LB-831

ON EXTENDING THE OPERATING VOLTAGE RANGE OF ELECTRON TUBE HEATERS

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Approved

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SUMMARY

A preliminary survey has been conducted of means whereby the permissible supply voltage variation for the heaters of radio receiving tubes with indirectly heated cathodes might be increased to ±20 per cent. The present tolerance is ±10 per cent. The latter figure corresponds to a cathode temperature change of ±4 per cent which may thus be considered a practical maximum with existing oxide cathode materials. The allowable voltage variation, ΔV, consistent with this temperature limitation, increases with the temperature exponent of resistance of the heater material. For example with a hypothetical constant-resistance heater, $\Delta V = \pm 8$ per cent; with the tungsten-molybdenum alloys usually used, $\Delta V = \pm 10$ per cent; but even with an arbitrarily high temperature exponent of resistance, $\Delta V_{max} = \pm 16$ per cent, corresponding to constant current, is a fundamental physical limit.

A ballast tube in series with a tungsten heater allows the desired ±20 per cent supply voltage variation, but existing ballast types use up at least 38 per cent additional power and often have shorter life than electron tubes. To decrease the ballast resistor power, suitable materials are required with higher temperature exponents of resistivity than appear in published data. A general analysis has been made for a heater in series with a stabilizing resistor and shows that an ideal ballast resistor material would require 6 per cent additional power when in series with a tungsten heater. Negative resistance heaters appear to offer no practical advantage.

An emission-controlled relay and a cathode-temperature-controlled internal thermostatic switch interrupting the heater current both provide unlimited voltage range with negligible power loss, but do not "fail safe". A relay which inserts a resistor in series with the heater at low supply voltages will nearly double the operating voltage range. The possibility of improving cathode materials to withstand a wider temperature range lies outside the scope of this study, but new ways of heating the cathode have been considered. A novel self-stabilizing electron-bombardment heater has been devised but has a very limited operating voltage range.

Present knowledge does not point to any way of making a significant improvement in the tube heater itself. Of known devices, an external ballast resistor or a relay seems most practical and economical of power in obtaining heater operation over a large voltage range.

On Extending the Operating Voltage Range of Electron Tube Heaters

Introduction

Certain applications of radio receiving tubes require operation over a wide range of heater supply voltage. This is often true in mobile installations where the power supply is a battery-generator combination whose output voltage fluctuates between wide limits. A particular instance of importance is the operation of aircraft electronic equipment at reduced battery voltage either with the engines idling or under emergency conditions of engine failure. This bulletin presents an analysis of means whereby the existing supply voltage limits for satisfactory operation may be extended.

I. General Considerations

Tubes, as they are made today, already have a small amount of inherent, built-in voltage compensation which is incidental to their normal operation. The metals used for cathode heaters are tungsten and tungsten-molybdenum alloys which have positive temperature coefficients of resistivity large compared with most other non-magnetic elements. Consequently, a temperature rise causes a resistance increase and thus the power input change due to a rise in voltage is less than for a constant resistance. In typical tubes the per cent resistance change is about four-tenths the per cent voltage change and, as a result, temperature changes are reduced to about four-fifths of the value for a constant resistance. This is discussed in Part III.

Another inherent factor making for temperature stability is the fact that radiated power is proportional to the fourth power of the absolute temperature. Consequently, if the cathode loses heat primarily by radiation, fractional changes in temperature are only one quarter of the fractional changes in power. Power losses by conduction, however, are proportional to the temperature differences and the temperature changes of a conduction-cooled cathode will be greater than those of a pure radiator for the same changes in power input. Most receiving tube cathodes are predominantly radiation cooled but particular types, for example, the cathodes of "lighthouse" and cathode-ray tubes, have considerable heat losses by conduction and so are somewhat worse in their ability to tolerate low and high heater voltage.

