R.K.

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APPLICATION OF LINEAR ACTIVE FOUR-TERMINAL NETWORK THEORY TO TRANSISTORS

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Approved

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Introduction

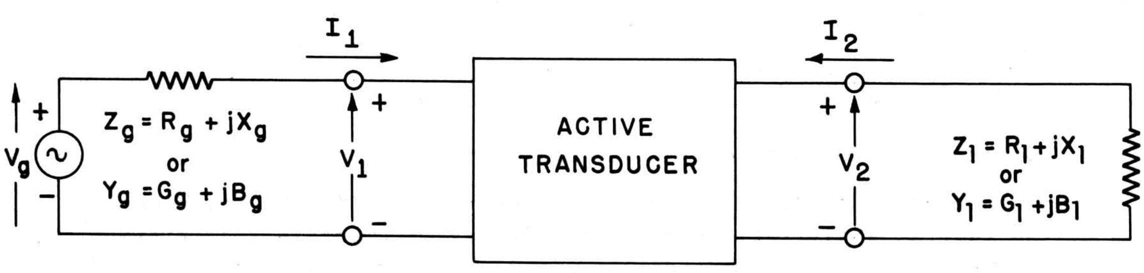
With the advent of transistors, considerably greater usage has been made of linear active four-terminal network theory. Some of this usage has been complicated because of differences in terminology. This bulletin is an attempt to express, in a unified system of nomenclature, the application of linear active four-terminal network theory to transistors. It is hoped that this will serve as a step towards a standard system of nomenclature.

Section I deals with the general properties of a linear active four-terminal network. Section II is devoted to the tabulation of circuits associated with transistor applications. For those transistor circuit properties not tabulated in this section, reference should be made to the appropriate equations in Section I. Several examples of the application of material in Sections I and II to transistor circuits are given in Section III.

Section I—Linear Active Four-Terminal Networks and Equations

In general, any linear active four-terminal network is characterized by two equations which interrelate the currents and voltages at its input and output terminals. These equations may be written in either nodal or loop equation form.

A. *Circuit With Associated Generator And Load, And Equations*



NODAL EQUATIONS

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

LOOP EQUATIONS

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

B. Definition Of Transducer Parameters

The above circuit equations, which completely describe the small-signal operation of the transducer, also serve as a guide to defining the parameters. Thus, for example, if $V_2 = 0$, i.e., the output is shorted, then $y_{11} = I_1/V_1$ defines y_{11} as the input admittance when the output is shorted. Continuing in this manner all of the parameters may be similarly defined.

NODAL

$y_{11} = g_{11} + jb_{11}$ = input admittance with output shorted

$y_{12} = g_{12} + jb_{12}$ = reverse transfer (feedback) admittance with input shorted

$y_{21} = g_{21} + jb_{21}$ = forward transfer admittance with output shorted

$y_{22} = g_{22} + jb_{22}$ = output admittance with input shorted

LOOP

$z_{11} = r_{11} + jx_{11}$ = input impedance with output open

$z_{12} = r_{12} + jx_{12}$ = reverse transfer (feedback) impedance with input open

$z_{21} = r_{21} + jx_{21}$ = forward transfer impedance with output open

$z_{22} = r_{22} + jx_{22}$ = output impedance with input open

C. Transformation Equations

In general, the parameters within each pair of equations are independent of each other. The two sets of parameters, however, are related by the following transformation equations:

NODAL

$$y_{11} = z_{22}/\Delta_z$$

$$y_{12} = -z_{12}/\Delta_z$$

$$y_{21} = -z_{21}/\Delta_z$$

$$y_{22} = z_{11}/\Delta_z$$

$$\Delta_z = \begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}$$

LOOP

$$z_{11} = y_{22}/\Delta_y$$

$$z_{12} = -y_{12}/\Delta_y$$

$$z_{21} = -y_{21}/\Delta_y$$

$$z_{22} = y_{11}/\Delta_y$$

$$\Delta_y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

D. Definition Of Amplification Factors

In addition to the parameters already discussed, it is convenient to define various amplification factors which serve as indexes of performance. Since the network may in general be bilateral, each factor must be defined in the reverse as well as in the forward direction.

Current Amplification Factors

The current amplification factor is defined as the ratio of the negative of the current at one pair of shorted terminals to the current introduced at the other pair of terminals.

α_{21} = *forward* current amplification factor

= ratio of the negative current at the shorted *output* terminals to the current introduced at the *input* terminals.

α_{12} = *reverse* current amplification factor

= ratio of the negative current at the shorted *input* terminals to the current introduced at the *output* terminals.

Voltage Amplification Factors

The voltage amplification factor is defined as the ratio of the voltage at one pair of open terminals to the voltage applied to the other pair of terminals. Note that voltage amplification as defined in reference (6) is only the magnitude of voltage amplification as defined here.

μ_{21} = *forward* voltage amplification factor

= ratio of the voltage at the open *output* terminals to the voltage introduced at the *input* terminals.

μ_{12} = *reverse* voltage amplification factor

= ratio of the voltage at the open *input* terminals to the voltage introduced at the *output* terminals.

Power Amplification Factors

The power amplification factor is defined as the maximum power amplification in a given direction when the transfer impedance or the transfer admittance in the opposite direction is zero. This definition is introduced for the first time since it is believed that the forward power amplification factor in particular may be more useful than either the forward current or the forward voltage amplification factor as a single index of performance.

φ_{21} = *forward* power amplification factor

= maximum power amplification from the *input to the output* terminals when the *reverse* transfer (feedback) impedance or the *reverse* transfer (feedback) admittance is zero.

φ_{12} = *reverse* power amplification factor

= maximum power amplification from the *output to the input* terminals when the *forward* transfer impedance or the *forward* transfer admittance is zero.

E. *Equation For Amplification Factors*

The amplification factors can be readily determined in terms of the impedance or admittance parameters already defined.

NODAL	LOOP
$\alpha_{21} = -y_{21}/y_{11}$	$\mu_{21} = z_{21}/z_{11}$
$\alpha_{12} = -y_{12}/y_{22}$	$\mu_{12} = z_{12}/z_{22}$
$\mu_{21} = -y_{21}/y_{22}$	$\alpha_{21} = .z_{21}/z_{22}$
$\mu_{12} = -y_{12}/y_{11}$	$\alpha_{12} = z_{12}/z_{11}$

$$\varphi_{21} = \frac{|y_{21}|^2}{4g_{11}g_{22}} \qquad \varphi_{21} = \frac{|z_{21}|^2}{4r_{11}r_{22}}$$

$$\varphi_{12} = \frac{|y_{12}|^2}{4g_{11}g_{22}} \qquad \varphi_{12} = \frac{|z_{12}|^2}{4r_{11}r_{22}}$$

F. *Input And Output Impedances*

The input and output impedances of the transducer under circuit conditions are, of course, dependent on the actual load and source impedances respectively. These impedances are readily determined by conventional analysis and are given by:

NODAL

$y_i = g_i + jb_i$ = input admittance

$$= y_{11} - \frac{y_{12}y_{21}}{y_{22}+Y_1}$$

$y_o = g_o + jb_o$ = output admittance

$$= y_{22} - \frac{y_{12}y_{21}}{y_{11}+Y_g}$$

LOOP

$z_i = r_i + jx_i$ = input impedance

$$= z_{11} - \frac{z_{12}z_{21}}{z_{22}+Z_1}$$

$z_o = r_o + jx_o$ = output impedance

$$= z_{22} - \frac{z_{12}z_{21}}{z_{11}+Z_g}$$

G. *Amplification Under Arbitrary Conditions*

The values of amplification for arbitrary values of generator and load impedances are given by the following equations:

NODAL

Forward Current Amplification

$$= \frac{y_{21}Y_1}{y_{11}(y_{22}+Y_1)-y_{12}y_{21}}$$

Reverse Current Amplification

$$= \frac{y_{12}Y_g}{y_{22}(y_{11}+Y_g)-y_{12}y_{21}}$$

Forward Voltage Amplification

$$= -\frac{y_{21}}{y_{22}+Y_1}$$

Reverse Voltage Amplification

$$= -\frac{y_{12}}{y_{11}+Y_g}$$

Forward Power Amplification

$$= \left|\frac{y_{21}}{y_{22}+Y_1}\right|^2 \frac{G_1}{g_1}$$

LOOP

Forward Voltage Amplification

$$= \frac{z_{21}Z_1}{z_{11}(z_{22}+Z_1)-z_{12}z_{21}}$$

Reverse Voltage Amplification

$$= \frac{z_{12}Z_g}{z_{22}(z_{11}+Z_g)-z_{12}z_{21}}$$

Forward Current Amplification

$$= -\frac{z_{21}}{z_{22}+Z_1}$$

Reverse Current Amplification

$$= -\frac{z_{12}}{z_{11}+Z_g}$$

Forward Power Amplification

$$= \left|\frac{z_{21}}{z_{22}+Z_1}\right|^2 \frac{R_1}{r_1}$$

Reverse Power Amplification

$$= \left|\frac{y_{12}}{y_{11}+Y_g}\right|^2 \frac{G_g}{g_o}$$

Reverse Power Amplification

$$= \left|\frac{z_{12}}{z_{11}+Z_g}\right|^2 \frac{R_g}{r_o}$$

H. Equation For Conjugate Impedance Match

The maximum power amplification is obtained when the generator and the load impedances are the complex conjugate of the input and output impedances respectively.

NODAL

Let:

$$G_N = \frac{g_{12}g_{21}-b_{12}b_{21}}{g_{11}g_{22}}$$

$$B_N = \frac{b_{12}g_{21}-b_{21}g_{12}}{2g_{11}g_{22}}$$

Then:

Input Admittance

$$= g_{11}\sqrt{1-G_N-B_N^2} + j(b_{11}-g_{11}B_N)$$

Output Admittance

$$= g_{22}\sqrt{1-G_N-B_N^2} + j(b_{22}-g_{22}B_N)$$

Forward Power Amplification

$$= \frac{|y_{21}|^2}{g_{11}g_{22}[2+2\sqrt{1-G_N-B_N^2}-G_N]}$$

Reverse Power Amplification

$$= \frac{|y_{12}|^2}{g_{11}g_{22}[2+2\sqrt{1-G_N-B_N^2}-G_N]}$$

LOOP

Let:

$$R_N = \frac{r_{12}r_{21}-x_{12}x_{21}}{r_{11}r_{22}}$$

$$X_N = \frac{x_{12}r_{21}-x_{21}r_{12}}{2r_{11}r_{22}}$$

Then:

Input Impedance

$$= r_{11}\sqrt{1-R_N-X_N^2} + j(x_{11}-r_{11}X_N)$$

Output Impedance

$$= r_{22}\sqrt{1-R_N-X_N^2} + j(x_{22}-r_{22}X_N)$$

Forward Power Amplification

$$= \frac{|z_{21}|^2}{r_{11}r_{22}[2+2\sqrt{1-R_N-X_N^2}-R_N]}$$

Reverse Power Amplification

$$= \frac{|z_{12}|^2}{r_{11}r_{22}[2+2\sqrt{1-R_N-X_N^2}-R_N]}$$

I. Equations For Image Impedance Match

The transducer is image impedance matched when the generator and load impedances are equal to the input and output impedances respectively.

NODAL

Input Admittance

$$= y_{11}\sqrt{1-\alpha_{12}\alpha_{21}}$$

Output Admittance

$$= y_{22}\sqrt{1-\alpha_{12}\alpha_{21}}$$

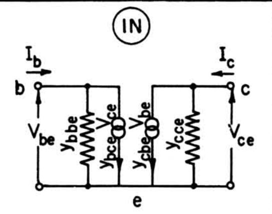
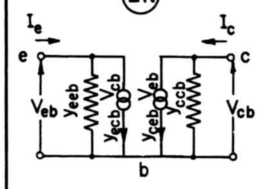
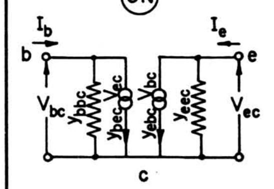
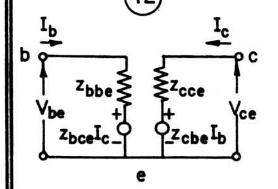
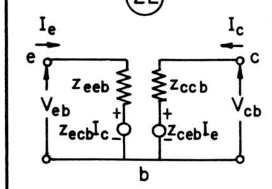
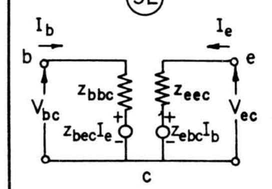
LOOP

Input Impedance

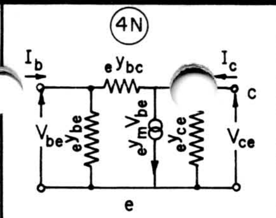
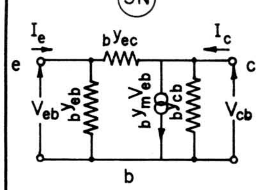
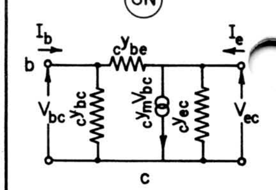
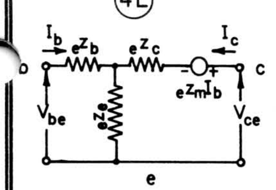
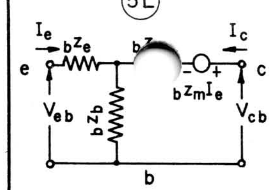
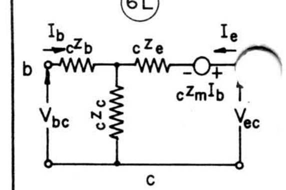
$$= z_{11}\sqrt{1-\alpha_{12}\alpha_{21}}$$

