LB-856

DESIGN DATA FOR HORIZONTAL RHOMBIC ANTENNAS

RADIO CORPORATION OF AMERICA

RCA LABORATORIES DIVISION

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Approved

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Introduction

The horizontal rhombic antenna is used extensively throughout the world for high-frequency transmission and reception. Though structurally simple its design is a relatively complicated procedure, due largely to the enormous labor of computation to determine its complete radiation pattern. Most engineering methods provide information on the main lobe of the pattern at one frequency. This information is insufficient because it omits consideration of other lobes that can be, and often are, very large and which greatly compromise its performance.

The popularity of the horizontal rhombic antenna is due to its relatively low cost, and to the fact that it is essentially aperiodic when correctly matched in its characteristic impedance at the distant end. This permits it to be used for more than one operating frequency. However, there is no relation between its input impedance characteristics and its radiation characteristics, andonly the radiation characteristics determine the useful range of operating frequencies for ahorizontal rhombic antenna.

To determine the significant radiation characteristics for a horizontal rhombic antenna over a band of frequencies the designer must examine the orientations and amplitudes of several of the secondary radiation lobes as well as the main lobe at each operating frequency. Data and charts are presented here which enable the designer to perform the necessary computations with a minimum of effort. By means of the tables given, secondary lobes through those of the fifth order can be computed with adequate engineering accuracy. Higher order lobes of the complete pattern are usually negligible for ordinary practical purposes, assuming the antenna is properly built and properly terminated. By means of the stereographic charts, the essential geometric parameters and the resulting patterns for a horizontal rhombic antenna can be determined with ease.

The charts were prepared according to the method described by Foster* who explains their theory and construction. Tables II to VII were compiled by using large-scale charts from which the appended charts were reproduced.

This bulletin was written primarily for communication applications, but is equally useful for VHF and UHF applications where there is every prospect that the rhombic antenna will find new uses. All data presented are correct for VHF or UHF designs.

*Donald Foster, "Radiation from Rhombic Antennas", *Proc. I.R.E.*, Vol. 25, p. 1326, Oct. 1937.

The Rhombic Element

Fig. 1 represents the coordinate system for a rhombic element in free space. The rhombus lies in the plane A-B, and the normal plane C-D is the principal vertical plane along the axis, which is the intersection of the two planes. The rhombus has two controlling parameters, the leg length L and the acute angle A. Pure unattenuated traveling waves are assumed to enter the system at the rear apex (1) and flow toward the front apex, (2) where the matching impedance is located.

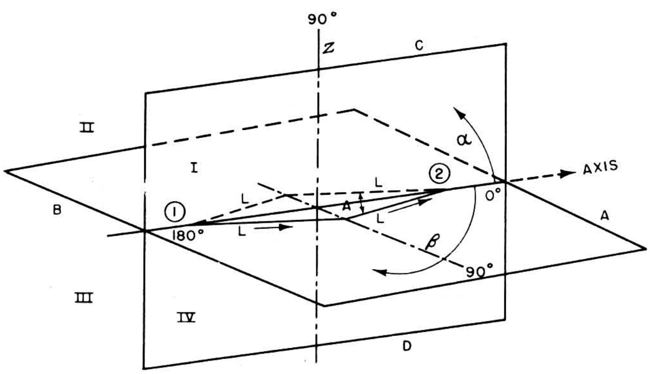

Fig. I - Coordinate system for a rhombus in free space.

Radiation patterns for the rhombus are symmetrical in quadrants I and II, and likewise in quadrants IV and III. This permits the entire pattern to be specified by the pattern in I only. The angle α, ranging from 0 to 90 degrees, always represents the angle of elevation with respect to the plane A-B. The angle β is an azimuth angle, ranging from 0 to 180 degrees, measured from the forward axis.

The radiation pattern for each leg is a function of its length in wavelengths. The radiation pattern for the rhombus is a composite of the patterns for the individual legs and the geometry of the rhombus.

The free-space radiation pattern for a single leg, which is a straight conductor guiding a unidirectional wave, consists of a number of cones of radiation centered on the conductor as an axis. There is a cone, or lobe, of radiation for each half wavelength of conductor length. Half of the lobes tilt forward in the direction of the wave movement in the conductor, and the other half tilt backward. Between these successive lobes of radiation there are conical zones of zero radiation. Table I gives the angles of maximums and zeros for straight conductors from 2 to 7 wavelengths long as a function of the angle to the conductor θ. There is always a zero in the direction of the conductor.

Table I also gives the amplitudes of the successive lobes in terms of field strength relative to that of the first (principal) lobe. These relations are correct for each leg length, but not for comparison with the amplitudes of lobes for different leg lengths.

Table I — Angles of Maximums and Zeros in the Free-Space Pattern for a Straight Wire With Traveling Waves

Order of Maximum or Zero	$L = 2\lambda$		$L = 3\lambda$		$L = 4\lambda$		$L = 5\lambda$		$L = 6\lambda$		$L = 7\lambda$	
	θ	E	θ	E	θ	E	θ	E	θ	E	θ	E
M-1	35.5	1.00	29.0	1.0	24.5	1.00	22.2	1.00	20.5	1.00	19.4	1.0
M-2	74.	0.426	58.	0.460	50.	0.479	45.	0.505	41.	0.520	37.	0.536
M-3	104.	0.252	80.	0.300	67.	0.340	60.	0.360	53.	0.383	49.	0.394
M-4	137.	0.127	99.	0.216	82.	0.251	73.	0.280	65.	0.305	59.	0.314
M-5			120.	0.153	97.	0.195	84.	0.228	75.	0.250	68.	0.264
M-6			147	0.076	112	0.149	96	0.182	85	0.207	76	0.226
M-7					129	0.107	108	0.148	94	0.175	86	0.190
M-8					151	0.058	120	0.118	106	0.146	94	0.165
M-9							134	0.086	116	0.121	102	0.145
M-10							153	0.050	127	0.097	111	0.124
M-11									139	0.066	120	0.103
M-12									155	0.044	130	0.083
M-13											141	0.063
M-14											157	0.037
0-0	0		0		0		0		0		0	
0-1	60		48		42		37		33		31	
0-2	90		70		59		52		47		44	
0-3	120		90		74		66		59		54	
0-4	180		110		90		78		70		63	
0-5			132		105		90		81		73	
0-6			180		121		102		90		81	
0-7					139		114		99		90	
0-8					180		127		110		98	
0-9							142		122		106	
0-10							180		147		116	
0-11									180		135	
0-12											148	
0-13												
0-14											180	

