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LB-917

ON THE VARIATION OF JUNCTION TRANSISTOR CURRENT-AMPLIFICATION FACTOR WITH EMITTER CURRENT

RADIO CORPORATION OF AMERICA

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Approved

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Introduction

Existing theories of the junction transistor fail to predict the very significant variation of current-amplification factor, α_{cb}, as the emitter current is varied. This variation has been very troublesome in power transistors, particularly at high emitter currents where the α_{cb} fall-off may be so severe as to limit usefulness. At low current, α_{cb} also drops off, an effect of importance in very low-power applications. By taking into account the modification of base region by the injected charge carriers, an explanation was found for the observed variation. Electric fields in the base region decrease the mean transit time for minority carriers on their way to the collector. This reduces the effect of surface recombination and increases current-amplification factor as the emitter current rises. Another effect, however, is in the opposite direction; this second effect is due to an increase in conductivity of the base material which increases the rate of volume recombination and also lowers emitter efficiency. The combination of these effects yields calculated curves which show a maximum and agree well with experiment. The work is applicable to both p-n-p and n-p-n types, and it is shown that the latter is inherently less sensitive to emitter current density.

General Discussion

As the current density in a junction transistor is increased, a number of second order effects make themselves felt. The most noticeable of these is the change in current-amplification factor. In the typical transistor, the current gain increases, goes through a maximum, and finally decreases steadily as the emitter current is increased. Fig. 1 shows this variation for a typical p-n-p alloy junction transistor.

Fig. 1 - Variation of current-amplification factor, α_{cb}, with emitter current for a typical p-n-p alloy transistor.

It is the purpose of this bulletin to show that the variation of current gain with emitter current can be accounted for by the change in the characteristics of the base material produced by the injected carriers. Previous transistor theory ignored such effects in that the injected charge density was assumed to be small compared to the density of ionized impurity atoms. This is not often the case. A simple calculation will show that the injected-charge density in the base region of a TA-153 transistor[1] is about equal to the impurity-charge density when the emitter current is of the order of one milliampere. (This is a current density of about one ampere per square centimeter.) Since similar transistors are sometimes used at currents in excess of ten milliamperes, the second order effects are not negligible.

The most important changes produced by the injected charge are these:

(1) A small field is developed in the base section which has the same effect as an increased diffusion coefficient for injected (minority) carriers.

(2) The conductivity of the base section is increased. This has an important effect on emitter efficiency and volume recombination.

The first sections of this bulletin will review briefly the aspects of transistor theory which need amendment and derive the proper corrections in simple terms. The remainder will consist of appendices where more complete derivations will be found.

Throughout, derivations will apply to the p-n-p transistor. The final equations have the same form for the n-p-n transistor and are obtained by simply interchanging a few subscripts. Data apply to germanium transistors.

II. Current-Amplification Factor

A current-amplification factor, α_{ce}, is defined as the variation of collector current, I_C, in response to a change in emitter current, I_E, with the collector voltage, V_C, held constant. These currents are defined in Fig. 2. This can be expressed mathematically as:

$$\alpha_{ce} \equiv \left.\frac{\partial I_C}{\partial I_E}\right|_{V_C} = \text{constant.} \qquad (1)$$

Another current-amplification factor, α_{cb}, can be defined as

$$\alpha_{cb} \equiv \frac{\partial I_C}{\partial I_B} = \frac{\alpha_{ce}}{1-\alpha_{ce}}$$

[1]This type is described in LB-868, *Germanium p-n-p Junction Transistors.*

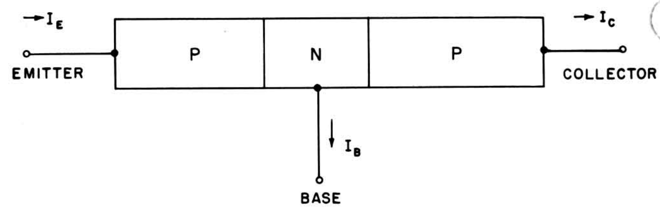

Fig. 2 - Current convention used to define α_{cb} and α_{ce}.

If α_{ce} is nearly unity, as is the case for a useful junction transistor,

$$\alpha_{cb} \doteq \frac{\partial I_E}{\partial I_B} \doteq \frac{1}{1-\alpha_{ce}} \qquad (2)$$

α_{cb} is a more sensitive parameter than α_{ce} and, since it is easier to measure accurately, permits a more reliable comparison between theory and experiment. Further, it is a little easier to connect α_{cb} with the physics of the junction transistor.