A fair generalization is that the average 6.3-volt receiving tube (with radiation-cooled cathode) may be operated satisfactorily with heater voltages ranging from 5.7 to 6.9 volts. This is a deviation of approximately ±10 per cent from the average value.[1] Because the attendant resistance change is about ±4.2 per cent, the current spread is reduced to ±5.8 per cent (not ±10 per cent as for constant resistance) and the heater power spread is ±15.8 per cent (not ±20 per cent).[2] From the fourth-power radiation law, the cathode temperature spread would then be ±4.0 per cent or ±40 degrees K for a cathode normally at 1000 degrees K. It would be desirable to keep this as a limit to the cathode temperature variation while pursuing the present objective of doubling the range of supply voltage variation that can be tolerated. In the following sections possible ways of achieving this will be described and analyzed briefly. Since any change in tube construction must be evaluated and compared with alternative means external to the tube, the present study also includes devices which are circuit elements to be used with tubes.

II. Ballast Tube

The gas-filled ballast resistance tube[3] is ideal for stabilizing the input current to a low-voltage, high-current load, its biggest drawback being limited life compared to electron tubes. The ballast resistor usually takes the form of an iron wire mounted in a glass envelope filled with hydrogen at a pressure of about 10 mmHg. The theory of such a system, outlined in Appendix I, predicts that the current will be nearly constant over a wide range of voltages.

Commercially available units are rated to have a current spread confined to ±5 per cent for a voltage change of ±50 per cent. The ambient temperature may vary from -40 degrees C to +50 degrees C. The actual ballast voltage must be chosen high enough so that this range will absorb the difference between a line-voltage change of ±20 per cent and the allowable heater voltage change of ±8.6 per cent which corresponds to ±5 per cent change in current. A simple calculation shows that the average ballast resistor voltage (or power) will be 0.38 of the average value for the heater.

Now the factors dictating the choice of materials for a ballast resistor will be considered in order to determine whether simpler construction or lower power loss than the above can be achieved. By Eq. (A-7) of Appendix I it is necessary to have

$$n > w \tag{1}$$

where

$$w \equiv \frac{T}{W}\frac{dW}{dT} \tag{2}$$

and

$$n \equiv \frac{T}{R}\frac{dR}{dT} \tag{3}$$

Here W is the power dissipated by the resistor, R is its resistance and T is its temperature in degrees Kelvin. It is seen that w is a property primarily of the operating conditions (i.e., the way in which heat is lost) of the ballast resistor, while n is a property of the material used.

At this point it would be well to eliminate some possible sources of confusion in terminology which exist in the literature. The "temperature coefficient of resistance" is commonly defined as $(1/R_0)\, dR/dT$ where R_0 is the resistance at 0 degrees C. This is the definition given in the "Handbook of Chemistry and Physics" although the tabulated values there are often based on an average $\Delta R/\Delta T$ from 0 to 100 degrees C. The resistivity is also sometimes given over a limited range by a power series expansion of the form $R = R'[1 + \alpha(T - T') + \beta(T - T')^2 + \ldots]$ where R' is the resistance at some specified temperature T'. This is the method of the "International Critical Tables". It is apparent that even if T' = 273 degrees K (0 degrees C.), α is not the "temperature coefficient of resistance" unless the resistance change is linear with temperature ($\beta = 0$).

The most generally useful coefficient in the present work, however, is the quantity, n, referred to above, which shall be called the "temperature exponent of resistance" defined as $n = (T/R)\, dR/dT$. This is the quantity tabulated by Jones and Langmuir[4] for tungsten; it is the fractional change in resistance divided by the fractional change in absolute temperature. If, over a limited range, $R = cT^n$ where c is some constant, n is the quantity just defined; n is thus the slope of a logarithmic graph of R against T.