The composite free-space pattern for a traveling-wave rhombus is the result of interference between the patterns for the individual legs as a result of their spacial separations, and their mutual orientations. The multiplicity of lobes in the leg patterns causes a large number of lobes in the composite pattern, and interference effects in space give each lobe a different orientation in azimuth and elevation. When the rhombus is placed above a reflecting surface, such as the earth, interference with the image pattern further complicates and modifies the basic pattern. If arrays of horizontal rhombic elements are used, still higher orders of complication are introduced by additional interference effects. The complete solution of such patterns for practical antenna designs by conventional methods involves an enormous amount of skillful computation, and is seldom attempted.

Reference Data for Computing the Complete Free-Space Pattern of a Rhombus

The essential information on the complete pattern for a rhombic element can be limited to a determination of the coordinates and relative amplitudes of the main and the various secondary lobes. The shapes of the secondary lobes are not important for practical design purposes.

Tables II to VII list the azimuths (β) and elevations (α) of the principal and the secondary lobes in the complete radiation pattern for a rhombus in free space. They are for leg lengths from 2 to 7 wavelengths in steps of one wavelength, and the acute angle (A) of the rhombus in steps of 5 degrees from 35 to 60 degrees. These limits embrace most of the values of practical interest. Intermediate data can be found by interpolation. The orientations listed have an accuracy of about 1 degree.

The relative amplitudes of the lobes given in Table VIII are idealized values for unattenuated traveling waves in the system. In practice there is attenuation, and also some standing waves, which are empirical and unpredictable, and which modify the lobe amplitudes. However, the idealized data are the only firm values available for the study and appraisal of a design.

Each pattern lobe is caused by the coincidence of a maximum in the pattern for one pair of parallel legs with a maximum in the pattern for the other pair of parallel legs of the rhombus. For example, the peak of the lobe 3-4 occurs where the third maximum (M-3) for one side coincides with the fourth maximum (M-4) for the other side. This occurs at some azimuth angle β and some elevation angle α that is not in the principal vertical plane. The coincidence of the first maximums for each side gives the main lobe (1-1), which has its peak in the principal vertical plane where $\beta = 0$. All intersections of lobes of equal order fall in the principal vertical plane, and it is only in this plane that the radiated fields are horizontally polarized. All intersections of lobes of unequal order occur outside of this plane, and the fields have both horizontally and vertically polarized components.

The tables of lobe orientations are used by choosing the order of maximum for one leg and the order of maximum for the second, for each value of A. The azimuth and elevation of the resulting lobes are then read from the table. Table VIII gives their relative field strengths with respect to the main lobe. The relative amplitudes are valueless if the rhombus parameters are such that the two first maximums do not coincide and therefore do not form the main lobe. Table VIII is to be used only when the main lobe exists and is fully formed.

In Tables II to VII the dashes put in place of a number indicate that the relevant first maximums do not intersect and that a resulting main lobe does not exist or is degenerate. Special caution is directed to those cases where the main lobe is shown not to exist, because then the main lobe has disintegrated to a mere vestigial form, or may be completely split by a zero on the principal plane at the lower angles. This indicates an impractical design that should always be avoided.

Table VIII shows that lobes due to intersections of maximums of order greater than the fifth are of vanishing importance and these lobes are therefore omitted from the orientation tables. However, as a practical engineering fact, it is impossible to build and adjust a rhombic antenna that is completely free of reflections from side angles and terminal load so there are always some standing waves. These

Table II — Orientation of Pattern Lobes for Free-Space Rhombus when $L = 2\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4	
		β	α	β	α	β	α	β	α
A = 71°	1	0	0	34	36	71	8	—	—
	2			0	70	88	64	108	24
	3					180	107	146	43
	4							180	155
A = 65°	1	0	16	38	36	—	—	—	—
	2			0	71	88	82	107	9
	3					180	107	142	43
	4							180	30
A = 60°	1	0	20	41	35	—	—	—	—
	2			0	72	88	59	—	—
	3					180	107	140	42
	4							180	33
A = 55°	1	0	24	43	33	—	—	—	—
	2			0	72	89	56	—	—
	3					180	107	138	41
	4							180	34
A = 50°	1	0	27	47	29	—	—	—	—
	2			0	72	89	52	—	—
	3					180	106	134	38
	4							180	37
A = 45°	1	0	28	50	24	—	—	—	—
	2			0	73	89	47	—	—
	3					180	106	131	35
	4							180	38
A = 40°	1	0	30	53	13	—	—	—	—
	2			0	73	89	40	—	—
	3					180	105	127	29
	4							180	39
A = 35°	1	0	32	—	—	—	—	—	—
	2			0	73	89	26	—	—
	3					180	105	124	19
	4							180	40

standing waves greatly augment the smaller backward lobes. For this reason it is safer to assume that no lobe is smaller than 34 decibels below the amplitude of the main lobe (0.02) under the best practical conditions.

Main Lobe Angles

Fig. 2 shows how the elevation of the main lobe of a rhombus varies with leg length and the acute angle. There is a value of A where the two first maximums intersect in the plane of the rhombus where the main lobe has the coordinates $\alpha = 0$, $\beta = 0$. This value of A is exactly twice the angle θ for the first maximum for the single-leg pattern. When A exceeds this limiting value the main lobe disintegrates, and finally splits into two side lobes as A gets greater. This is what may happen when a given rhombus, designed for one frequency, is used at a frequency that is much greater.