Fig. 3 shows a sketch of the potential distribution through a p-n-p transistor and, schematically, the paths taken by the holes and electrons therein. The arrows on the paths indicate the direction of particle motion. The emitter current is composed of holes injected into the base (I_{Ep}) and electrons extracted from the base (I_{Ee}). Some of the holes which constitute I_{Ep} recombine on their way through the base with electrons which enter through the base lead. The recombination rate defines a current I_R. The collector current is composed of the holes which did not recombine and a "saturation" current, I_{co}, composed primarily of holes and electrons produced spontaneously by thermal energy in both the base and collector region and a leakage current across the collector junctions. For a good transistor, I_{co}, I_R, and I_{Ee} should all be small compared to I_{Ep}. Now,

$$I_B = I_R + I_{Ee} - I_{co}$$

and

$$I_E = I_{Ep} + I_{Ee} \doteq I_{Ep}$$

since $I_{Ep} >> I_{Ee}$. The equation which describes the change in I_B for an incremental change in I_E with collector voltage constant is:

$$\frac{\partial I_B}{\partial I_E} \doteq \frac{\partial I_R}{\partial I_{Ep}} + \frac{\partial I_{Ee}}{\partial I_{Ep}} - \frac{\partial I_{co}}{\partial I_{Ep}}$$

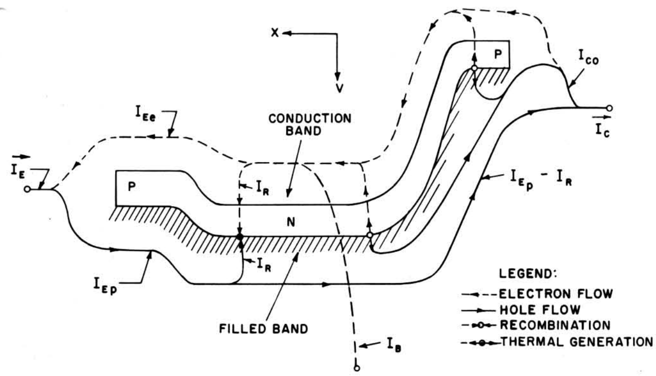

Fig. 3 - Potential distribution in a p-n-p transistor showing, schematically, the paths followed by holes and electrons.

There is no physical reason why the thermal generation and leakage should vary with I_{Ep} so that $\partial I_{co}/\partial I_{Ep}$ may be set equal to zero. Thus,

$$\frac{\partial I_B}{\partial I_E} \doteq \frac{\partial I_R}{\partial I_{Ep}} + \frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{1}{\alpha_{cb}} \tag{3}$$

This means that α_{cb} can be determined if it is known how recombination and the fraction of emitter current composed of electrons vary with the number of holes injected.

The recombination term should be separated into two parts, one due to surface recombination and the other to volume recombination because the two mechanisms obey different equations. If this is done, an expression may be written for $1/\alpha_{cb}$ which contains three terms which may be investigated independently.

$$1/\alpha_{cb} = \frac{\partial I_{SR}}{\partial I_{Ep}} + \frac{\partial I_{VR}}{\partial I_{Ep}} + \frac{\partial I_{Ee}}{\partial I_{Ep}} \tag{4}$$

Here, SR refers to surface recombination and VR to volume recombination. The partial first-order theory previously published[2] gives the following values:

$$\frac{\partial I_{VR}}{\partial I_{Ep}} = \frac{I_{VR}}{I_{Ep}} = \frac{1}{2}\left(\frac{W}{L_b}\right)^2 \tag{5}$$

and

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{I_{Ee}}{I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} \tag{6}$$

[2] W. Shockley, M. Sparks, and G. K. Teal, "The p-n Junction Transistors", *Phys. Rev.*, Vol. 83, No. 1, pp. 151-162, July 1951.

where W is the width of the base region, L_b is the diffusion length for holes in the base region, σ_b and σ_e are the conductivities of base and emitter regions respectively and L_e is the diffusion length for electrons in the emitter region.

An expression for the surface recombination term, $\partial I_{SR}/\partial I_{Ep}$, is now needed to complete the first-order theory. It may be derived as follows: Nearly all of the surface recombination in an alloy junction transistor occurs in an area which is ring-shaped and surrounds the emitter "dot". The number of holes recombining depends on the product of s, the surface recombination velocity, the effective area, A_s; and the density of holes present near the surface, p. This is expressed by the equation $I_{SR} = esA_s p$ where e is the electronic charge. Since the area where major surface recombination takes place is very near the emitter, one sets p equal to the hole density at the emitter junction which will be called p_e. The first order theory assumes that holes flow purely by diffusion according to the law: $J_p = -eD_p \text{ grad } p = I_{Ep}/A$ where J_p is the hole current density and A is the cross-sectional area of the conduction path (which is about equal to the emitter area). For plane-parallel geometry where volume recombination may be neglected as far as its effect on hole density is concerned (the case for any useful transistor), this may be integrated to yield $p_e = I_{Ep}W/AeD_p$ which is substituted into the expression for I_{SR}. (The density of holes near the collector is so much smaller than the density near the emitter that it may be set equal to zero in evaluating the integral.) The result is:

$$\frac{I_{SR}}{I_{Ep}} = \frac{sA_s W}{D_p A} = \frac{\partial I_{SR}}{\partial I_{Ep}} \tag{7}$$

Since electric fields in the base region were neglected in solving for p_e, this expression is not exact. It is, in fact, of approximation comparable to the terms quoted above as Eqs. (5) and (6).