In a similar manner, w--defined in Eq. (2)--may be called the "temperature exponent of power dissipation". For example, a black body radiator obeys the Stefan-Boltzmann equation $w = k_1 eT^4$ and for it $w = 4$ since k_1 and e are constants. For a metal, however, e increases with temperature and w is larger than 4. Since, as has been mentioned, n must be greater than w, the problem of good ballast action is to achieve a sufficiently large n, or a sufficiently small w.

For the case of a radiation-cooled resistor at temperature T with the surroundings at T_0,

$$W = k_1 (T^4 - T_0^4)\, e$$

where k_1 is a constant and e is the thermal emissivity. At temperatures high enough to give large values of n, the term in T_0 is usually negligible while for pure metals e is approximately proportional to T leading to the relation

$$W = k_2 T^5$$

It follows from this and Eq. (2) above that, for radiation cooling of a metal,

$$w = 5 \tag{4}$$

Even iron, which assumes one of the largest values of n known, shows a maximum of n = 2.3 at 970 degrees K, and tungsten, which can be used at very high temperatures has n = 1.3 at all temperatures above 273 degrees K. It is thus apparent on comparing Eq. (4) with the condition of Eq. (1) that radiation cooling is inadequate for constructing a ballast resistor unless one can find metals with a value of n about twice that now known.

On the other hand, if there could be constructed a resistance completely cooled by conduction to a sink at temperature T_0, without having the cooling medium also short out the electrical resistance, then

$$W = k_3 (T - T_0)$$

(for one-dimensional flow and constant heat conductivity) and again using the definition of Eq. (2) for w

$$w = \frac{1}{1 - (T_0/T)} \tag{5}$$

Note that w is infinite at $T = T_0$, but drops to 2 at $T = 2T_0$ (600 degrees K, if T_0 is room temperature).

The use of a gas-filled tube[19,20] is a successful maneuver to lower the value of w by what is largely thermal conduction, effected without danger of electrical conduction. Hydrogen and helium may be used, the former working better and reaching its optimum cooling effect at a pressure of 10 mmHg. The value for w is then 2 at 550 degrees K, decreasing continuously and reaching 1.3 at 1400 degrees K. From 570 degrees K to 1070 degrees K the w at this pressure is less than the n for iron, making possible the commercial ballast tube. These data are shown graphically for iron in Fig. 1 and are plotted for iron, nickel, silver, platinum, and tungsten in the paper by Jones.[18]

Of course, the discovery of a material with a value of n greater than w over a larger temperature range than exhibited by iron will lead to a better ballast resistor even if the maximum value of n is not increased. In fact, iron is now used in preference to nickel, which, over a narrow range of temperatures, has a greater n than the maximum for iron. However, a metal with still greater n must be found before a vacuum ballast will be possible.

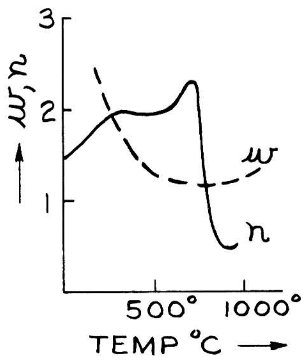

Fig. 1 - Functions governing the ballast action of an iron wire in hydrogen at low pressure.

It is difficult to find resistivity data on many metals and alloys for temperatures above 100 degrees C.[5] The behavior of the ferromagnetic elements (cobalt also exhibits a large value of n) suggests that better ballast resistor materials might be found among the many magnetic alloys now known, probably in the vicinity of the Curie point. It may be concluded that simpler construction (e.g., elimination of gas cooling) and lower power loss in the ballast resistor await the discovery of such better materials.

III. Resistance Temperature Exponent of Heater

As mentioned in Part I, there is already some stabilizing action built into standard tubes because the resistance goes up with the voltage. A very satisfactory solution of the present problem would result if this effect could be sufficiently enhanced by the production of a heater with a higher temperature exponent of resistivity. No additional components would be required and all the power would be delivered to the cathode; the stabilization should be such that the remaining extreme variations of heater power would be tolerable.