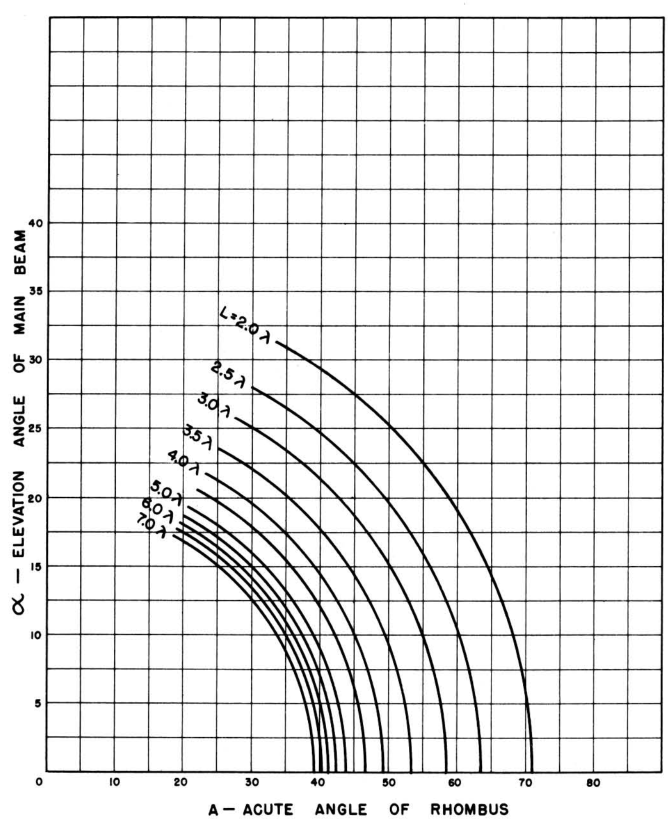

Fig. 2 - Main beam elevation with respect to the plane of a rhombus in free space.

Special attention is directed to the very rapid variation of α with A for low main-lobe angles. When a low-angle lobe is desired, the value of A becomes very critical. This also implies that such an antenna would have almost no tolerance in the direction of higher frequencies, for which the electrical leg length of a fixed structure would be increasing, with the main beam tending to split.

Fig. 2 also reveals the variation of the main lobe angle with frequency for a structure of fixed size and acute angle. For A constant, the locus of variation with electrical leg length is a vertical line running across the various curves. This variation of lobe angle with frequency is an important factor in high-frequency applications, where a fixed structure designed for one optimum frequency is to be used over a range of frequencies.

The Horizontal Rhombic Antenna

A rhombic element whose plane is parallel to ground is called a horizontal rhombic antenna. When the ground is perfectly conducting (as usually assumed for design purposes) it acts as a perfect mirror surface. The pattern for the horizontal rhombic antenna is then the rhombus pattern multiplied by a height factor introduced by the mirror image.

Height factor = $f(\alpha)$ = $\sin(H \sin \alpha)$, expressed in terms of relative field strengths. This factor is independent of azimuth. For each half wavelength (180 degrees) in the height H in electrical degrees, there is a lobe in the height factor. When $H<180$ degrees, $f(\alpha)$ has but one lobe that varies in shape until, for very small values of H, it approaches a tangent sphere in shape. It is usually drawn in figures as a tangent circle. There are zeros between successive lobes in the height factor.

There are an unlimited number of choices of the height of a rhombic antenna, each causing a different effect on the radiation pattern, so that there are no unique conditions that can be tabulated in the manner of the free-space rhombus pattern. It is therefore necessary to compute the effect of the height factor on the 3-dimensional pattern for each electrical height H.

Table III — Orientation of Pattern Lobes for Free-Space Rhombus when $L = 3\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4		5		6	
		β	α	β	α	β	α	β	α	β	α	β	α
A = 65°	1	—	—	22	27	46	26	—	—	—	—	—	—
	2			0	52	38	59	71	49	89	21	—	—
	3					0	78	89	73	107	49	—	—
	4							180	79	141	60	133	31
	5									180	53	158	32
	6											180	7
A = 60°	1	—	—	24	28	49	22	—	—	—	—	—	—
	2			0	53	41	58	73	46	90	0	—	—
	3					0	78	89	72	106	46	—	—
	4							180	79	137	59	130	27
	5									180	55	157	33
	6											180	15
A = 55°	1	0	9	27	29	52	17	—	—	—	—	—	—
	2			0	54	44	57	74	42	—	—	—	—
	3					0	78	89	69	104	42	—	—
	4							180	80	135	58	127	22
	5									180	56	154	33
	6											180	19
A = 50°	Opt. 1	0	15	29	29	29	0	—	—	—	—	—	—
	2			0	55	47	57	86	36	—	—	—	—
	3					0	79	89	68	103	36	—	—
	4							180	80	132	57	125	14
	5									180	57	152	33
	6											180	22
A = 45°	1	0	19	32	27	—	—	—	—	—	—	—	—
	2			0	56	50	54	78	28	—	—	—	—
	3					0	79	89	65	102	28	—	—
	4							180	80	128	56	—	—
	5									180	57	148	33
	6											180	25
A = 40°	1	0	22	36	25	—	—	—	—	—	—	—	—
	2			0	57	54	52	79	5	—	—	—	—
	3					0	79	89	62	110	5	—	—
	4							180	80	125	53	—	—
	5									180	57	145	30
	6											180	27
A = 35°	1	0	23	40	20	—	—	—	—	—	—	—	—
	2			0	57	58	48	—	—	—	—	—	—
	3					0	80	89	58	—	—	—	—
	4							180	80	122	48	—	—
	5									180	58	142	27
	6											180	28

Fig. 3 shows the elevation angles of unity (maximums) and zeros in the height factor. When applied to a horizontal rhombic antenna, the best radiation efficiency is obtained when the first maximum in the height factor occurs at the same elevation angle as that of the main lobe in the rhombus pattern. It is most desirable that this angle should also be the optimum angle of propagation for the space radio path. However, for a fixed physical height, size and shape of antenna, the variation of the angle of the first maximum in the height factor does not follow the variation in the angle of the free-space rhombus as frequency is varied, even though the directions of the variations are the same. The result is that the height-factor maximum does not track the rhombus maximum for any considerable change of frequency.