Now, the complete expression for $1/\alpha_{cb}$ according to the first-order theory may be written combining Eqs. (5), (6), and (7).

$$1/\alpha_{cb} = \frac{sA_s W}{D_p A} + \frac{\sigma_b W}{\sigma_e L_e} + \frac{1}{2}\left(\frac{W}{L_b}\right)^2 \tag{8}$$

None of the three terms depends on emitter current. Thus to explain experimental results, the basic assumptions which may be insufficiently exact must be further investigated. Two assumptions require revision. One of these states that electric fields in the base region may be neglected, the other, that the change in base region conductivity due to injected charge is trivial. These are both indirect consequences of the assumption that the injected hole density is small compared to the density of ionized donor atoms which, as indicated above, is not often the case.

III. Fields in the Base Region and Surface Recombination

In the first-order theory, where electric fields in the base region are neglected, hole current density in the base is given by:

$$J_p = -eD_p \text{ grad } p \qquad (9)$$

where J_p is the hole current density, e is the electronic charge, D_p is the diffusion coefficient for holes in the base region, and p is the hole density at any point. Electron current density is assumed equal to zero.

To include effects due to the electric fields in the base two equations are required:

$$J_e = ne\mu_e \text{ grad } V - eD_e\text{grad } n \qquad (10)$$

$$J_p = -pe\mu_p \text{ grad } V - eD_p \text{ grad } p \qquad (11)$$

Here, J_e is the current density of electrons in the base, n is the electron density, μ_e and μ_p are electron and hole mobilities, D_e and D_p are electron and hole diffusion coefficients and V is the electric potential.

To these equations can be added $n + N_a \doteq p + N_d$ where N_a and N_d are acceptor and donor ion densities. This equation stems from the fact that the net charge density in the base region must be essentially zero. In n-type material, N_a may be set equal to zero without serious loss of accuracy and N_d assumed to be a constant throughout the base. Thus $n = p + N_d$ and grad N = grad p.

If the transistor is to be useful, J_e must be very small compared to J_p. If $J_e = 0$, n replaced with $N_e + p$, and grad n with grad p Eqs. (10) and (11) may be combined as:

$$J_p = -(eD_p \text{ grad } p)\left(1 + \frac{p}{N_d+p}\right) \qquad (12)$$

Also,

$$\text{grad } V = \frac{D_e}{\mu_e} \cdot \frac{1}{(N_d+p)} \text{ grad } p = \frac{kT \text{ grad } p}{e(N_d+p)} \qquad (13)$$

If $p<<N_d$, Eq. (12) reduces to the first-order equation given as Eq. (9). However, as p increases, the current density increases more rapidly than grad p until, when $p>>N_d$, $J_p \doteq -2eD_p$ grad p. When this happens, half the hole current is carried by diffusion and half by the electric field. Under these conditions, reasonably accurate results may be obtained if the diffusion coefficient is multiplied by two wherever it appears in equations derived from the first-order theory. In the transition region where $0.01<p/N_d<100$, more careful calculation is required.

The origin of the electric field can be described in physical terms as follows: In order to pass a certain hole current, a hole density gradient is required. The condition of space-charge neutrality stipulates an equal electron-density gradient. The electron-density gradient would like to induce a flow of electrons in the same direction as the flow of holes. This happens, momentarily, until an unbalance of charge sets up an electric field to hold the electrons in place against their density gradient. The same field, however, acts in a direction to encourage hole flow and, in the limit, doubles the hole-current density for a given density gradient.

The field in the base region has a considerable effect on the surface recombination term. At low currents, the expression given above applies, i.e., sA_sW/D_pA. At high current density, however, D_p is effectively doubled and the per cent of the hole current lost by surface recombination is divided by two. The transition between the two cases accounts for the initial rise in current gain with emitter current. A calculation of the behavior in the transition region is given in Appendix I.

