that have a major dimension larger than the mesh holes. If such openings are present, the discharge will inevitably choose these for exit and become noisy, as well as extremely difficult to interrupt by grid action.

The effect of tube deterioration on the voltage drop and the maximum noiseless current, noted in Part II, is largely tied up with cathode behavior. Either sputter deposit on this electrode or damage by ion bombardment will act to increase the ion losses of the plasma in the cathode-grid region. This results in an increase in tube drop since there has to be a compensating increase in the rate of ion generation. Along with this, the maximum noiseless current will decrease because the ratio of ion to electron current density in Eqs. (1) and (2) will more closely approach the critical value. Gas cleanup simply moves the operation of the tube to the left along the curve in Fig. 13 and results in an increase in tube drop. However, as can be seen from the dotted curve, it will not normally decrease the noiseless current.

Cutoff Operation

Cutoff action results from the closure of the grid holes by a space-charge sheath which is opaque to electrons attempting to move from the cathode to the anode region. The thickness of this sheath can be derived from the relation

$$j_p = 3.43 \times 10^{-7} \frac{V_y^{3/2}}{r_g B^2 M^{1/2}}$$

which is the familiar 3/2 power law[10] for cylindrical geometry. Here j_p is the ion current density in amperes per square centimeter to a grid wire of radius r_g centimeters, V_g is the potential in volts across the sheath, B is a function[10] whose value depends upon the ratio of the sheath radius r_s to the grid wire radius r_g, and M is the ion mass (oxygen 16). The solution of this equation is facilitated by nomograph technique,[11] and by this means the empirical equation

$$Z = \frac{r_s}{r_g} = 1 + \frac{1}{10}\left[\frac{V_g^{3/2}}{r_g j_p}\right]^{1/2} \qquad (11)$$

was derived.

The transparency ϕ_e of the mesh grid to electrons from the cathode plasma can be ap-

[10] I. Langmuir and K. B. Blodgett, "Currents Limited by Space Charge Between Coaxial Cylinders", *Phys. Rev.*, Vol. 22, pp. 347-356, October 1923.

[11] L. Malter and W. M. Webster, "Rapid Determination of Gas Discharge Constants from Probe Data", *RCA REVIEW*, Vol. 12, pp. 191-210, June 1951.

proximately represented in terms of geometrical considerations by the relation*

$$\phi_e = (1 - Znd)^2 , \qquad (12)$$

where d is the mesh wire diameter in inches, and n is the number of mesh openings per linear inch of a mesh with square holes.

Before Eq. (11) can be substituted into Eq. (12), to get an explicit relation for ϕ_e in terms of grid geometry and the grid bias V_g[12], the quantity j_p has to be replaced by a more convenient parameter. It will become obvious later that j_p is a feedback term, since an increase in V_g sets up a reaction whereby j_p also increases. The ultimate behavior of ϕ_e hence depends upon the nature of this feedback process as well as upon V_g.

Using Eq. (2) of the Appendix, along with the usual kinetic theory value for the ratio of ion and electron current densities (at the grid edge of the cathode plasma) it is found that

$$j_p = \frac{1}{A\sqrt{M/m}\,\sqrt{T_e/T_p}} \frac{I}{\phi_e}$$

or

$$j_p = Y \frac{I}{\phi_e}$$

where the quantity Y is introduced for convenience. Here, A is the grid mesh area facing the cathode plasma, I is the anode current in milliamperes, M/m is the ratio of ion to electron mass, and T_e/T_p is the ratio of electron to ion temperature in the cathode plasma. For the tubes considered in this bulletin $A \cong 1$ square centimeter, $\sqrt{M/m} = 490$ (xenon), $\sqrt{T_e/T_p} \cong 1.7$ and, therefore $Y \cong 10^{-3}$. Now, if the above value for j_p is introduced into Eq. (11) one obtains:

$$Z = 1 + \frac{1}{10}\left[\frac{V_g^{3/2}\phi_e}{r_g Y I}\right]^{1/2}$$

or

$$Z = 1 + \frac{1}{10}\left[U\phi_e\right]^{1/2} \qquad (13)$$

The quantity U, which equals $\left[\frac{V_g^{3/2}}{r_g\, YI}\right]$, is introduced for convenience. By substituting Eq. (13) into Eq. (12) there is finally obtained

$$\phi_e = \frac{(1 - nd)^2}{(1 + nd/10\,\sqrt{U})^2} \qquad (14)$$

This expression makes it clear that the mesh transparency decreases monotonically as the U increases. Thus, if the grid bias V_g is increased and I is either held constant or allowed to decrease, the mesh transparency must always decrease. The numerator of the equation will be recognized as the expression for the optical transparency of the mesh.