It may be said at the outset that by this means the solution of the problem posed in the introduction is impossible. This follows from the analysis of ballast resistors given in the Appendix I. It is explained there that the resistance of a wire may increase very rapidly with voltage because of the attendant temperature rise; that the dynamic resistance may even become infinite resulting in a constant current device; but that, because of the instability which results in the ballast action, the dynamic resistance never becomes negative to give a falling current characteristic. This means that the greatest stability a heater can provide is to keep the current *constant* with rising voltage. Under such conditions and assuming radiation cooling with a constant emissivity[a], it is clear that

$$\frac{dP}{P} = \frac{dV}{V} \quad dP/P = dV/V = 4\ dT_c/T_c$$

where P is power input, V is voltage and T_c is cathode temperature in degrees Kelvin. A voltage variation of ±20 per cent with a constant-current heater thus means a spread of ±50 degrees K for a cathode at 1000 degrees K. Consequently, the goal of keeping the temperature variation within ±40 degrees K is unrealizable by this method. For the latter figure (±40 degrees K), the permissible voltage variation would be ±16 per cent or from 5.3 to 7.3 volts. Nevertheless, significant improvement in performance may be achieved by increasing the temperature exponent of resistance of the heater and the relationships involved will now be treated in some detail in order to determine what resistance characteristics the heater wire should have for a given result.

Consider the effect of a heater voltage rise on the temperature of a cathode where all the heat losses are by radiation to surroundings near room temperature. Assume that under normal operating conditions, the cathode is at 1000 degrees K and the heater wire is at 1400 degrees K.

Let P be power input; V, heater voltage; T_h, heater temperature; T_c, cathode temperature; R, heater resistance.

$$P = \frac{V^2}{R}$$

For not-too-large changes these differentials[7] may be used,

$$dP = \frac{2V\ dV}{R} - \frac{V^2\ dR}{R^2}$$

whence

$$dP = \frac{2dV}{V} - \frac{dR}{R} \qquad (6)$$

P may be related to the temperature of the cathode coating by the radiation law, giving

$$P = k_1\ T_c^4,$$

$$\frac{dP}{P} = 4\frac{d\ T_c}{T_c} \qquad (7)$$

It is known, using the definition for n given in Part II, that

$$\frac{dR}{R} = n\frac{dT_h}{T_h} \qquad (8)$$

To determine the relation between dT_h and dT_c, one must assume a mechanism for the heat transfer between coated heater and cathode. If this is entirely by radiation, with constant emissivity to an equal area,

$$P = k_2\ (T_h^4 - T_c^4)$$

and by differentiating and then substituting from Eq. (7) it is found that

$$\frac{d\ T_h}{T_h} = \frac{d\ T_c}{T_c} \qquad (9)$$

Other reasonable assumptions for radiative transfer give substantially the same result. Substituting Eqs. (7), (8), and (9) into Eq. (6) leads to the relations

$$\frac{dV}{V} = (2 + \frac{n}{2})\ \frac{d\ T_c}{T_c} \qquad (10)$$

and

$$\frac{dR}{R} = \frac{2n}{4 + n}\ \frac{dV}{V} \qquad (11)$$

On the other hand, if the heat transfer from heater to cathode is primarily by conduction (across the insulation coating).

$$P = k_3\ (T_h - T_c)$$

and again differentiating and using Eq. (7)

there obtains

$$\frac{d\,T_h}{T_h} = \frac{d\,T_c}{T_c}\left(4 - \frac{3\,T_c}{T_h}\right) \qquad (12)$$

Now substitute Eqs. (7), (8), and (12) into Eq. (6) to get the equations

$$\frac{dV}{V} = \left(2 + 2n - \frac{3n}{2}\frac{T_c}{T_h}\right)\frac{dT_c}{T_c} \qquad (13)$$

and

$$\frac{dR}{R} = \left(2 - \frac{4}{2 + 2n - \frac{3n}{2}\frac{T_c}{T_h}}\right)\frac{dV}{V} \qquad (14)$$

Using Jones' and Langmuir's value of $n = 1.182$ for tungsten at 1400 degrees K, Eq. (11) gives $dR/R = 0.45\ dV/V$ for radiative transfer; Eq. (14) gives $dR/R = 0.71\ dV/V$ for conductive transfer. Measurements on a number of tungsten heater tubes give 0.42 for this factor indicating that radiation probably accounts for most of the heat transfer.