The result of the analysis of Appendix I is:

$$\frac{\partial I_{SR}}{\partial I_{Ep}} = \frac{sWA_s}{D_p A} g\ (z) \qquad (14)$$

where the function g (z), which will be called the "field factor", is plotted vs z in Fig. 4 and

$$z \equiv \frac{W\mu_e}{AD_p\sigma_b} I_E$$

For high values of z, i.e., greater than about 20, g (z) approaches 0.5; for z = 0, g (z) has the value of unity. Thus, the loss in current-amplification factor due to surface recombination decreases as emitter current is increased and approaches a final value of one-half the initial value. This causes the current gain to rise, initially, with emitter-current density at a rate which depends on the base width, W, the impurity type and conductivity of the base material.

In passing, it might be mentioned that the so-called diffusion capacitance of a junction transistor is reduced by the field in the base region by a factor of two as emitter current is increased. This has been observed to occur at about the same current as saturation of the curve for g (z) as it should, since it also depends inversely on the diffusion coefficient.

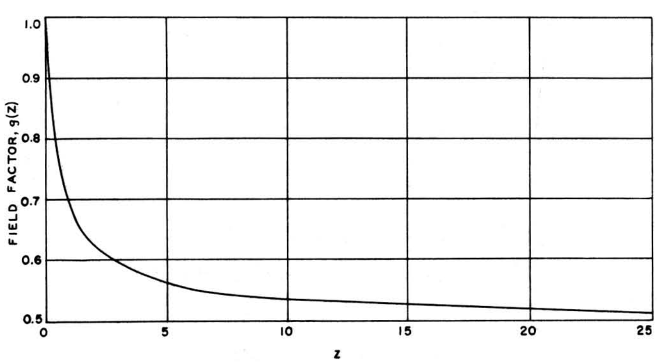

Fig. 4 – The field factor, g(z), as a function of z.

IV. Modulation of Base Region Conductivity by Injected Carriers

As pointed out before, the assumption that $p \ll N_d$ is incorrect in a transistor which is operating at a current density in excess of about 0.1 amperes per square centimeter. It is usually assumed that the number of electrons in the conduction band of the base region is N_d. Instead, $N_d + p$ should be used. A reasonably good correction to the first-order theory consists of simply multiplying the base conductivity by the ratio $(p+N_d)/N_d$ where it appears in the above equations for d-c current flow*. This ratio is that of the electron density in the base with injected holes present to the density in the absence of holes. One should use the value of p/N_d at the emitter which, for large currents, is given approximately by $I_{Ep}W\mu_e/2D_pA\sigma_b = z/2$. (see Appendix I). Thus, wherever σ_b appears in the d-c equations it should be multiplied by:

$$\frac{p+N_d}{N_d} = 1 + z/2$$

V. Emitter Efficiency

The partial first-order theory predicts[2]

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{I_{Ee}}{I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} \qquad (6)$$

This may be amended by writing instead,

$$\frac{I_{Ee}}{I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} (1+z/2)$$

Since z is a function of I_{Ep}, one cannot simply equate $\partial I_{Ee}/\partial I_{Ep}$ to this but must take the derivative as follows:

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} \left(1+z/2+\frac{I_{Ep}}{2}\frac{\partial z}{\partial I_{Ep}}\right)$$

$I_{Ep}\frac{\partial z}{\partial I_{Ep}}$, however, equals z since z and I_{Ep} are linearly related. Thus, the effect of differentiation is simply to double the term in z and

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} (1+z) \qquad (15)$$

*The reader is cautioned that this substitution is not valid for all transistor equations but only where σ_b has been substituted for the density of conduction electrons (or holes, in p-type material.) The underlying physics should be examined for each equation to make certain that this is the case.

This equation states that the emitter efficiency decreases, and hence, the current-amplification factor drops as emitter current is increased. Further, at high currents, the current-amplification factor α_{cb} should vary inversely with the emitter current as is observed experimentally.

A more exact calculation is made in Appendix II which also takes into account the changing field in the base region. The accurate calculation and the approximate one given here are both plotted in Fig. 5 against z. The difference between them is not great, being at most about 25 per cent and less than 20 per cent for z greater than about 4. Another approximation which is very good for z>2 may be obtained by extrapolating the exact solution to z = 0. This gives a straight line whose equation is (2+z). Use of this instead of (1+z) in the expression for $\partial I_{Ee}/\partial I_{Ep}$ gives pessimistic results for current gain at low currents but very good values for high currents.

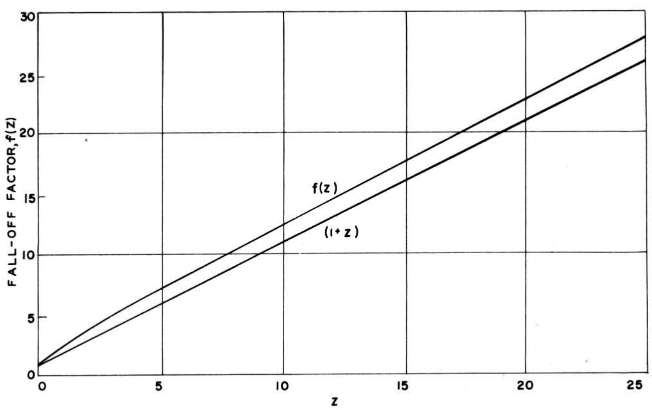

Fig. 5 - The fall-off factor, f(z), and (1+z) as functions of z.