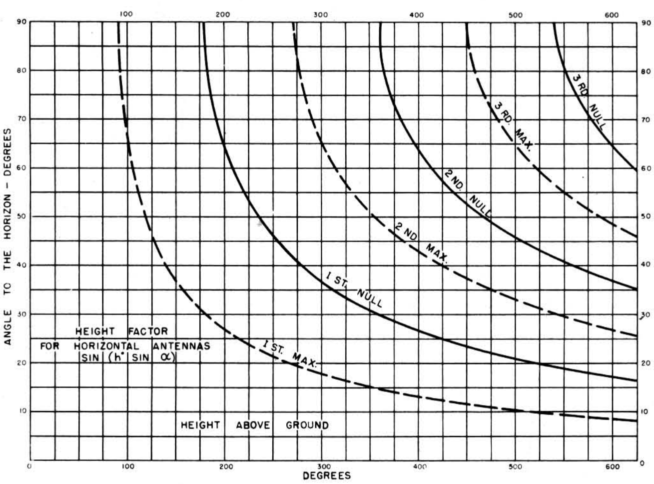

Fig. 3 - Angles of maximums and zeros in the height factor for a horizontal antenna.

The height factor is always zero at $\alpha = 0$, so that any rhombus lobes occurring at $\alpha = 0$ are suppressed. Any rhombus lobes occurring at elevation angles corresponding to zeros in the height factor are also suppressed, or split into much smaller pairs of lobes. Any rhombus lobes occurring at elevation angles corresponding to maximums (relative value unity) in the height factor remain unchanged in amplitude or orientation. Other rhombus lobes occurring at elevation angles where the height factor has values anywhere between 1.0 and 0 are reduced to the value which is the product of the relative amplitudes of the rhombus lobe and the height factor at that elevation angle. The rhombus lobe amplitudes are read from Tables II to VII, and the value of $f(\alpha)$ at any angle is computed from the equation for the height factor.

In dealing with only the peaks of the rhombus lobes, there often is a small error caused by a shift in the orientation and amplitude of a lobe where the actual pattern for the rhombus lobe is diminishing while the height factor is increasing with α, or vice versa. However, this is usually negligible as a design factor except when the main lobe is involved.

By this method, therefore, one can quickly determine all the essential values of a pattern for a horizontal rhombic antenna of chosen dimensions at any given frequency by working only with the peaks of the various lobes up through those of fifth order. By repeating this analysis for several different frequencies the performance over a band of frequencies can be evaluated.

The principal fault with rhombic antennas, in common with all long-wire types, is that there are inevitably a multiplicity of secondary lobes. The best design is one where the desired main lobe is properly oriented for the propagation path and all other lobes reduced as far as possible. The largest lobes are seen from Table VIII to be those formed by the intersection of the first maximum for one leg pattern with the second and higher order maximums for the other leg pattern. In practical rhombic antennas the first maximum for one side never crosses maximums higher than the fourth for the other side.

The largest secondary lobes are usually the pair 1-2. By proper choice of A, it is possible to have their elevation occur at an angle about twice that of the main lobe. This condition usually brings the next pair of sidelobes (1-3) at or near zero elevation. By choosing a height H to bring the first maximum in the height factor at the elevation of the main lobe, the first zero in the height factor will be at or very near to the elevation of the 1-2 lobes, and they are suppressed quite completely. At the same time, the 1-3 lobes, at 0 degrees elevation, are suppressed by the zero in the height factor at this angle. With the first two dominant pairs of sidelobes suppressed in this way, the resultant pattern for the horizontal rhombic antenna is the cleanest

Table IV — Orientation of Pattern Lobes for Free-Space Rhombus when $L = 4\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4		5		6	
		β	α	β	α	β	α	β	α	β	α	β	α
A = 60°	1	—	—	17	21	35	24	52	11	—	—	—	—
	2			0	42	23	50	48	48	69	36	—	—
	3					0	63	40	67	74	58	90	41
	4							0	81	88	76	105	58
	5									180	82	140	68
	6											180	64
A = 55°	1	—	—	18	23	37	23	—	—	—	—	—	—
	2			0	44	25	50	51	46	71	30	—	—
	3					0	64	42	67	75	56	90	36
	4							0	81	88	74	104	56
	5									180	82	137	67
	6											180	65
A = 50°	1	—	—	21	24	41	19	—	—	—	—	—	—
	2			0	45	27	50	54	44	73	20	—	—
	3					0	64	45	66	77	52	90	27
	4							0	81	88	73	102	52
	5									180	82	134	66
	6											180	65
A = 45°	1	0	10	23	25	44	12	—	—	—	—	—	—
	2			0	46	30	50	57	40	—	—	—	—
	3					0	65	48	65	78	47	99	10
	4							0	81	89	70	101	47
	5									180	82	130	65
	6											180	66
A = 40°	1	0	14	26	24	—	—	—	—	—	—	—	—
	2			0	47	34	48	61	34	—	—	—	—
	3					0	65	52	63	80	41	—	—
	4							0	81	89	68	100	42
	5									180	82	127	64
	6											180	66
A = 35°	1	0	17	29	22	—	—	—	—	—	—	—	—
	2			0	48	37	47	64	24	—	—	—	—
	3					0	65	55	61	81	31	—	—
	4							0	81	89	64	98	33
	5									180	82	123	61
	6											180	66

possible. This interesting case was found and explained by Foster. The gain in the main beam is therefore at its maximum. This particular condition prevails for one frequency only, or a very narrow band of frequencies near the optimum. The parameters for optimum designs of this type are shown in Fig. 4. The word "optimum" in this regard pertains solely to the horizontal rhombic antenna as a radiator, and does not include any considerations of propagation requirements, or how well it would fit the requirements of any particular radio path. However, where the main lobe characteristics fit the propagation requirements, this optimum design is to be preferred.