To see how much the resistance increment reduces the temperature change of the heater, this last value for dR/R may be substituted in Eq. (6), at the same time replacing $\frac{dP}{P}$ by its value from Eq. (7). This gives

$$\frac{dT_c}{T_c} = \tfrac{1}{2}\,(1 - 0.21)\,\frac{dV}{V} \qquad (15)$$

where the second term in the parenthesis arises from the resistance change and is the basis for the statement made in Part I that temperature changes are reduced about one-fifth in practice because the resistance increases with temperature.

Eq. (10) is useful in determining what temperature exponent of resistance (n) is necessary for a given performance. For example, if a material with $n = 3.0$ is made available, $2 + \frac{n}{2} = 3.5$. Then in order to limit $d\,T_c/T_c$ to ± 4.0 per cent, $dV/V \leq \pm 15$ per cent. This represents a 50 per cent extension of the operating range over tungsten, obtained by increasing drastically the temperature exponent of resistance almost threefold. It should be noted that ballast action (constant current) will start at a value of $n = 4$ (for a heater material with radiation cooling and constant emissivity). For such values the tacit assumption of a heater with a uniform temperature distribution is no longer valid and the analysis breaks down. With n's greater than this, one cannot, however, exceed the ±16 per cent voltage tolerance, as discussed in the second paragraph of this section.

The industry considers only tungsten and its alloys with molybdenum as being suitable for heaters, but a review of other materials will be instructive. One may start by examining the pure elements because a higher temperature exponent of resistance n is desired and the alloying of metals generally causes a decrease in n. A first selection may be made according to melting points. With present-day techniques the heater is fired at 1800 degrees C in processing the insulation coating and reaches 1600 degrees C in breaking down the cathode coating. High melting point, however, is not enough since many of the metals which remain solid have insufficient tensile strength at these temperatures. This is the reason why pure molybdenum, with a melting point of 2620 degrees C, is not used for heaters. In addition to high n, a large value for the actual resistivity at the operating temperature is also a desirable heater property since it is often a serious problem to keep the total resistance of the heater sufficiently high.

As already mentioned in Section II, it is difficult to find reliable published data on resistivity of metals at elevated temperatures particularly above 1100 degrees C. The exception is tungsten which is treated exhaustively in the papers by Jones and Langmuir. The available data on the platinum group of metals have been assembled in book form by Vines[8] but even here high-temperature data on resistivity are given for only platinum and palladium. Information on tantalum can be found in the paper by Malter and Langmuir[9]. Zirconium has been promoted in this country as a getter material by the Foote Mineral Company and some data and references can be found in a paper by Alnutt and Scheer.[10] Niobium (Columbium) is covered nicely by Grant and Reimann[11] and tungsten-molybdenum alloys are treated briefly by Bossart.[12] Handbook tables provide handy references but are incomplete and disagree with each other and within themselves (although usually the sources of the data are listed).