Output Impedance

$$= z_{22}\sqrt{1-\alpha_{12}\alpha_{21}}$$

	NODAL DERIVED EQUIVALENT CIRCUITS			LOOP DERIVED EQUIVALENT CIRCUITS		
	COMMON EMITTER	COMMON BASE	COMMON COLLECTOR	COMMON EMITTER	COMMON BASE	COMMON COLLECTOR
EQUATIONS	$I_b=y_{bbe}V_{be}+y_{bce}V_{ce}$ $I_c=y_{cbe}V_{be}+y_{cce}V_{ce}$	$I_e=y_{eeb}V_{eb}+y_{ecb}V_{cb}$ $I_c=y_{ceb}V_{eb}+y_{ccb}V_{cb}$	$I_b=y_{bbc}V_{bc}+y_{bec}V_{ec}$ $I_e=y_{ebc}V_{bc}+y_{eec}V_{ec}$	$V_{be}=z_{bbe}I_b+z_{bce}I_c$ $V_{ce}=z_{cbe}I_b+z_{cce}I_c$	$V_{eb}=z_{eeb}I_e+z_{ecb}I_c$ $V_{cb}=z_{ceb}I_e+z_{ccb}I_c$	$V_{bc}=z_{bbc}I_b+z_{bec}I_e$ $V_{ec}=z_{ebc}I_b+z_{eec}I_e$
TWO - GEN	(1N) 	(2N) 	(3N) 	(1L) 	(2L) 	(3L) 

	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE
AMP. FACTORS	$\alpha_{cb}=-\frac{y_{cbe}}{y_{bbe}}$ $\mu_{cb}=-\frac{y_{cbe}}{y_{cce}}$ $\phi_{cb}=\frac{\lvert y_{cbe}\rvert^2}{4g_{bbe}g_{cce}}$	$\alpha_{bc}=-\frac{y_{bce}}{y_{cce}}$ $\mu_{bc}=-\frac{y_{bce}}{y_{bbe}}$ $\phi_{bc}=\frac{\lvert y_{bce}\rvert^2}{4g_{bbe}g_{cce}}$	$\alpha_{ce}=-\frac{y_{ceb}}{y_{eeb}}$ $\mu_{ce}=-\frac{y_{ceb}}{y_{ccb}}$ $\phi_{ce}=\frac{\lvert y_{ceb}\rvert^2}{4g_{eeb}g_{ccb}}$	$\alpha_{ec}=-\frac{y_{ecb}}{y_{ccb}}$ $\mu_{ec}=-\frac{y_{ecb}}{y_{eeb}}$ $\phi_{ec}=\frac{\lvert y_{ecb}\rvert^2}{4g_{eeb}g_{ccb}}$	$\alpha_{eb}=-\frac{y_{ebc}}{y_{bbc}}$ $\mu_{eb}=-\frac{y_{ebc}}{y_{eec}}$ $\phi_{eb}=\frac{\lvert y_{ebc}\rvert^2}{4g_{bbc}g_{eec}}$	$\alpha_{be}=-\frac{y_{bec}}{y_{eec}}$ $\mu_{be}=-\frac{y_{bec}}{y_{bbc}}$ $\phi_{be}=\frac{\lvert y_{bec}\rvert^2}{4g_{bbc}g_{eec}}$	$\alpha_{cb}=+\frac{z_{cbe}}{z_{cce}}$ $\mu_{cb}=+\frac{z_{cbe}}{z_{bbe}}$ $\phi_{cb}=\frac{\lvert z_{cbe}\rvert^2}{4r_{bbe}r_{cce}}$	$\alpha_{bc}=+\frac{z_{bce}}{z_{bbe}}$ $\mu_{bc}=+\frac{z_{bce}}{z_{cce}}$ $\phi_{bc}=\frac{\lvert z_{bce}\rvert^2}{4r_{bbe}r_{cce}}$	$\alpha_{ce}=+\frac{z_{ceb}}{z_{ccb}}$ $\mu_{ce}=+\frac{z_{ceb}}{z_{eeb}}$ $\phi_{ce}=\frac{\lvert z_{ceb}\rvert^2}{4r_{eeb}r_{ccb}}$	$\alpha_{ec}=+\frac{z_{ecb}}{z_{eeb}}$ $\mu_{ec}=+\frac{z_{ecb}}{z_{ccb}}$ $\phi_{ec}=\frac{\lvert z_{ecb}\rvert^2}{4r_{eeb}r_{ccb}}$	$\alpha_{eb}=+\frac{z_{ebc}}{z_{eec}}$ $\mu_{eb}=+\frac{z_{ebc}}{z_{bbc}}$ $\phi_{eb}=\frac{\lvert z_{ebc}\rvert^2}{4r_{bbc}r_{eec}}$	$\alpha_{be}=+\frac{z_{bec}}{z_{bbc}}$ $\mu_{be}=+\frac{z_{bec}}{z_{eec}}$ $\phi_{be}=\frac{\lvert z_{bec}\rvert^2}{4r_{bbc}r_{eec}}$