If the electron transparency ϕ_e decreases the tube drop V must go up according to the relation*

$$\frac{V}{V_i} = 1 + \frac{1}{\left[(\alpha V_i) \quad f\lambda(1-e^{-\frac{d}{\lambda}}) \frac{AC\tau}{4v\psi} \quad (\phi_p\phi_e) \quad p\right]} \qquad (15)$$

where all the terms other than ϕ_e remain roughly constant. As will be shown, the grid transparency to ions ϕ_p undergoes little change because, compared to the tenth-volt mean electron energy of the electrons in the cathode plasma, the ions falling out of the anode plasma each have about V volts of energy, and hence can pass through most of sheath even when the grid bias potential is somewhat greater than V. As the tube drop increases, the tube current decreases in the circuit of Fig. 2 according to the relation

$$I = \frac{V_b - V}{R} \qquad (16)$$

where, as noted in the figure, V_b is the supply potential and R is the circuit resistance.

When V reaches some value, say about one-half of V_b, the discharge in the tube becomes unstable and the anode current suddenly falls

[12]Aside from a possible discrepancy of several volts due to contact potentials and several other second order effects, the voltage across the sheath can be taken as being equal to the grid bias. It is to be noted that both V_g and j_p must be related to the cathode plasma, since the sheath conditions on the cathode side of the grid are the only ones directly operative in blocking electron flow from the cathode plasma.

*The transparency ϕ_e, as here defined, has a somewhat different meaning than ϕ_e as used in Eq. (4).

*Appendix, Eq. (4).

to zero no matter how slowly the bias had been applied. In particular, this instability is found to arise when[5,13]

$$\frac{dV}{dI} = -R \qquad (17)$$

This behavior can be traced out in Fig. 14, wherein the volt-ampere characteristics are plotted for different grid bias potentials. The straight line connecting the supply potential V_b and the zero bias operating point o corresponds to the load resistance R. As the grid

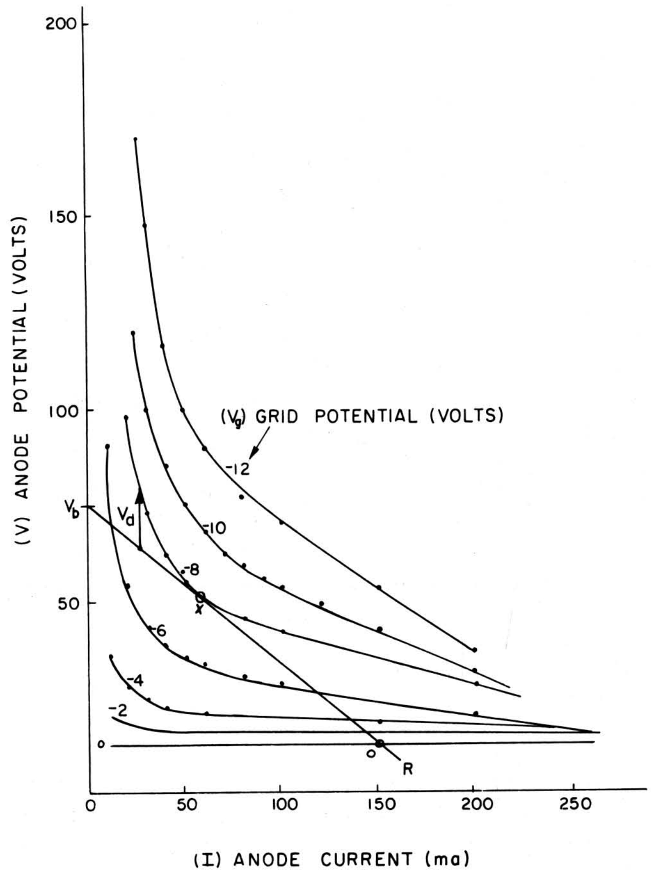

Fig. 14 - Volt-ampere characteristics of an experimental Tacitron for various grid potentials.

bias is increased the tube has current and voltage drop values which mark the intersections between the load line R and the appropriate tube curve for the grid bias potential in question. At the point x, where the grid bias potential is -8 volts and the load line is tangent to the tube curve, Eq. (17) applies and the current I suddenly and spontaneously falls to zero. The process proceeds regeneratively to the left of point x because a decreasing current causes an increase in the tube's voltage deficit (V_d in the figure) which, in turn, causes a decrease in ion generation that leads to increased sheath size, hence further current reduction, and so forth.