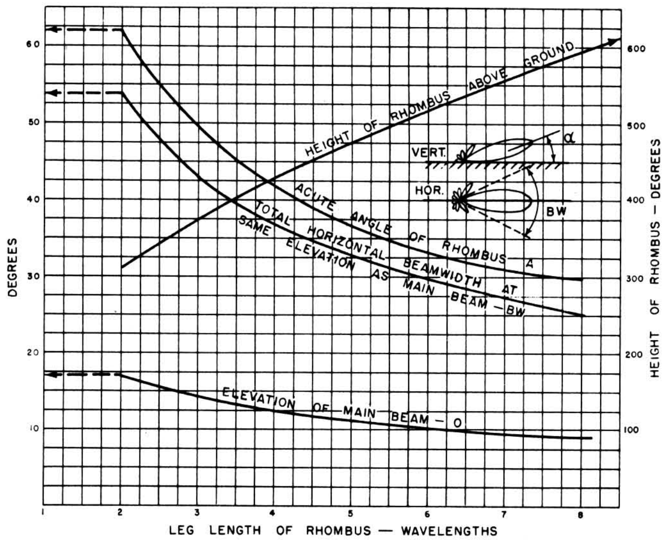

Fig. 4 – Parameters and principal characteristics for horizontal rhombic antennas of optimum design.

Example

The application of these data for the design analysis of a horizontal rhombic antenna of one element will be made clear by the following examples:

Assume that a rhombic antenna is wanted that will have its main beam at an angle = 12.5 degrees at a specified frequency. In order to take advantage of an optimum design at this frequency, Fig. 4 is used and the following dimensions are read:

Leg length	L	= 4λ
Acute angle	A	= 42 degrees
Height	H	= 428 degrees (1.19λ)
Total beam width	BW	= 37 degrees between first zeros.

This optimum design is known to suppress lobes 1-2 and 1-3 at the frequency f_0, leaving vestigial lobes of a low order of amplitude that are relatively negligible.

The height factor for this array at f_0 is

$$f(\alpha) = \sin\ (428 \sin \alpha)$$

There are maximums in this height factor at

$$428 \sin \alpha = 90 \text{ and } 270 \text{ degrees and zeros at}$$
$$428 \sin \alpha = 0,\ 180 \text{ and } 360 \text{ degrees.}$$

These can be computed or read from Fig. 3 which shows that two maximums occur at 12.5 and 39.5 degrees, and three zeros at 0, 25 and 57 degrees.

Table IV for L = 4λ shows that we must interpolate between the values for A = 40 and A = 45 to find the coordinates of the various lobes of the 42-degree rhombus. The facts about lobes 1-1, 1-2 and 1-3 are already known sufficiently from the characteristics of the optimum design. There are no higher order lobes formed with M-1 for one side only. The next lobe of interest is 2-2, occurring at 46.5 degrees at zero azimuth. At this point the height factor lies between a maximum (39.5 degrees) and the next zero (57 degrees) so we compute its value at 46.5 degrees to obtain

$$\sin\ (428 \sin 46.5) = 0.7660$$

The relative amplitude for the lobe 2-2, from Table VIII is 0.26. With the height factor, this becomes 0.26 x 0.7660 = 0.20.

In the same way the other lobes are examined, and we obtain: lobe 2-3, 0.023; lobe 2-4, 0.00049; lobe 3-3, 0.0014. Other lobes are smaller, and need not be studied. For engineering purposes it will be assumed that no lobe is less than 0.02 (-34 db).

The next step is to determine the high-frequency limit of this antenna. In order not to split the main beam, Fig. 2 shows that when A = 42 degrees the greatest leg length that can be used is 5.5λ, at which length the main beam is at 3 degrees. At this corresponding

Table V—Orientation of Pattern Lobes for Free-Space Rhombus when $L = 5\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4		5		6	
		β	α	β	α	β	α	β	α	β	α	β	α
A = 60°	1	—	—	13	15	27	22	42	19	53	0	—	—
	2			0	36	17	44	35	45	52	41	67	29
	3					0	54	24	59	49	58	69	50
	4							0	69	42	72	74	66
	5									0	84	90	78
	6											180	84
A = 55°	1	—	—	15	18	30	22	44	15	—	—	—	—
	2			0	38	18	44	38	45	54	38	69	22
	3					0	55	27	60	52	57	71	47
	4							0	70	45	72	76	64
	5									0	84	91	78
	6											180	84
A = 50°	1	—	—	16	20	32	22	48	0	—	—	—	—
	2			0	39	20	45	41	43	58	34	71	0
	3					0	56	28	60	54	55	73	43
	4							0	71	47	72	77	62
	5									0	84	91	77
	6											180	84
A = 45°	1	0	0	18	22	36	18	—	—	—	—	—	—
	2			0	41	22	45	44	41	61	27	—	—
	3					0	57	32	59	58	52	75	36
	4							0	71	50	70	78	58
	5									0	84	91	76
	6											180	84
A = 40°	1	0	10	21	22	39	12	—	—	—	—	—	—
	2			0	42	25	45	48	37	64	14	—	—
	3					0	57	35	58	61	48	77	25
	4							0	72	54	69	80	54
	5									0	84	90	74
	6											180	84
A = 35°	1	0	14	24	22	40	0	—	—	—	—	—	—
	2			0	42	28	44	52	32	—	—	—	—
	3					0	58	39	57	64	43	—	—
	4							0	72	58	67	82	48
	5									0	84	90	71
	6											180	84

frequency, the electrical height changes to 588 degrees. The first maximum for this height is at 8.5 degrees and the height factor at 3 degrees is 0.512. Since the height factor is increasing slowly between 3 and 8 degrees while the rhombus beam is decreasing rapidly in this range, we can estimate that their continuous product would give a maximum at about 4 degrees with a relative amplitude of about 0.75. The first zero in the height factor is at α = 17.5 degrees. Interpolating between Tables V and VI, and between A = 40 and 45, we find the lobe 1-2 at α = 21 degrees. The height factor at this angle is 0.515. The lobe 1-2 is 0.544, so their product is 0.28. After normalizing the main lobe, the ratio of lobe 1-2 changes from 0.28/0.75 = 0.374, to 0.50 (-6 db). This is a large sidelobe and one would consider it to be intolerable. Therefore the frequency ratio 5.5/4.0 is too high for this antenna. If we make the same examination of the 1-2 lobe for A = 5λ, we find that its value is of the order of 0.20 at α = 22 degrees, β = 19.5 degrees. The lobe 1-3 is found to be larger and of the order of 0.30 at α = 14.5, β = 38. These lobes are rather large, being down only about 14 and 10.5 db respectively. It is a matter of engineering judgment to decide if a frequency ratio of 5/4 is admissible. The sidelobe amplitudes are not large in terms of power leakage so the gain would not be greatly increased by reducing them. However, from an interference standpoint, the pattern for this frequency is not good.