VI. Volume Recombination

The expression for the variation of base current with emitter current due to volume recombination (Eq. 5) does not involve σ_b explicitly

$$\frac{\partial I_{VR}}{\partial I_{Ep}} = \frac{I_{VR}}{I_{Ep}} = 1/2\left(\frac{W}{L_b}\right)^2$$

However, L_b is dependent on base conductivity through the average hole lifetime since $L_b^2 = D_p\tau$. τ, when it applies to holes lost by bi-molecular* recombination, is inversely proportional to the number of electrons in the base region and, hence, inversely proportional to σ_b. Thus the expression for volume recombination may be corrected in the same way as the expression for emitter efficiency, i.e.,

$$\frac{\partial I_{VR}}{\partial I_{Ep}} = \frac{1}{2}\left(\frac{W}{L_b}\right)^2 (1 + z) \qquad (16)$$

Note that the volume recombination term exhibits the same dependence on emitter current as the emitter efficiency term.

Actually the volume recombination term ought to involve an integral of the base conductivity over the entire base region, whereas simply the value of base conductivity near the emitter was used. Appendix III gives a somewhat more exact treatment which is trivially different from the result obtained here. This is because most of the volume recombination takes place near the emitter where hole and electron densities are greatest. Both calculations apply to bi-molecular volume recombination. There is some evidence that volume recombination becomes mono-molecular at high values of z.[3] In this case, the variation with emitter current vanishes and the volume recombination term is replaced by $\partial I_{VR}/\partial I_{Ep} = W^2/4D_p\tau_m$ where τ_m is the lifetime for mono-molecular volume recombination at high injection levels.

VII. Current-Amplification Factor vs Emitter-Current Theory and Experiment

According to the preceding sections the approximate equation for current gain as a function of emitter current can now be written by combining Eqs. (14), (15) and (16):

*The use of the word bi-molecular in this case applies to recombination proceedings at a rate proportional to the product n-p. Mono-molecular refers to a recombination rate proportional only to p. The details of the recombination mechanism are not of concern at the moment since capture cross-sections and the like may be lumped into a single recombination coefficient.

[3] R. N. Hall, "Electron-Hole Recombination in Germanium", *Phys. Rev.*, Vol. 87, p. 387; 1952.

$$\frac{1}{\alpha_{cb}} \doteq \frac{sWA_s}{D_p A} g(z) + \left[\frac{\sigma_b W}{\sigma_e L_e} + \frac{1}{2}\left(\frac{W}{L_b}\right)^2\right](1 + z) \quad (17)$$

where

$$z = \frac{W\mu_e I_E}{D_p A\sigma_b} \quad (17a)$$

and g (z) is given by Fig. 4. The accuracy of this equation may be improved somewhat by using the calculated curve of Fig. 5 for the emitter efficiency term but the difference is not usually significant.

The above equation is for the p-n-p junction transistor. To apply it to an n-p-n transistor, all that is required is to change the subscripts on mobility and diffusion coefficients from e to p and vice-versa. Thus for the n-p-n transistor:

$$\frac{1}{\alpha_{cb}} \doteq \frac{sWA_s}{D_e A} g(z) + \left[\frac{\sigma_b W}{\sigma_e L_e} + \frac{1}{2}\left(\frac{W}{L_b}\right)^2\right](1+z) \quad (18)$$

where

$$z = \frac{W\mu_p I_E}{D_e A\sigma_b} \quad (18a)$$

Into the expressions for z (17a and 18a) may be substituted $D_p = kT\mu_p/e$ and $D_e = kT\mu_e/e$ where k is Boltzmann's constant and T is the temperature of the transistor. Also, μ_e/μ_p may be set equal to the constant b which has a value of about 2 for germanium. Then:

$$z = \frac{WeI_E}{kTA\sigma_b} \cdot b \text{ for the p-n-p transistor and}$$

$$z = \frac{WeI_E}{kTA\sigma_b} \cdot \frac{1}{b} \text{ for the n-p-n.}$$

One sees immediately that, if all quantities except conductivity type are the same (i.e., identical geometry, diffusion length and conductivities) α_{cb} should vary less with emitter current in an n-p-n transistor than in a p-n-p transistor by a factor of $b^2 \doteq 4$ (for germanium).

All the derivations above, and those in the appendices, apply to a geometry wherein the emitter and collector are assumed to be parallel planes and "end effects" are neglected.