TRANSFORMATION EQUATIONS	(4N)→(1N) $y_{bbe}={}_ey_{be}+{}_ey_{bc}$ $y_{bce}=-{}_ey_{bc}$ $y_{cbe}={}_ey_m-{}_ey_{bc}$ $y_{cce}={}_ey_{ce}+{}_ey_{bc}$	(5N)→(2N) $y_{eeb}={}_by_{eb}+{}_by_{ec}$ $y_{ecb}=-{}_by_{ec}$ $y_{ceb}={}_by_m-{}_by_{ec}$ $y_{ccb}={}_by_{cb}+{}_by_{ec}$	(6N)→(3N) $y_{bbc}={}_cy_{bc}+{}_cy_{be}$ $y_{bec}=-{}_cy_{be}$ $y_{ebc}={}_cy_m-{}_cy_{be}$ $y_{eec}={}_cy_{ec}+{}_cy_{be}$	(4L)→(1L) $z_{bbe}={}_ez_b+{}_ez_e$ $z_{bce}={}_ez_e$ $z_{cbe}={}_ez_m+{}_ez_e$ $z_{cce}={}_ez_c+{}_ez_e$	(5L)→(2L) $z_{eeb}={}_bz_e+{}_bz_b$ $z_{ecb}={}_bz_b$ $z_{ceb}={}_bz_m+{}_bz_b$ $z_{ccb}={}_bz_c+{}_bz_b$	(6L)→(3L) $z_{bbc}={}_cz_b+{}_cz_c$ $z_{bec}={}_cz_c$ $z_{ebc}={}_cz_m+{}_cz_c$ $z_{eec}={}_cz_e+{}_cz_c$
	(2N)→(1N) $V_{be}=-V_{eb}$; $V_{ce}=V_{cb}-V_{eb}$ $y_{bbe}=y_{eeb}+y_{ecb}+y_{ceb}+y_{ccb}$ $y_{bce}=-(y_{ccb}+y_{ecb})$ $y_{cbe}=-(y_{ccb}+y_{ceb})$ $y_{cce}=y_{ccb}$	(3N)→(2N) $V_{eb}=V_{ec}-V_{bc}$; $V_{cb}=-V_{bc}$ $y_{eeb}=y_{eec}$ $y_{ecb}=-(y_{eec}+y_{ebc})$ $y_{ceb}=-(y_{eec}+y_{bec})$ $y_{ccb}=y_{bbc}+y_{bec}+y_{ebc}+y_{eec}$	(1N)→(3N) $V_{bc}=V_{be}-V_{ce}$; $V_{ec}=-V_{ce}$ $y_{bbc}=y_{bbe}$ $y_{bec}=-(y_{bbe}+y_{bce})$ $y_{ebc}=-(y_{bbe}+y_{cbe})$ $y_{eec}=y_{bbe}+y_{cbe}+y_{bce}+y_{cce}$	(2L)→(1L) $V_{be}=-V_{eb}$; $V_{ce}=V_{cb}-V_{eb}$ $z_{bbe}=z_{eeb}$ $z_{bce}=z_{eeb}-z_{ecb}$ $z_{cbe}=z_{eeb}-z_{ceb}$ $z_{cce}=z_{eeb}-z_{ecb}-z_{ceb}+z_{ccb}$	(3L)→(2L) $V_{eb}=V_{ec}-V_{bc}$; $V_{cb}=-V_{bc}$ $z_{eeb}=z_{bbc}-z_{bec}-z_{ebc}-z_{eec}$ $z_{ecb}=z_{bbc}-z_{ebc}$ $z_{ceb}=z_{bbc}-z_{bec}$ $z_{ccb}=z_{bbc}$	(1L)→(3L) $V_{bc}=V_{be}-V_{ce}$; $V_{ec}=-V_{ce}$ $z_{bbc}=z_{bbe}-z_{bce}-z_{cbe}+z_{cce}$ $z_{bec}=z_{cce}-z_{bce}$ $z_{ebc}=z_{cce}-z_{cbe}$ $z_{eec}=z_{cce}$
	(3N)→(1N) $V_{be}=V_{bc}-V_{ec}$; $V_{ce}=-V_{ec}$ $y_{bbe}=y_{bbc}$ $y_{bce}=-(y_{bbc}+$ [illegible]$)$ $y_{cbe}=-(y_{bbc}+y_{ebc})$ $y_{cce}=y_{bbc}+y_{bec}+y_{ebc}+y_{eec}$	(1N)→(2N) $V_{eb}=-V_{be}$; $V_{cb}=V_{ce}-V_{be}$ $y_{eeb}=y_{bbe}+y_{cbe}+y_{bce}+y_{cce}$ $y_{ecb}=-(y_{cce}+y_{bce})$ $y_{ceb}=-(y_{cce}+y_{cbe})$ $y_{ccb}=y_{cce}$	(2N)→(3N) $V_{bc}=-V_{cb}$; $V_{ec}=V_{eb}-V_{cb}$ $y_{bbc}=y_{eeb}+y_{ecb}+y_{ceb}+y_{ccb}$ $y_{bec}=-(y_{eeb}+y_{ceb})$ $y_{ebc}=-(y_{eeb}+y_{ecb})$ $y_{eec}=y_{eeb}$	(3L)→(1L) $V_{be}=V_{bc}-V_{ec}$; $V_{ce}=-V_{ec}$ $z_{bbe}=z_{bbc}-z_{ebc}-z_{bec}+z_{eec}$ $z_{bce}=z_{eec}-z_{bec}$ $z_{cbe}=z_{eec}-z_{ebc}$ $z_{cce}=z_{eec}$	(1L)→(2L) $V_{eb}=-V_{be}$; $V_{cb}=V_{ce}-V_{be}$ $z_{eeb}=z_{bbe}$ $z_{ecb}=z_{b}$[illegible]$_{be}$ $z_{ceb}=z_{bbe}-z_{cbe}$ $z_{ccb}=z_{bbe}-z_{bce}-z_{cbe}+z_{cce}$	(2L)→(3L) $V_{bc}=-V_{cb}$; $V_{ec}=V_{eb}-V_{cb}$ $z_{bbc}=z_{ccb}$ $z_{bec}=z_{ccb}-z_{ceb}$ $z_{ebc}=z_{ccb}-z_{ecb}$ $z_{eec}=z_{eeb}-z_{ecb}-z_{ceb}+z_{ccb}$
ONE - GEN.	(4N) 	(5N) 	(6N) 	(4L) 	(5L) 	(6L) 

	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE	FORWARD	REVERSE
AMP. FACTORS	$\alpha_{cb}=-\frac{{}_ey_m-{}_ey_{bc}}{{}_ey_{be}+{}_ey_{bc}}$ $\mu_{cb}=-\frac{{}_ey_m-{}_ey_{bc}}{{}_ey_{ce}+{}_ey_{bc}}$	$\alpha_{bc}=+\frac{{}_ey_{bc}}{{}_ey_{be}+{}_ey_{bc}}$ $\mu_{bc}=+\frac{{}_ey_{bc}}{{}_ey_{be}+{}_ey_{bc}}$	$\alpha_{ce}=-\frac{{}_by_m-{}_by_{ec}}{{}_by_{eb}+{}_by_{ec}}$ $\mu_{ce}=-\frac{{}_by_m-{}_by_{ec}}{{}_by_{cb}+{}_by_{ec}}$	$\alpha_{ec}=+\frac{{}_by_{ec}}{{}_by_{cb}+{}_by_{ec}}$ $\mu_{ec}=+\frac{{}_by_{ec}}{{}_by_{eb}+{}_by_{ec}}$	$\alpha_{eb}=-\frac{{}_cy_m-{}_cy_{be}}{{}_cy_{bc}+{}_cy_{be}}$ $\mu_{eb}=-\frac{{}_cy_m-{}_cy_{be}}{{}_cy_{ec}+{}_cy_{be}}$	$\alpha_{be}=+\frac{{}_cy_{be}}{{}_cy_{ec}+{}_cy_{be}}$ $\mu_{be}=+\frac{{}_cy_{be}}{{}_cy_{ec}+{}_cy_{be}}$	$\alpha_{cb}=\frac{{}_ez_m+{}_ez_e}{{}_ez_c+{}_ez_e}$ $\mu_{cb}=\frac{{}_ez_m+{}_ez_e}{{}_ez_b+{}_ez_e}$	$\alpha_{bc}=\frac{{}_ez_e}{{}_ez_b+{}_ez_e}$ $\mu_{bc}=\frac{{}_ez_e}{{}_ez_c+{}_ez_e}$	$\alpha_{ce}=\frac{{}_bz_m+{}_bz_b}{{}_bz_c+{}_bz_b}$ $\mu_{ce}=\frac{{}_bz_m+{}_bz_b}{{}_bz_e+{}_bz_b}$	$\alpha_{ec}=\frac{{}_bz_b}{{}_bz_e+{}_bz_b}$ $\mu_{ec}=\frac{{}_bz_b}{{}_bz_c+{}_bz_b}$	$\alpha_{eb}=\frac{{}_cz_m+{}_cz_c}{{}_cz_e+{}_cz_c}$ $\mu_{eb}=\frac{{}_cz_m+{}_cz_c}{{}_cz_b+{}_cz_c}$	$\alpha_{be}=\frac{{}_cz_c}{{}_cz_b+{}_cz_c}$ $\mu_{be}=\frac{{}_cz_c}{{}_cz_e+{}_cz_c}$
	$\phi_{cb}=\frac{\lvert {}_ey_m-{}_ey_{bc}\rvert^2}{4({}_eg_{be}+{}_eg_{bc})({}_eg_{ce}+{}_eg_{bc})}$ $\phi_{bc}=\frac{\lvert {}_ey_{bc}\rvert^2}{4({}_eg_{be}+{}_eg_{bc})({}_eg_{ce}+{}_eg_{bc})}$		$\phi_{ce}=\frac{\lvert {}_by_m-{}_by_{ec}\rvert^2}{4({}_bg_{eb}+{}_bg_{ec})({}_bg_{cb}+{}_bg_{ec})}$ $\phi_{ec}=\frac{\lvert {}_by_{ec}\rvert^2}{4({}_bg_{eb}+{}_bg_{ec})({}_bg_{cb}+{}_bg_{ec})}$		$\phi_{eb}=\frac{\lvert {}_cy_m-{}_cy_{be}\rvert^2}{4({}_cg_{bc}+{}_cg_{be})({}_cg_{ec}+{}_cg_{be})}$ $\phi_{be}=\frac{\lvert {}_cy_{be}\rvert^2}{4({}_cg_{bc}+{}_cg_{be})({}_cg_{ec}+{}_cg_{be})}$		$\phi_{cb}=\frac{\lvert {}_ez_m+{}_ez_e\rvert^2}{4({}_er_b+{}_er_e)({}_er_c+{}_er_e)}$ $\phi_{bc}=\frac{\lvert {}_ez_e\rvert^2}{4({}_er_b+{}_er_e)({}_er_c+{}_er_e)}$		$\phi_{ce}=\frac{\lvert {}_bz_m+{}_bz_b\rvert^2}{4({}_br_e+{}_br_b)({}_br_c+{}_br_b)}$ $\phi_{ec}=\frac{\lvert {}_bz_b\rvert^2}{4({}_br_e+{}_br_b)({}_br_c+{}_br_b)}$		$\phi_{eb}=\frac{\lvert {}_cz_m+{}_cz_c\rvert^2}{4({}_cr_b+{}_cr_c)({}_cr_e+{}_cr_c)}$ $\phi_{be}=\frac{\lvert {}_cz_c\rvert^2}{4({}_cr_b+{}_cr_c)({}_cr_e+{}_cr_c)}$	