When computations of this kind are made for horizontal rhombic antennas one is impressed by the rapid increase in the major sidelobes as the frequency departs from those that give optimum performance. This leads to the conclusion that the range of frequencies that should be used with a horizontal rhombic antenna is very restricted. By omitting to study the complete radiation pattern the efficiency of the antenna system may be low in the desired direction, and cause unnecessary interference toward or from other directions. The purpose for which one uses a directive antenna is to give the best possible radiation intensity in the desired direction and to reduce radiation or pickup as much as possible in all other directions.

Further Remarks on Rhombic Antennas

The majority of rhombic antennas in service use heights less than those that provide the optimum design. This places the first maximum in the height factor at an angle that may be much higher than the main lobe elevation for the free-space rhombus. The combined effect is to raise the main lobe and to decrease its unnormalized value. This in turn tends to give all other lobes a relatively larger value, making the radiation efficiency lower, the gain lower, and the spurious radiation higher. Initial economy may well prove to be unwise, a fact that is not evident from a study of the main lobe only.

Since the values of the main lobe at angles off its maximum are not available in tables, it is necessary to compute the shape of the main beam in the vertical plane for the free-space rhombus. The pattern for the rhombus in the principal vertical plane C-D is ideally

$$f_r\ (\alpha) = N \frac{\sin^2[180L\ (1 - \cos A/2 \cos \alpha)]}{1 - \cos A/2 \cos \alpha}$$

where N is a normalizing factor for the main lobe, and L is the leg length in wavelengths.

By using the appropriate pair of appended charts for a leg length L, and setting them at a selected acute angle A, the elevation of the first zero above the main beam can be measured. This equation can then be applied to compute the vertical shape of the main lobe and the relative field strength at any angle up to the first zero. Then, multiplexing this series of values for the main lobe by the series of values for the height factor for the chosen height, the peak of the resulting main lobe for the horizontal rhombic antenna can be located exactly. With this pivotal information, one proceeds as before to compute the relative values of the important secondary lobes, finally determining their value relative to that of the main lobe.

This operation is tedious and slow for exploratory computations. It usually suffices for preliminary studies to determine, from the charts, the angle of the main lobe and the first zero above it, and, using polar or Cartesian coordinates, to sketch a curve to fit the known maximum and zero angles. One

Table VI—Orientation of Pattern Lobes for Free-Space Rhombus When $L = 6\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4		5		6	
		β	α	β	α	β	α	β	α	β	α	β	α
A = 60°	1	—	—	10	7	22	19	34	20	45	14	—	—
	2			0	28	13	37	27	40	41	39	55	32
	3					0	46	17	52	35	53	53	49
	4							0	61	23	65	50	63
	5									0	72	45	74
	6											0	85
A = 55°	1	—	—	11	13	25	20	37	18	47	0	—	—
	2			0	31	14	38	29	40	43	37	57	28
	3					0	47	19	52	37	52	56	47
	4							0	62	26	65	53	62
	5									0	73	46	64
	6											0	85
A = 50°	1	—	—	13	17	26	21	39	15	—	—	—	—
	2			0	33	15	39	32	40	47	15	60	21
	3					0	48	21	53	41	51	59	44
	4							0	62	28	65	56	60
	5									0	73	49	74
	6											0	85
A = 45°	1	—	—	14	19	28	20	43	6	—	—	—	—
	2			0	34	17	40	35	38	50	30	63	5
	3					0	49	23	53	44	50	62	40
	4							0	62	30	64	59	58
	5									0	73	52	72
	6											0	85
A = 40°	1	0	5	16	20	32	17	—	—	—	—	—	—
	2			0	36	20	40	39	36	54	23	—	—
	3					0	60	26	53	47	47	65	33
	4							0	63	34	64	63	55
	5									0	74	57	71
	6											0	85
A = 35°	1	0	11	18	21	35	10	—	—	—	—	—	—
	2			0	37	22	40	43	33	57	2	—	—
	3					0	51	29	52	51	44	67	22
	4							0	63	38	63	66	50
	5									0	74	60	69
	6											0	85

accustomed to working with radiation patterns can judge typical shapes well enough for an approach. The resulting function is then used with the height factor to solve for the angle and the amplitude of the main lobe for the antenna, and the relative amplitudes of other lobes.

Let us assume that a rhombic antenna of leg length 375 feet, acute angle of 40 degrees and height 65 feet is to be analyzed for performance at 13 Mc. Then $L = 5\lambda$ and $H = 0.866\lambda = 312$ degrees. From Table V the rhombus main beam peaks at 10 degrees. From Fig. 3 it is seen that the height factor is 1.0 at 17 degrees (first max.). With the 5λ charts we find the first zero to be at 32 degrees. If we plot the height factor in rectangular coordinates and sketch in a probability pattern that has its maximum (unity) at 10 degrees and a zero at 32 degrees the resultant maximum obtained by multiplying the two functions should occur at approximately 14 degrees, with an amplitude of about 0.9. (By precise computation it is 0.925 at 14.3 degrees before normalizing.)