[13] See, for example, J. D. Cobine, GASEOUS CONDUCTORS, pp. 207-209, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

The negative resistance of the tube, obvious from Fig. 14, has its origin in Eqs. (14) and (15). In these, a decreasing tube current results in a decreasing mesh transparency and, so, a higher tube drop. The process is stable as long as the circuit resistance exceeds the negative resistance of the tube. The tube curve corresponding to zero grid bias is seen to have no indication of negative resistance. This is as it should be, because if V_g is zero, the sheath parameter U is zero regardless of current, and so the tube drop cannot change with current. On the other hand, a large value of grid bias allows a large variation in U so that the tube drop can change rapidly with I.

If the cutoff criterion, Eq. (17), is used in conjunction with the set of curves in Fig. 14, the cutoff characteristics displayed in Fig. 4 can be reconstructed fairly closely.

It is instructive to consider the curves shown in Fig. 15. The anode current I, the tube drop V, the positive ion current to a negative probe in the cathode region, and the positive ion current to a negative probe in the anode region are plotted as a function of grid potential. The positive ion current to these probes, in accordance with usual probe theory, is a measure of the ion density in the region surrounding the probes. Now, if the grid bias is increased, the grid becomes more opaque to electrons by virtue of Eq. (14), and the anode voltage must rise according to Eq. (15). Under the conditions presented in the figure the ion generation goes up more rapidly with tube drop than it goes down with tube current, so that the ion current measured by the probe in the anode region is seen to rise. The ion current to the probe in the cathode region is seen to rise in almost the same proportion. This indicates, as previously mentioned, that the transparency of the grid to the ions from the anode region changes very little with grid bias.

By combining Eq. (16) with the ion generation equation* it is easily shown that the maximum in ion generation occurs when

$$V = V_m = \frac{V_b + V_i}{2} \tag{18}$$

The generation increases with V when V is less than V_m: it decreases when V exceeds V_m. Experiment has verified this behavior. In fact, it can be checked in Fig. 14 where the probe current, and hence ion generation, is seen to be approaching a maximum when V = 57 volts. Since V_b = 110 volts and V_i = 10 volts, V_m = 60 volts, a value in agreement with the observed value.

The tendency for a tacitron to become noisy at high grid bias, as indicated by the wavy lines in Fig. 4, can be understood from a consideration of Fig. 15. From this latter figure it is apparent that the ion current flowing from the grid-anode plasma into the cathode-grid plasma must increase with grid bias since the ion density, as measured by the cathode probe ion current is increasing. The anode electron current I, however, is seen to be decreasing. Under these conditions the current ratio J_p/J_e in Eqs. (1) and (2) must increase and tend to reach the critical value for the onset of noise. From Eq. (3)* which specifies the rate of ion flow through the grid wires into the cathode plasma, there is obtained

$$\frac{I_p}{I} = f\phi_p \alpha p (V - V_i)\lambda\left(1 - e^{-\frac{d}{\lambda}}\right) \tag{19}$$

First of all, the appropriate parameter values† inserted into the right hand member of this equation show that it has a value of about 4.7×10^{-3} which is the same order of magnitude as the approximate value, $\sqrt{m/M} = 2 \times 10^{-3}$, for the critical current ratio in xenon. Secondly, since all the factors on the right except $(V - V_i)$ remain roughly constant for a particular tube, the onset of noise should ensue at a given value of $(V - V_i)$. This can be shown to be approximately true by making the appropriate graphical construction in Fig. 14 with the noise onset parameter values shown in Fig. 4. In making this graphical construction the curves in Fig. 14 have to be extrapolated and the current and voltage scales extended. The practical consequences of the onset of noise at a given value of $(V - V_i)$ dictate that the alterable factors in the right member of Eq. (19) should be decreased if practical operation is to be achieved at high values of V_b. The alterable factors are the mesh transparency to ions ϕ_p, the gas pressure p, and the grid anode distance d. The electron mean free path λ cancels out if it is large compared to d. Experience with tacitrons of various designs bears out the above conclusions.