The alloy junction transistor is a reasonable approximation to this case. Most of the quantities involved in the expression can be measured fairly exactly for the alloy transistor. "A" should be the actual emitter area and W approximately the base thickness measured by the capacitance methods.[4,5] σ_b and L_b are determined by the material used. D_p and μ_e are known constants. However, no accurate method for measuring the quantities s, A_s, σ_e and L_e in alloy junction transistors has so far been developed. What is done, then, is to use values for sA_s and $\sigma_e L_e$ which give the best agreement between theory and experiment. These values are reasonable ones and the agreement is good over a wide current range.

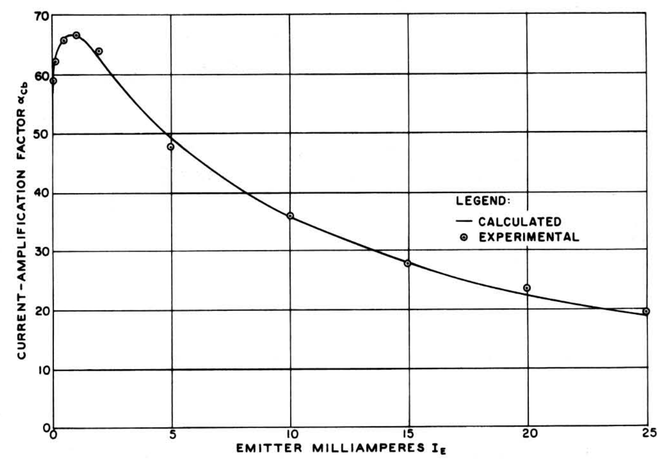

Fig. 6 - Comparison of theory and experiment for a typical p-n-p alloy transistor.

Fig. 6 shows the variation of current-amplification factor with emitter current for a TA-153 (p-n-p) transistor. The solid line is the computed curve and the points are experimental data. The following values were used for computation:

$W = 4.8 \times 10^{-3}$ cm (measured by the capacitance method)
$A = 1.1 \times 10^{-3}$ cm^2 (known emitter area)
$\sigma_b = 0.45$ mho/cm (known base conductivity)
$L_b = 0.14$ cm (calculated from lifetime of 500 μsec)
$\sigma_e L_e = 1.55$ mho (obtained from best fit)
$sA_s = .147$ cm^3/sec (obtained from best fit)

[4] See discussion in LB-915, *A p-n-p Alloy Junction Transistor for Radio-Frequency Amplification.*

[5] LB-900, *Equipment for Measurement of Junction Transistor Small-Signal Parameters for a Wide Range of Frequencies.*

D_p = 44 cm^2/sec and μ_e = 3600 cm^2/sec volt (known values)

from these:

$$\frac{\sigma_b W}{\sigma_e L_e} = .0014, \quad \frac{1}{2}\left(\frac{W}{L_b}\right)^2 = 6\times10^{-4}\,*$$

and

$$z = \frac{W\mu_e}{D_p A \sigma_b} I_E = 800\ I_E \text{ where } I_E \text{ is in amperes.}$$

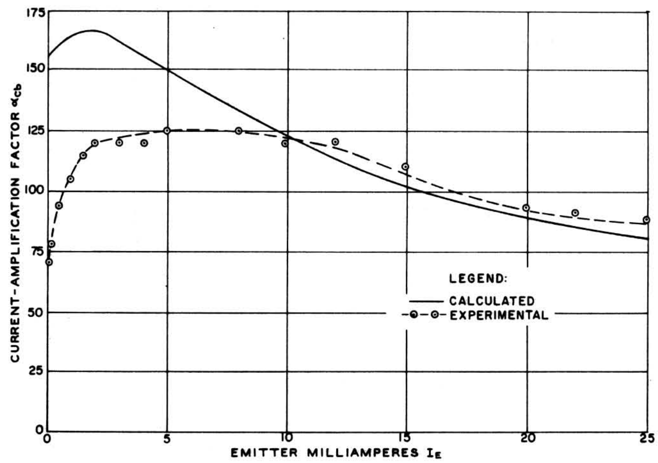

Fig. 7 – Comparison of theory and experiment for a typical n-p-n alloy transistor.

The value of sA_s is quite reasonable. Fig. 8 shows the paths of hole flow through a junction transistor as calculated using an analogue mapping technique.[8] The average TA-153 has a surface recombination velocity, s, of about 350 cm/sec. The analogue maps show that nearly all of the surface recombination occurs in a ring of width Δd around the emitter. This has an area, A_s, of about πd Δd where d is the emitter diameter. Further, the width Δd is of the order of W. For Δd = W = 4.8×10^{-3} cm and πd = 0.12 cm, A_s = $.58\times10^{-3}$ cm^2. Using s = 350 cm/sec, sA_s = 0.2 cm^3/sec. This is in very good agreement with the value 0.147 cm^3/sec used to match theory and experiment.