TRANSFORMATION EQUATIONS	(1N)→(4N) ${}_ey_{be}=y_{bbe}+y_{bce}$ ${}_ey_{bc}=-y_{bce}$ ${}_ey_{ce}=y_{cce}+y_{bce}$ ${}_ey_m=y_{cbe}-y_{bce}$	(2N)→(5N) ${}_by_{eb}=y_{eeb}+y_{ecb}$ ${}_by_{ec}=-y_{ecb}$ ${}_by_{cb}=y_{ccb}+y_{ecb}$ ${}_by_m=y_{ceb}-y_{ecb}$	(3N)→(6N) ${}_cy_{bc}=y_{bbc}+y_{bec}$ ${}_cy_{be}=-y_{bec}$ ${}_cy_{ec}=y_{eec}+y_{bec}$ ${}_cy_m=y_{ebc}-y_{bec}$	(1L)→(4L) ${}_ez_b=z_{bbe}-z_{bce}$ ${}_ez_e=z_{bce}$ ${}_ez_c=z_{cce}-z_{bce}$ ${}_ez_m=z_{cbe}-z_{bce}$	(2L)→(5L) ${}_bz_e=z_{eeb}-z_{ecb}$ ${}_bz_b=z_{ecb}$ ${}_bz_c=z_{ccb}-z_{ecb}$ ${}_bz_m=z_{ceb}-z_{ecb}$	(3L)→(6L) ${}_cz_b=z_{bbc}-z_{bec}$ ${}_cz_c=z_{bec}$ ${}_cz_e=z_{eec}-z_{bec}$ ${}_cz_m=z_{ebc}-z_{bec}$
	(5N)→(4N) $V_{be}=-V_{eb}$; $V_{ce}=V_{cb}-V_{eb}$ ${}_ey_{be}={}_by_{eb}+{}_by_m$ ${}_ey_{bc}={}_by_{cb}$ ${}_ey_{ce}={}_by_{ec}$ ${}_ey_m=-{}_by_m$	(6N)→(5N) $V_{eb}=V_{ec}-V_{bc}$; $V_{cb}=-V_{bc}$ ${}_by_{eb}={}_cy_{be}-{}_cy_m$ ${}_by_{ec}={}_cy_{ec}+{}_cy_m$ ${}_by_{cb}={}_cy_{bc}$ ${}_by_m={}_cy_m$	(4N)→(6N) $V_{bc}=V_{be}-V_{ce}$; $V_{ec}=-V_{ce}$ ${}_cy_{bc}={}_ey_{bc}$ ${}_cy_{be}={}_ey_{be}$ ${}_cy_{ec}={}_ey_{ce}+{}_ey_m$ ${}_cy_m=-{}_ey_m$	(5L)→(4L) $V_{be}=-V_{eb}$; $V_{ce}=V_{cb}-V_{eb}$ ${}_ez_b={}_bz_b$ ${}_ez_e={}_bz_e$ ${}_ez_c={}_bz_c-{}_bz_m$ ${}_ez_m=-{}_bz_m$	(6L)→(5L) $V_{eb}=V_{ec}-V_{bc}$; $V_{cb}=-V_{bc}$ ${}_bz_e={}_cz_e$ ${}_bz_b={}_cz_b-{}_cz_m$ ${}_bz_c={}_cz_c+{}_cz_m$ ${}_bz_m={}_cz_m$	(4L)→(6L) $V_{bc}=V_{be}-V_{ce}$; $V_{ec}=-V_{ce}$ ${}_cz_b={}_ez_b-{}_ez_m$ ${}_cz_c={}_ez_c$ ${}_cz_e={}_ez_e$ ${}_cz_m=-{}_ez_m$
	(6N)→(4N) $V_{be}=V_{bc}-V_{ec}$; $V_{ce}=-V_{ec}$ ${}_ey_{be}={}_cy_{be}$ ${}_ey_{bc}={}_cy_{bc}$ ${}_ey_{ce}={}_cy_{ec}+{}_cy_m$ ${}_ey_m=-{}_cy_m$	(4N)→(5N) $V_{eb}=-V_{be}$; $V_{cb}=V_{ce}-V_{be}$ ${}_by_{eb}={}_ey_{be}+{}_ey_m$ ${}_by_{ec}={}_ey_{ce}$ ${}_by_{cb}={}_ey_{bc}$ ${}_by_m=-{}_ey_m$	(5N)→(6N) $V_{bc}=-V_{cb}$; $V_{ec}=V_{eb}-V_{cb}$ ${}_cy_{bc}={}_by_{cb}$ ${}_cy_{be}={}_by_{eb}+{}_by_m$ ${}_cy_{ec}={}_by_{ec}-{}_by_m$ ${}_cy_m={}_by_m$	(6L)→(4L) $V_{be}=V_{bc}-V_{ec}$; $V_{ce}=-V_{ec}$ ${}_ez_b={}_cz_b-{}_cz_m$ ${}_ez_e={}_cz_e$ ${}_ez_c={}_cz_c$ ${}_ez_m=-{}_cz_m$	(4L)→(5L) $V_{eb}=-V_{be}$; $V_{cb}=V_{ce}-V_{be}$ ${}_bz_e={}_ez_e$ ${}_bz_b={}_ez_b$ ${}_bz_c={}_ez_c-{}_ez_m$ ${}_bz_m=-{}_ez_m$	(5L)→(6L) $V_{bc}=-V_{cb}$; $V_{ec}=V_{eb}-V_{cb}$ ${}_cz_b={}_bz_b+{}_bz_m$ ${}_cz_c={}_bz_c-{}_bz_m$ ${}_cz_e={}_bz_e$ ${}_cz_m={}_bz_m$