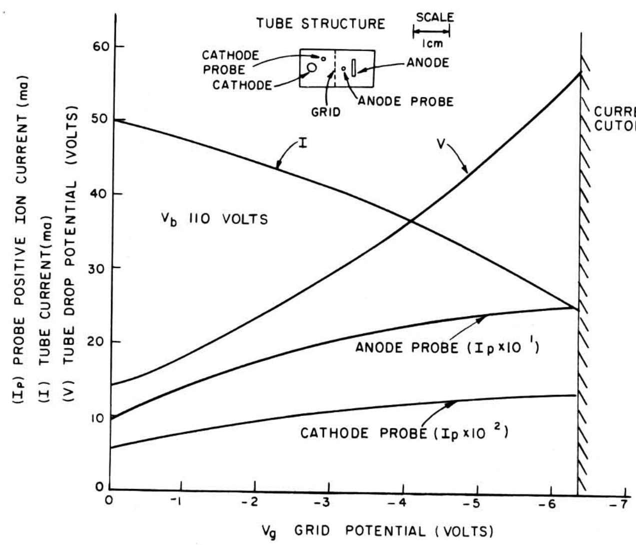

Fig. 15 – Anode potential and current, and probe currents in experimental Tacitron as functions of grid bias.

Next comes the question of why practical current cutoff is not observed in conventional thyratrons when operated at appreciable values of anode current. In short, this question can be answered by the statement that such tubes do not operate in the tacitron mode. Compared to the other possible discharge modes for a three-element hot-cathode gas tube, the tacitron mode possesses three important advantages with respect to current cutoff. These are: (1) the ion generation mechanism is such that the feedback action on the sheath expansion process seems to be a minimum, (2) the sheath expansion takes place in a plasma which is characterized by low particle density and low electron

*Appendix

†Here f ≅ 1/6 from geometrical considerations, ϕ_p ≅ 0.44 for a 50 x 50 mesh, α ≅ 1.0 for xenon, p ≅ 0.05 mm, $(V-V_i)$ ≅ 10 for a typical case, λ ≅ 0.25 cm, and d ≅ 0.5 cm.

energies, and (3) the tube current is distributed over an interelectrode path of relatively large cross-section area.

With respect to the first feature, it is always observed in a tacitron that practical cutoff ceases almost simultaneously with the onset of noise and the abrupt appearance of the luminous protuberance shown in Fig. 10. Observation of conventional thyratron operation has shown the existance of an analogous situation. As the grid bias is increased in an attempt to cut off the tube current, this protuberance is seen to increase both in brilliance and size. Accompanying probe measurements show that the ion density in the tube increases so rapidly that the factor $V^{3/2}/j_p$ in Eq. (11) stays constant, or even decreases. This implies that the mesh transparency is staying constant, or even decreasing. Apparently, then, the application of grid bias causes such an increase in ion generation, amounting to a strong negative feedback on the sheath expansion process, that cutoff is impossible with any reasonable bias. This fact could well explain the very large biases of hundreds of volts[5] required to cut off such discharges.

The second feature has to do with the fact that the cathode plasma of the tacitron is populated with free electrons having a mean energy of about a tenth volt. This is a consequence of their being in thermal equilibrium with the hot cathode. It is hard to conceive of any discharge mode where the mean electron energy, and hence case of repulsion from the sheath, could be any less than this. On the other hand, in any plasma where ion generation is taking place there must be electrons present with energies of the order of ten volts. Accompanying this consideration of electron energy is the one involving plasma density. The density in the cathode plasma of the tacitron, and consequently the j_p term in Eq. (11), must be a minimum because ion generation is taking place elsewhere in the tube. The probe measurements presented in Fig. 15 illustrate this point by showing how much greater the positive ion currents in the anode plasma are compared to those in the cathode plasma.

When the tube current is distributed over a large cross-section area, then the plasma density, hence j_p, can be relatively low at each grid hole. This obviously makes for ease of sheath closure compared to the luminous protuberance case where all of the tube current passes through one, or at best, a few grid holes. Aside from the triode mode equivalent of the Langmuir mode in the diode, which is impractical from the standpoint of the second feature mentioned above, the tacitron mode is the only one which allows thisdesirable spreading of the current path.