*R. N. Hall, in his paper "Power Rectifiers and Transistors" in the November, 1952 issue of the *Proceedings of the IRE*, ascribes the fall-off of current gain to the increase in volume recombination only. For the transistors described here, the decrease of emitter efficiency is more significant.

[8] LB-916, *The Variation of Current Gain with Junction Shape and Surface Recombination in Alloy Transistors.*

The value of $\sigma_e L_e$, however, seems to be a bit large. The existing evidence indicates a value of σ_e of about 10^3 mho/cm. An intelligent estimate based on an extrapolation of L from measurements made on relatively pure material to this value of conductivity would suggest that L_e should be about 10^{-4} cm. However, such an extrapolation is probably to be questioned more than the result obtained above.

Fig. 7 shows a similar comparison for a TA-154 (n-p-n) transistor[7] of essentially the same geometry. Here, the following values were used:

W = 4.6×10^{-3} cm (measured as before)
A = 1.1×10^{-3} cm^2 (known)
σ_b = 0.33 mho/cm (known)
L_b = .28 cm (assumed)
$\sigma_e L_e$ = 1.1 mho } obtained from best fit
sA_s = .11 cm^3/sec } obtained from best fit
D_e = 93 cm^2/sec and μ_p = 1700 cm^2/sec volt. (known values)

From these $\sigma_b W/\sigma_e L_e$ = .0014

$W^2/2L_b^2$ = 1.3×10^{-4}, and z = 215 I_E.

The values of $\sigma_e L_e$ and sA_s are about the same as for the p-n-p transistor, which is reassuring. However, the agreement at low currents is not nearly as good for the n-p-n as for the p-n-p. The logical interpretation is that some mechanism exists in addition to surface recombination which becomes less detrimental as the current density increases. It is quite conceivable that this may be either the transition from bi-molecular to mono-molecular volume recombination or evidence of a patch-effect at the emitter. In all, however, the agreement between the analysis and experimental data is not too bad.

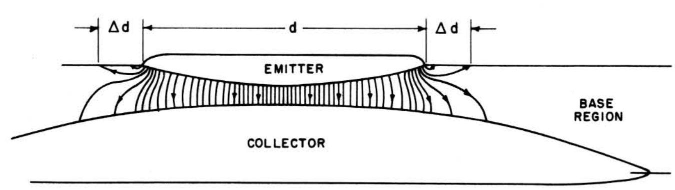

Fig. 8 – Analogue map of hole flow in an alloy transistor.

[7] LB-903, *A Germanium n-p-n Junction Transistor by the Alloy Process.*

VIII. Conclusion

A number of corrections to the first-order theory for current gain are required to explain the variation of current gain with emitter current. Three of these have been obtained by considering the change in base material conductivity with injected minority carriers and the fields produced in the base region by the injected current density. New terms for surface recombination, volume recombination, and emitter efficiency have been derived. The more complete theory gives reasonable agreement with experiment.

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Appendix I

Surface Recombination

From maps of hole flow in the alloy type transistor such as the one shown in Fig. 8 made by an analogue technique[8], it has been determined that nearly all the surface recombination takes place in a ring around the emitter "dot". If this ring has an area A_s, the number of holes recombining there per second gives rise to a current $I_{SR} = esA_s p_e$ where s is the surface recombination velocity and p_e is the hole density at the emitter. Further,

$$\frac{\partial I_{SR}}{\partial I_{Ep}} = seA_s \frac{\partial p_e}{\partial I_{Ep}}$$

In Section III the corrected expression for hole current density was derived:

$$J_p = -eD_p \left(1+\frac{p}{N+p}\right) \text{grad } p.$$

For the plane parallel case then,

$$\left[1 + \frac{p/N_d}{1+p/N_d}\right] \frac{d(p/N_d)}{dx} = \frac{-I_{Ep}}{eD_p N_d A} = \frac{-I_{Ep}\mu_e}{D_p A\sigma_b} = -\frac{z}{W}$$

where A is the cross-sectional area of the transistor for conduction and σ_b is the base material conductivity (without injected hole current). This equation assumes I_{Ep} is constant throughout the base region and is sufficiently accurate for a typical transistor.