Proceeding as outlined before, this antenna has normalized values of the important secondary lobes, and their orientations, of:

Lobe	Relative Amplitude		
1-1	1.00	14	0
1-2	0.54	21	21
1-3	0.39	12	39
2-2	0.14	42	0
2-3	0.028	45	25
All others	0.02 (allowed)	-	-

One sees that this antenna must have low gain because of the large secondary lobes, and that considerable power is radiated in undesired directions.

Rhombic Antenna Arrays

The ordinary single-element horizontal rhombic antenna may be regarded as a free-space array of two antiphased rhombic elements with parallel planes spaced twice the height above ground. In reality its pattern is eliminated in the lower half space by the ground. The principle is the same when rhombic elements are stacked into multiplane arrays, or variously disposed in a common plane.

Computation of such patterns is made relatively easy if we regard the geometric center of each rhombic element, real or image, as an isotropic point source of radiation and compute the space pattern resulting from this array of isotropic radiators, taking into account the amplitude and phase differences in their excitation. The pattern for an array of isotropic sources is then multiplied by the pattern for the rhombic element used, to give the pattern for the rhombic array. When the rhombic elements are all parallel to ground, the image currents are inverted, and this fact is recognized in assigning an inverted polarity to the isotropic images.

It is very evident that high-angle lobes can be reduced or suppressed by stacking rhombic elements one above the other to provide more destructive interference for the higher lobes than is provided by the height factor of the single-element horizontal rhombic antenna.

Destructive interference can be applied to important sidelobes (usually those at the lower elevation angles around the main lobe) by coplanar rhombic elements spaced along a common major axis, or side by side along a common minor axis, or both. The elements may overlap or not, according to the choice of spacing parameters for their centers. Christiansen* has shown how such arrays of rhombic elements can provide relatively clean radiation patterns, comparable to broadside dipole arrays, but with the additional feature of aperiodicity which is one of the most desired properties of rhombic antennas. In taking advantage of aperiodicity, a compromise choice is necessary to accommodate favorably a band of frequencies. The engineering of such arrays becomes a formidable task, although the method of computation given here minimizes the labor involved in computing the patterns.

Rhombic Antenna Design Charts

There are included with this bulletin thirteen transparent charts for computing rhombic antenna patterns. There are six pairs of charts for rhombus leg lengths of 2, 3, 4, 5, 6 and 7 wavelengths.

*W. N. Christiansen, "Directional Patterns for Rhombic Antennas", *A.W.A. Tech. Rev.*, Vol. 7, p. 33, Sept. 1946.

Table VII—Orientation of Pattern Lobes for Free-Space Rhombus When $L = 7\lambda$ (in degrees)

	Order of Maximums For 2nd Side	For 1st Side 1		2		3		4		5		6	
		β	α	β	α	β	α	β	α	β	α	β	α
A = 60°	1	—	—	—	—	18	15	28	19	48	17	48	8
	2			0	23	10	32	21	37	33	37	45	35
	3					0	42	12	47	27	49	41	48
	4							0	54	16	58	34	59
	5									0	64	24	68
	6											0	75
A = 55°	1	—	—	10	7	20	18	31	19	42	14	—	—
	2			0	27	11	34	23	37	36	36	48	32
	3					0	43	14	47	29	49	44	47
	4							0	55	18	58	37	59
	5									0	65	27	68
	6											0	76
A = 50°	1	—	—	11	13	22	19	33	17	45	0	—	—
	2			0	29	13	36	26	37	39	35	51	27
	3					0	44	15	48	31	49	47	46
	4							0	56	20	59	41	58
	5									0	66	29	68
	6											0	76
A = 45°	1	—	—	12	16	24	19	37	13	—	—	—	—
	2			0	31	14	36	28	37	42	33	54	20
	3					0	45	17	49	34	48	50	43
	4							0	56	22	59	43	67
	5									0	66	32	68
	6											0	76
A = 40°	1	—	—	13	18	27	17	—	—	—	—	—	—
	2			0	32	16	37	32	36	47	27	57	0
	3					0	46	20	49	38	36	54	38
	4							0	57	24	59	48	55
	5									0	67	35	67
	6											0	76
A = 35°	1	0	8	16	19	31	14	—	—	—	—	—	—
	2			0	33	18	37	36	33	50	19	—	—
	3					0	47	22	49	42	44	57	32
	4							0	48	27	58	52	52
	5									0	67	40	66
	6											0	76

Each chart is a stereographic map of the radiation pattern for a straight wire in free space guiding a pure traveling wave. The arrow on the axis indicates the direction of the wave (current) flow. In stereographic projection, one looks down on the long wire and sees the enclosing hemisphere. The outer circle is the "horizon" in the plane of the wire, looking at it from the zenith. Each cone of radiation is a broken line, and each is numbered in sequence M1, M2, etc. starting with the main lobe nearest to the wire in the direction of current flow. Between successive lobes are cones representing the nulls, or directions of "zero" radiation. Actually, due to attenuation of current along the wire due to radiation and heat losses, these are minimums and not true zeros. They are numbered in sequence 01, 02, etc. starting with the zero between the first and second lobes. There is also a zero in the direction of the wire which we can call 00 if it requires identification.

Thus each chart is the radiation pattern for one leg of a rhombus. Since only the *angles* of maximums and zeros are shown, a chart also represents the maximum and zeros in the radiation pattern for two opposite parallel sides of a rhombus. Each chart has small cross lines on the axis which are in stereographic 10-degree intervals from 0 (the horizon circle) to 90(the zenith and center). A circle drawn from the center represents an angle of constant elevation, or latitude, with respect to the horizon (equatorial) circle.

A separate chart is provided which is a stereographic hemisphere calibrated in latitude and longitude angles in 10-degree steps. It is to the same scale as the other charts and is used as a cocentered overlay when one wishes to read the coordinates of any point of a pattern.

The radiation pattern for a rhombus in free space is obtained by setting two identical charts on the same centers with their axes at an angle that is the acute angle of the rhombus. The intersection of M1 on one chart with M1 of the other gives the maximum radiation lobe of the rhombic pattern, which lies in the principal vertical plane. There are other lobes of radiation where various orders of maximums for the two charts intersect, such as where M1 crosses M2.