TRANSISTOR CIRCUITS AND EQUATIONS

$$\text{Forward current amplification} = -\frac{\alpha_{21}}{1+\sqrt{1-\alpha_{12}\alpha_{21}}}$$

$$\text{Reverse current amplification} = -\frac{\alpha_{12}}{1+\sqrt{1-\alpha_{12}\alpha_{21}}}$$

$$\text{Forward voltage amplification} = \frac{\mu_{21}}{1+\sqrt{1-\alpha_{12}\alpha_{21}}}$$

$$\text{Reverse voltage amplification} = \frac{\mu_{12}}{1+\sqrt{1-\alpha_{12}\alpha_{21}}}$$

If all transducer parameters are real,

$$\text{Forward power amplification} = \frac{\alpha_{21}\mu_{21}}{(1+\sqrt{1-\alpha_{12}\alpha_{21}})^2}$$

$$\text{Reverse power amplification} = \frac{\alpha_{12}\mu_{12}}{(1+\sqrt{1-\alpha_{12}\alpha_{21}})^2}$$

J. Equations For Iterative Impedance Match

The transducer is iterative impedance matched if it is an element of an infinite number (or the equivalent thereof) of cascaded identical transducers.

NODAL

Let:

$$y_m = \frac{y_{11}+y_{22}}{2}$$

Then:

Input Admittance

$$= -y_{22}+y_m\left[1\pm\sqrt{1-\frac{y_{12}y_{21}}{y_m}}\right]$$

Output Admittance

$$= -y_{11}+y_m\left[1\pm\sqrt{1-\frac{y_{12}y_{21}}{y_m}}\right]$$

LOOP

Let:

$$z_m = \frac{z_{11}+z_{22}}{2}$$

Then:

Input Impedance

$$= -z_{22}+z_m\left[1\pm\sqrt{1-\frac{z_{12}z_{21}}{z_m}}\right]$$

Output Impedance

$$= -z_{11}+z_m\left[1\pm\sqrt{1-\frac{z_{12}z_{21}}{z_m}}\right]$$

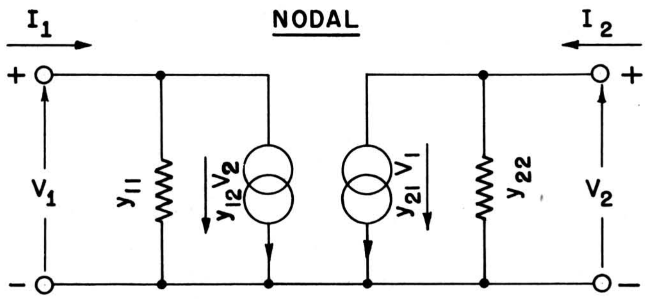
The equations in Section IG may be used to obtain the various circuit amplifications.

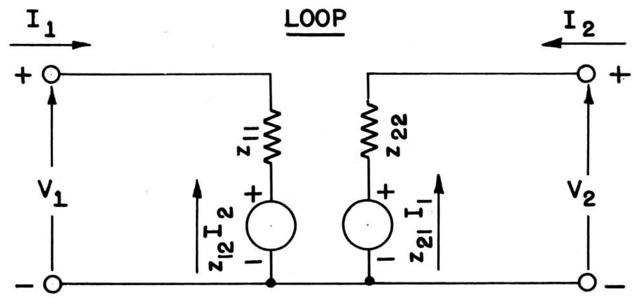
K. Equivalent Circuits

While formal manipulation of the admittance and impedance parameters of a network will result in the correct algebraic expression for whatever quantity is desired, it is nevertheless convenient to be able to express the basic relationships of Section IA in terms of an equivalent circuit.

Two-Generator Equivalent Circuits

When two generators are used the circuit follows directly from the equations of Section IA.





Single-Generator Equivalent Circuits

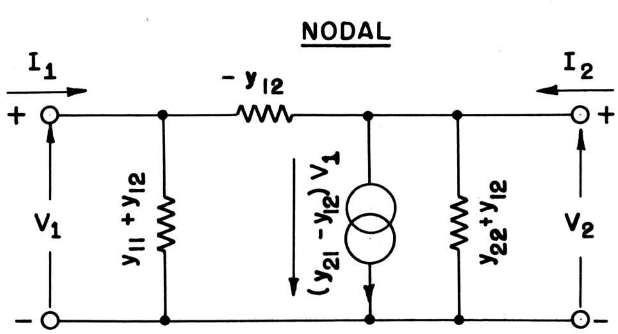
It is often desirable to express the network in terms of a single-generator equivalent circuit. This may be done in several ways. For example, z_{21} may be separated into two parts:

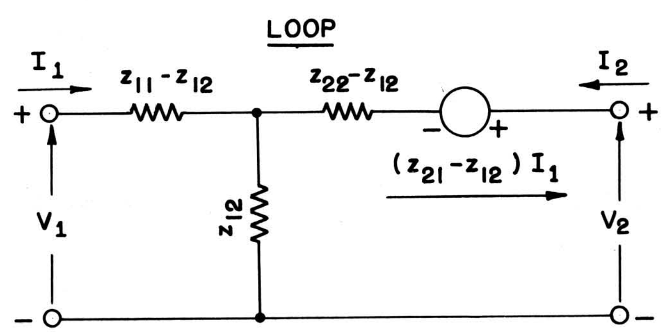
$$z_{21} = z_{12} + (z_{21} - z_{12}).$$

The basic equations then become

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{12}I_1 + z_{22}I_2 \quad \Big| \quad + (z_{21} - z_{12})I_1 \end{aligned}$$

The expressions to the left of the dotted line represent a passive network with a common mutual impedance, and hence is representable by a T network. The remainder of the expression represents a voltage source. Hence the corresponding loop network, shown below, can readily be drawn. For an indication of the numerous other equivalent circuits that can be derived, see reference (3).





Measurement of the actual parameters of a given network may be made by the methods outlined in reference (5).

Section II — Equivalent Circuits of Transistors

A compilation of the more useful transistor equivalent circuits is given on pages eight and nine. Included with the circuits are the amplification factors and the transformation equations which enable one to go readily from a nodal (or loop) derived equivalent circuit to any other nodal (or loop) derived equivalent circuit. However, in order to go from a nodal to the corresponding loop derived equivalent circuit, or vice versa, the transformation equations of section IC are required.

The notation used in this tabulation was chosen in order to avoid ambiguity among the various circuit parameters. When applied to the two-generator equivalent circuits this notation basically consists of the conventional two letter subscript. To this conventional notation, a third subscript has been added, which designates the common terminal between the input and output circuit. Thus z_{bbe} is the base self impedance of the transistor in a common-emitter circuit, and y_{ceb} is the transfer admittance to the collector from the emitter in a common-base circuit. If only one circuit connection is being considered or if no ambiguity arises, the last subscript designating the common electrode may be omitted.