The grid input impedance of the tube can be roughly approximated by a two-branch circuit containing two resistances and one capacitor. It is not practical to generalize the values of these elements because they are a complicated function of time, tube design, and operating conditions. Accordingly, the values for one specific case are presented here to give an idea of what might be expected. Here, the tube is as shown in Fig. 1, V_b = 200 volts, R = 1900 ohms, and I = 100 milliamperes. In this case, the first branch is composed of a 620-ohm resistor R_1: the second branch is composed of an 1800-ohm resistor R_2 in series with a 3300-μμf capacitor C. Now, if the grid bias is applied slowly, the positive ion current from the anode plasma accounts for R_1. If the bias is applied suddenly, the sheath capacitance coupling the grid to the anode plasma gives rise to the capacitance C. The load resistance R, which connects the anode back to ground, and hence the anode plasma, is mainly responsible for R_2. To a first approximation the role played by the cathode plasma can be neglected, because it has a far smaller density than the anode plasma. This lower density obviously means that there is little effect on R_1. It also means, because of thicker sheaths between it and the grid, that the contribution to C must be small. As can be deduced from the grid current curves in Fig. 4, the resistance R_1 varies inversely as the tube current. To a first order, C is unaffected by tube current.

* * *

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Appendix

In deriving an expression for tube voltage drop as a function of the gas and geometrical parameters the approach in this simple analysis will be to equate ion generation to ion loss. The ion loss current I_{pl} from the cathode plasma can be expressed in terms of the mean ion lifetime τ due to ambipolar diffusion, and the total number of ions present, by the relation

$$I_{pl} = \frac{e\int_v n\,dv}{\tau} = \frac{e n_g v\psi}{\tau} = \frac{4I}{CA\phi_e}\,\frac{v\psi}{\tau} \qquad (1)$$

where e is the electronic charge, n is plasma density, v is the total plasma volume, n_g is the average plasma density at the grid edge of the cathode plasma, I is the anode electron current, C is the mean thermal velocity of the plasma electrons, A is the total grid area facing the cathode plasma, and ϕ_e is the effective transparency of the grid to electrons from the cathode plasma. It is found convenient to introduce the plasma density distribution factor ψ, which is a dimensionless number defined by the relation

$$\psi = \frac{1}{n_g v}\int_v n\,dv$$

The factors A, ϕ_e, n_g, and C are introduced by means of the usual kinetic theory notion of space current by the relation

$$A\,\phi_e\left(\frac{n_g e c}{4}\right) = I \qquad (2)$$

where the term in the parenthesis is the electron space current density j_e at the grid edge of the cathode plasma.

The ion loss current I_{pl} is balanced by an equal ion current I_{pg} that flows through the grid holes from the anode plasma, the site of all ion generation in the tube. The latter current can be expressed, in terms of differential ionization by the relation

$$I_{pg} = f\phi_p I\alpha p(V-V_i)\int_0^d \varepsilon^{-\frac{x}{\lambda}}dx = f\phi_p I\alpha p(V-V_i)\lambda(1-\varepsilon^{-\frac{d}{\lambda}}) \qquad (3)$$

where f is the fraction of ions generated in the anode plasma that get to the grid edge of this plasma, ϕ_p is the fraction of these ions that are not lost to the grid wires and which find their way into the cathode plasma, α is the pressure normalized differential ionization coefficient, p is the gas pressure, V is the potential between the cathode and anode plasmas (which is closely equal to the tube drop) V_i is the gas ionization potential, ε is the base of natural logarithms, x is the distance from the grid plane to the differential distance dx, d is the effective grid-anode distance, and λ is the electron mean free path for the voltage V and the pressure p.

If Eq. (1) is equated to Eq. (2) there obtains:

$$\frac{V}{V_i} = 1 + \frac{1}{\left[(\alpha V_i)\left(f\lambda(1-\varepsilon^{-\frac{d}{\lambda}})\right)\left(\frac{AC\tau}{4v\psi}\right)(\phi_p\phi_e)\;p\right]} \qquad (4)$$

which gives a curve of the form shown in Fig. 13. The factors within the brackets can be compared to the factors which compose the parameter K of Eq. (10). Thus, to a fair approximation, (αV_i) corresponds to W, $(f\lambda(1-\varepsilon^{-\frac{d}{\lambda}}))$ to $\sqrt{a}$, $(AC\tau/4v\psi)$ to r, and $(\phi_p\phi_e)$ to ϕ. The product of the factors within the brackets is independent of pressure to the first order. The quantities f and λ depend upon the tube drop V: the quantities τ, ψ, and ϕ_e depend upon the tube current I. It should also be noted that when V exceeds about twice V_i the ionization increases less rapidly than predicted by the factor $(V - V_i)$.