There is a zero in the array pattern wherever there is a zero in the pattern for one leg. These zeros are lines (solid) associated with each chart. Zones between these zero lines are zones through which some radiation occurs, however small. The location of the null zones is a very important part of the design of the radiation patterns for practical use.

By changing the acute angle of the rhombus it can be seen that the angle of the main beam can be adjusted to occur at any desired angle with respect to the plane of the rhombus. If the angle is too large, the two M_1's do not intersect at all and the beam either is not fully formed, or it is actually split into two symmetrical lobes, directed off the array axis. This often happens when a rhombic antenna designed for one frequency is operated at a much higher frequency. It is instructive to check this important fact in the following way: Set the two charts for a leg length of 2 wavelengths for an acute angle of 65 degrees and note the main beam at an angle of 17 degrees above the plane of the rhombus. This may be a very desirable pattern for a particular application where a vertical beam angle of 17 degrees is called for. If the antenna is constructed to give this pattern, how will it work for higher frequencies? This can be found by setting the pairs of charts for 3, 4, 5, etc. wavelengths successively at 65 degrees acute angle, and observing the results at frequencies where the leg lengths correspond to these electrical lengths. Take, for example, the frequency at which the length is 6 wavelengths. It will be seen that the two M_1's do not meet at all (missing by 20 degrees) and there is a null at low angles in the pattern along the main axis. There is a secondary beam along the array axis at 26 degrees above the horizontal plane of the rhombus. A main beam therefore does not exist, the resulting pattern is very poor, most of the energy being radiated at angles where it can cause interference.

Table VIII gives the mathematical relations of the amplitudes of the various lobes up through those of sixth order, in terms of relative field strength when used as a transmitting antenna, or antenna current for a receiving antenna, based on the assumption that all parts of the antenna are immersed in a field of uniform strength. The fact that this is not true in reality is the basis for diversity reception.

Table VIII—Relative Field Strengths of Lobes in Radiation Pattern for Rhombus in Free Space

Order of Maximum for 2nd side \ Order of Maximum for 1st side	1	2	3	4	5	6
1	1.000					
2	0.544	0.26				
3	0.420	0.038	0.003			
4	0.354	0.007	0.00054	0.0001		
5	—	0.002	0.000156	0.000028	10^{-6}	
6	—	—				

To study a particular rhombic antenna pattern, make a print of the two relevant charts in a chosen angular relationship, using a dry process in which the paper does not shrink or stretch. The black-line printing processes are excellent for this reason. This gives a worksheet on which the various lobes and null zones are seen. On this worksheet, draw circles from the center at the elevation angles of maximums and nulls in the chosen height factor, using broken lines for the maximums. The print can then be marked with any notations necessary. The hemispheric coordinate chart may be used to measure any orientations desired.

The charts enable one to arrive at an antenna design for a particular frequency with a minimum of effort. They also facilitate the analysis of the performance of an antenna of fixed mechanical form and dimension at other working frequencies by using the charts for other leg lengths set at the acute angle of the antenna. For leg lengths that are not an integral number of wavelengths it is necessary to interpolate between the available charts, or, if necessary, to construct intermediate charts. Most engineering needs will be satisfied by those provided.

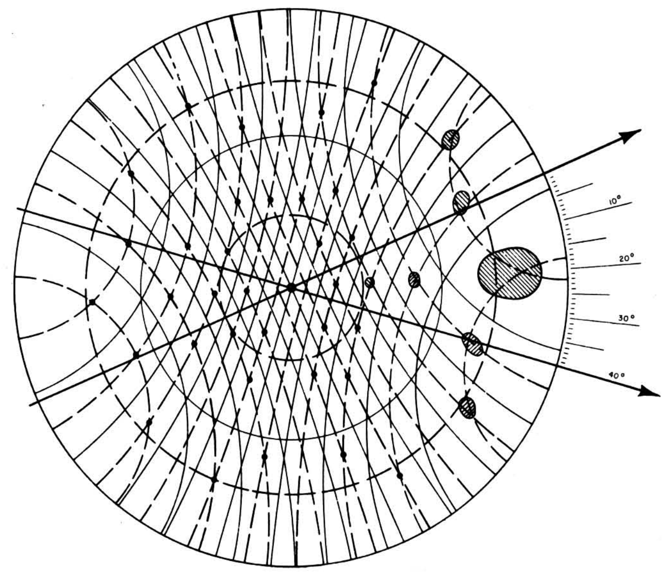

Fig. 5 – Example of a stereoscopic map for a horizontal rhombic antenna where L = 5λ, A = 40 degrees and H = 312 degrees corresponding to the second example. The main beam and all other lobes are represented by shaded circles or dots. Broken lines represent maximums and solid lines zeros.

Fig. 5 is an example of a design worksheet for one set of conditions, illustrating the use of the charts and the application of height factor information.

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Listed below are the titles of the transparent stereographic charts which will be found inside the back cover of this bulletin.

Fig. 6 - Hemispherical coordinates in stereographic projection.

Figs. 7a and 7b - (Two identical charts) Stereographic pattern for a straight wire 2 wavelengths long with a pure traveling wave propagated in the direction of the arrow.

Figs. 8a and 8b - (Identical pair) Stereographic pattern for a straight wire 3 wavelengths long with a pure traveling wave propagated in the direction of the arrow.

Figs. 9a and 9b - (Identical pair) Stereographic pattern for a straight wire 4 wavelengths long with a pure traveling wave propagated in the direction of the arrow.

Figs. 10a and 10b - (Identical pair) Stereographic pattern for a straight wire 5 wavelengths long with a pure traveling wave propagated in the direction of the arrow.

Figs. 11a and 11b - (Identical pair) Stereographic pattern for a straight wire 6 wavelengths long with a pure traveling wave propagated in the direction of the arrow.

Figs. 12a and 12b - (Identical pair) Stereographic pattern for a straight wire 7 wavelengths long with a pure traveling wave propagated in the direction of the arrow.