When applied to the single-generator equivalent circuit, a pre-subscript designates the common electrode between the input and output circuits. For the π-equivalent or nodal-derived network, admittances are used with the two post-subscripts designating the terminals between which the admittance is located. Thus ${}_{c}y_{be}$ is the admittance between the base and the emitter in a common-collector circuit. For the T-equivalent or loop-derived network there is one node in common with all the elements of the T. Thus only one post-subscript is necessary to designate the terminal to which the impedance is connected. Accordingly ${}_{b}z_{c}$ is the impedance in a common-base circuit between the collector and the center of the T. In both the π and T networks the generator parameter is designated by the subscript m. If only one circuit connection is being considered or if no ambiguity arises, the pre-subscript designating the common electrode may be omitted.

When it is desirable to use admittances in place of impedances, or vice-versa, ambiguity may be avoided by using reciprocal notation. Thus $1/y_{eeb}$ is the emitter self impedance in the two-generator, common-base, nodal-derived, equivalent circuit, and $1/{}_{e}z_{m}$ is the generator admittance in the single-generator, common-emitter, loop-derived equivalent circuit.

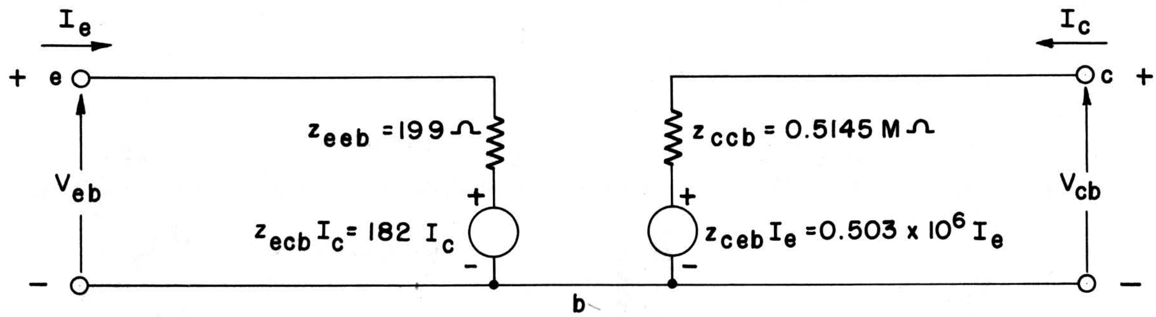
Section III — Illustrative Calculations

Consider a junction transistor in a common-base circuit for which the following low-frequency open-circuited impedance measurements have been made.

$z_{11} = z_{eeb} = 199$ ohms $\qquad z_{21} = z_{ceb} = 0.503$ megohm

$z_{12} = z_{ecb} = 182$ ohms $\qquad z_{22} = z_{ccb} = 0.5145$ megohm

Therefore the two-generator equivalent circuit (2L) of Section II is:



Using these data, the amplification factors can be computed from Section IE,

$$\mu_{21} = \mu_{ce} = z_{ceb}/z_{eeb} = 2{,}530$$

$$\mu_{12} = \mu_{ec} = z_{ecb}/z_{ccb} = 354 \times 10^{-6}$$

$$\alpha_{21} = \alpha_{ce} = z_{ceb}/z_{ccb} = 0.977$$

$$\alpha_{12} = \alpha_{ec} = z_{ecb}/z_{eeb} = 0.915$$

$$\varphi_{21} = \varphi_{ce} = \frac{|z_{ceb}|^2}{4 r_{eeb} r_{ccb}} = 620$$

$$\varphi_{12} = \varphi_{ec} = \frac{|z_{ecb}|^2}{4 r_{eeb} r_{ccb}} = 81 \times 10^{-6}$$

Since all elements are real, the image match equations of Section II can be used. Thus the input and output match impedances are:

$$\text{Input Impedance} = z_{eeb}\sqrt{1-\alpha_{ec}\alpha_{ce}} = 64.8 \text{ ohms}$$

$$\text{Output Impedance} = z_{ccb}\sqrt{1-\alpha_{ec}\alpha_{ce}} = 167{,}000 \text{ ohms}$$

and the amplification values for these conditions are:

$$\text{Forward current amplification} = -\frac{\alpha_{ce}}{1+\sqrt{1-\alpha_{ec}\alpha_{ce}}} = -0.737$$

$$\text{Reverse current amplification} = -\frac{\alpha_{ec}}{1+\sqrt{1-\alpha_{ec}\alpha_{ce}}} = -0.690$$

$$\text{Forward voltage amplification} = \frac{\mu_{ce}}{1+\sqrt{1-\alpha_{ec}\alpha_{ce}}} = 1{,}910$$

$$\text{Reverse voltage amplification} = \frac{\mu_{ec}}{1+\sqrt{1-\alpha_{ec}\alpha_{ce}}} = 267 \times 10^{-6}$$

$$\text{Forward power amplification} = \frac{\alpha_{ce}\mu_{ce}}{(1+\sqrt{1-\alpha_{ec}\alpha_{ce}})^2} = 1{,}408 \text{ (31.5 db)}$$

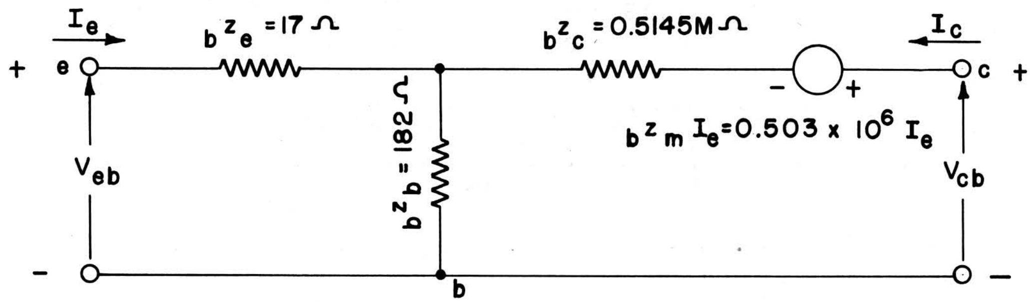
$$\text{Reverse power amplification} = \frac{\alpha_{ec}\mu_{ec}}{(1+\sqrt{1-\alpha_{ec}\alpha_{ce}})^2} = 184 \times 10^{-6}$$

The one-generator equivalent circuit can be obtained by using the (2L)→(5L) transformation equations of Section II. Thus:

$_b z_e = z_{eeb} - z_{ecb} = 17$ ohms $\qquad$ $_b z_c = z_{ccb} - z_{ecb} = 0.5145$ megohm

$_b z_b = z_{ecb} = 182$ ohms $\qquad$ $_b z_m = z_{ceb} - z_{ecb} = 0.503$ megohm

so that the equivalent circuit (5L) of Section II is:



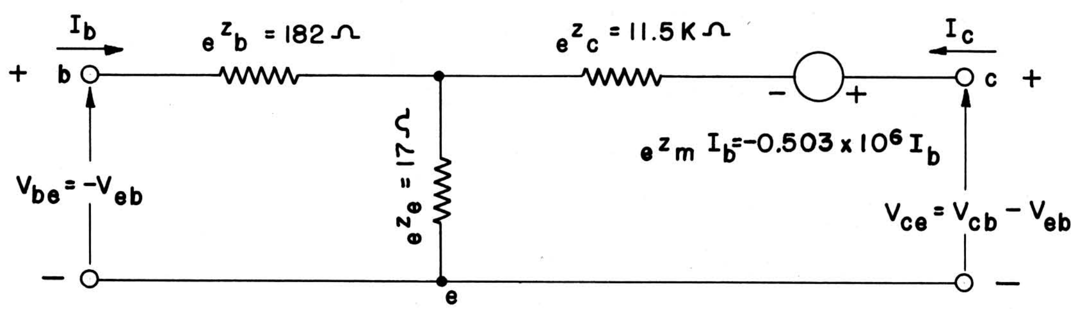
If the common-emitter one-generator equivalent circuit is desired, the (5L)→(4L) transformation equations of Section II are used. Thus:

$V_{be} = -V_{eb}$ $\qquad$ $V_{ce} = V_{cb} - V_{eb}$

$_e z_b = {_b z_b} = 182$ ohms $\qquad$ $_e z_c = {_b z_c} - {_b z_m} = 11.5$ K ohms

$_e z_e = {_b z_e} = 17$ ohms $\qquad$ $_e z_m = -{_b z_m} = -\ 0.503$ megohm

so that the equivalent circuit (4L) of Section II is:



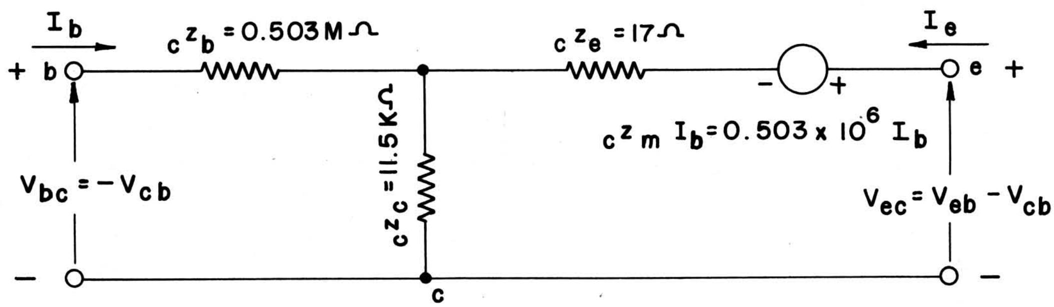
If the common-collector one-generator equivalent circuit is desired, the (5L)→(6L) transformation equations of Section II are used. Thus:

$V_{bc} = -V_{cb}$ $\qquad$ $V_{ec} = V_{eb} - V_{cb}$

$_c z_b = {_b z_b} + {_b z_m} = 0.503$ megohm $\qquad$ $_c z_e = {_b z_e} = 17$ ohms

$_c z_c = {_b z_c} - {_b z_m} = 11.5$ K ohms $\qquad$ $_c z_m = {_b z_m} = 0.503$ megohm

so that the equivalent circuit (6L) of Section II is:



The amplification factors and the circuit amplifications for the common-emitter and the common-collector circuits will of course be different from those values computed above for the common-base circuit. The appropriate equations can be used to compute the new values.

If a nodal-derived equivalent circuit is desired, the transformation equations of Section IC must be used. Thus, suppose that the common-emitter one-generator equivalent circuit (4N) of Section II is desired. There are several different successive transformations that can be employed to arrive at circuit (4N). The following successive transformation will be employed: (2L)→(1L)→(1N)→(4N).

Thus, (2L)→(1L)

$V_{be} = -V_{eb}$ $\qquad$ $V_{ce} = V_{cb} - V_{eb}$

$z_{bbe} = z_{eeb} = 199$ ohms $\qquad$ $z_{cbe} = z_{eeb} - z_{ceb} = -0.503 \times 10^6$ ohms

$z_{bce} = z_{eeb} - z_{ecb} = 17$ ohms $\qquad$ $z_{cce} = z_{eeb} - z_{ecb} - z_{ceb} + z_{ccb} = 11.5$ K ohms

And (1L)→(1N) $\qquad$ $\Delta_z = z_{bbe}z_{cce} - z_{bce}z_{cbe} = 10.84 \times 10^6$

$$y_{bbe} = z_{cce}/\Delta_z = 1.06 \times 10^{-3} \text{ mho}$$

$$y_{bce} = -z_{bce}/\Delta_z = -1.568 \times 10^{-6} \text{ mho}$$

$$y_{cbe} = -z_{cbe}/\Delta_z = 0.0464 \text{ mho}$$

$$y_{cce} = z_{bbe}/\Delta_z = 18.37 \times 10^{-6} \text{ mho}$$

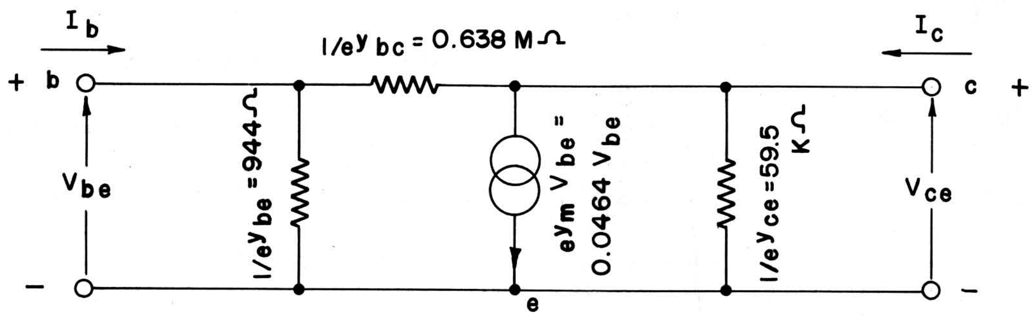
Finally, (1N)→(4N)

$_ey_{be} = y_{bbe} + y_{bce} = 1.059 \times 10^{-3}$ mho $\qquad$ $1/_ey_{be} = 944$ ohms

$_ey_{bc} = -y_{bce} = 1.568 \times 10^{-6}$ mho $\qquad$ $1/_ey_{bc} = 0.638$ megohm

$_ey_{ce} = y_{cce} + y_{bce} = 16.8 \times 10^{-6}$ mho $\qquad$ $1/_ey_{ce} = 59.5$ K ohms

$_ey_m = y_{cbe} - y_{bce} = 0.0464$ mho so that the equivalent circuit (4N) is:



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[1] ASA C42-1941, "American Standard Definitions of Electrical Terms", American Institute of Electrical Engineers, 33 West 39th St., N.Y.C., 1941.

[2] "Standards on Abbreviations, Graphical Symbols, Letter Symbols, and Mathematical Signs", The Institute of Radio Engineers, 1 East 79 St., N.Y.C., 1948.

[3] L. C. Peterson, "Equivalent Circuits of Linear Active Four-Terminal Networks", *B.S.T.J.*, Vol. 27, pp. 593-622; October, 1948.

[4] "Standards on Electron Tubes; Definition of Terms, 1950", *Proc. of I.R.E.*, Vol. 38, pp. 426-438; April, 1950.

[5] "Standards on Electron Tubes: Methods of Testing, 1950", *Proc. of I.R.E.*, Vol. 38, pp. 917-948; August, 1950.

[6] "Standards on Transducers: Definitions of Terms, 1951", *Proc. of I.R.E.*, Vol. 39, pp. 897-899; August 1951.