This equation can be integrated to yield:

$$2\frac{P}{N_d} - \ln(1+p/N_d) = \frac{zx}{W}$$

if x = 0, when p = 0 (i.e., at the collector). For P/N_d small, this reduces to $P/N_d = zx/W$. For P/N_d large, it becomes $p/N_d \doteq zx/2W$. The density of holes at the emitter, i.e., when x = W, is given by

$$2\frac{p_e}{N_d} \cdot \ln(1+p_e/N_d) = z$$

In order to substitute this quantity into the expression for $\partial I_{SR}/\partial I_{Ep}$, $\partial p_e/\partial I_{Ep}$ is required. This is obtained as follows:

$$\frac{\partial p_e}{\partial I_{Ep}} = \frac{\partial p_e}{\partial z}\frac{\partial z}{\partial I_{Ep}} = \frac{\partial(p_e/N_d)}{\partial z} \cdot \frac{N_d W\mu_e}{D_p A\sigma_b} = \frac{\partial(p_e/N_d)}{\partial z} \cdot \frac{W}{eD_p A}$$

From the equation which relates p_e/N_d and z, one obtains,

$$\frac{\partial p_e/N_d}{\partial z} = \frac{1+p_e/N_d}{1+2p_e/N_d} \equiv g(z)$$

g(z) is plotted in Fig. 4 against z. Combining, one obtains:

$$\frac{\partial I_{SR}}{\partial I_{Ep}} = \frac{WsA_s}{D_p A} g(z)$$

As shown in Fig. 8, the area A_s should be $\pi d\Delta d$ where d is the diameter of the emitter dot and Δd is the width of the "absorbing" ring. A fair estimate of Δd for the TA-153 type alloy transistor may be obtained from the analogue maps as is discussed in Section VII.

Appendix II

Emitter Efficiency

Since the effective diffusion coefficient is changing with emitter current, p_e is not a linear function of I_{Ep} as was assumed earlier. The more exact d-c equation for the ratio of electron to hole flow across the emitter junction may be written as:

$$\frac{I_{Ep}}{I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e}\left(1+\frac{p_e}{N_d}\right)$$

where p_e/N_d is obtained from the equation $2p_e/N_e - \ln(1+p_e/N_d) = z$. Now,

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \sigma_b W/\sigma_e L_e \left[1+(p_e/N_d)+I_{Ep}\frac{\partial(p_e/N_d)}{\partial z}\frac{\partial z}{\partial I_{Ep}}\right]$$

$\frac{\partial(p_e/N_d)}{\partial z}$, as before, $= \frac{1+p_e/N_e}{1+2p_e/N_d}$ and $I_{Ep}\frac{\partial z}{\partial I_{Ep}} = z$

where $z = W\mu_e I_{Ep}/D_p A\sigma_b$.

Combining,

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} = \frac{\sigma_b W}{\sigma_e L_e} f(z)$$

where $f(z) = 1+p_e/N_d + z(1+p_e/N_d)/(1+2p_e/N_d)$. $f(z)$ is called the "fall-off" factor and is plotted in Fig. 5 where it may be compared to $(1+z)$, the approximate expression derived in Section V.

Appendix III

Volume Recombination

An analytic expression for volume recombination is impossible, even for the simple, plane-parallel, case considered here. If the process is "bi-molecular", as most experiments indicate, the d-c volume recombination current is given by:

$I_{VR} = AR\int_0^W p(N_d+p)\,dx$ where x is the distance from the collector toward the emitter and R is a volume recombination coefficient. p is obtained from the equation:

$$2p/N_d - \ln(1+p/N_d) = zx/W.$$

Integration must be performed by graphical techniques.

Volume recombination, however, is not usually important until I_{Ep} (and z) are large. Thus, the approximate expression which neglects the logarithmic term may be used for p,

i.e., $p = N_d\, zx/2W = I_{Ep}x/2eD_pA$

Now, $I_{VR} = eAR\left[\frac{N_d I_{Ep}}{2D_p Ae}\int_0^W x\,dx + \left(\frac{I_{Ep}}{2eD_pA}\right)^2\int_0^W x^2 dx\right].$

Integration yields:

$$\frac{I_{VR}}{I_{Ep}} = \frac{RN_d W^2}{4D_p}\left[1+\frac{I_{Ep}W}{3eD_pAN_d}\right] = \frac{RN_d W^2}{4D_p}\left[1+\frac{z}{3}\right]$$

It can be shown that τ, the average hole lifetime at a low injection level is equal to $1/RN_d$. Further $L_b^2 = 2D_p\tau$. (The factor 2 comes in as an approximation of the effect of the electric field in the base region which, at high current densities, essentially doubles the diffusion coefficient). Combining these relations, one obtains:

$$\frac{I_{VR}}{I_{Ep}} = \frac{1}{2} \left(\frac{W}{L_b}\right)^2 (1+z/3).$$

Taking partial derivatives:

$$\frac{\partial I_{VR}}{\partial I_{Ep}} = \frac{1}{2} \left(\frac{W}{L_b}\right)^2 (1+\frac{2}{3} z).$$

The approximate expression given in Section VI differs from this by the absence of the factor 2/3. Since even this result is only approximate, the improvement in accuracy may usually be considered trivial.