The pertinent data found in these sources have been assembled in Table I. Some remarks

Table I

Element	Symbol	Melt'g Pt. °C	Resistivity: μohm -cms	Resistivity: Temp. °C	Temp. Variation*: Coeff., α Per °C	Temp. Variation*: Expon. n	Temp. Variation*: Temp. °C	Remarks
Tungsten	W	3370	5.00	0	0.0047	1.275	0	
			37.19	1123	0.0063	1.182	1123	
Molybdenum	Mo	2620	5.14	0	0.0048	1.32	0	
Vanadium	V	1715						Hard to work
Platinum Group								
Platinum	Pt	1773	9.83	0	0.0039	1.06	0-100	Low hot strength
			45.8	1100	0.0027	0.79	1100	
Iridium	Ir	2454	5.3	0	0.0039		0-100	Hard to work
Osmium	Os	2700?	9.5	0	0.0042		0-100	Unworkable
Rhodium	Rh	1985	4.3	0	0.0044		0-100	
Ruthenium	Ru	>2450	14.4	18				Hard to work
Reactive Group								
Boron	B	2300						
Carbon	C	>3500	3500	0	-0.0005			Brittle
Thorium	Th	1845	18	20	0.0021		20-1800	Radioactive
Titanium	Ti	1800	Data conflicting					
Zirconium	Zr	1860	39	0	0.0045		0-100	Absorbs Gases
Tantalum	Ta	2850	47	0	0.0035		0-100	
Rare Group								
Columbium	Cb	2500				0.735	1510-2240	
Rhenium	Re	3000?						
Yturbium	Yb	1800						

*See Part II of text for definitions

have been added, but these are not meant to be complete. Elements with a melting point above 1600 degrees C have been chosen and are listed in several groups. Although the platinum metals look most promising because of their chemical inertness, there is no indication from the data on platinum and palladium (M.P. 1554 degrees C and so not listed) that a large value of n is to be expected. The next group of elements marked "reactive", are usually rejected because of their chemical activity which would cause reduction of the alumina insulating coating. Even cladding with a less active metal would probably be inadequate because of diffusion at the high temperatures involved.

If a material is available as wire, its resistance behavior for heater use can be determined by simple experiment. The wire is mounted in a vacuum and heated to about 1400 degrees K by an electric current. A voltage-current characteristic is then taken and compared with similar data for other materials. Actual temperatures need not be measured accurately since it is the voltage-current behavior that ultimately determines the material's usefulness as a wide-voltage range heater.

IV. Arbitrary Combinations of Heater and Stabilizing Resistance

In the preceding two sections the physical characteristics of the ballast tube and of the heater were considered separately. It is interesting to know what performance can be achieved by using two series elements of arbitrary current-voltage relationship. This opens the question of using thermistors[13]-- semi-conductors with very large negative temperature exponents of resistance. These can be made to exhibit negative dynamic resistance over a portion of their characteristic.

At the outset it may be said that there is no advantage to a dynamic negative resistance for the *stabilizing* element. If one were used, the heater would have to exhibit positive resistance for stability. Since a rise in line voltage (and hence in current) would reduce the voltage drop on the negative resistance, the heater would experience a greater voltage change than if it had been connected directly across the line. A bridge circuit using thermistor (or thyrite) elements will, however, stabilize the heater and is considered later.

It is possible to give a general treatment[14] of the circuit consisting of two arbitrary elements connected in series. The graphical representation of Fig. 2 will be very helpful. The logarithm of the voltage (V) is plotted against the logarithm of the current (I). An arbitrary operating point has been chosen at a line voltage (V_{lo}) of 1. Line voltages of 0.8 and 1.2 (V_{lo} ±20 per cent) have also been indicated since these are the extremes for which stabilization is desired. Loci of constant power are straight lines with a slope of −1 and a normal heater power (P_{ho}) of 1 has been arbitrarily chosen. Lines are also drawn for 1.16 P_{ho} and 0.84 P_{ho} since these are the limits already proposed for an oxide-coated cathode.

Loci of constant resistance are straight lines with a slope of +1. A non-ohmic relation between current and voltage will be indicated by some other curve. If V increases with I (positive dynamic resistance), the slope will be positive; if V decreases, the characteristic has a negative slope. In general, the slope gives the ratio of the dynamic resistance (R_d) to static resistance (R_o) at any point. This can be shown as follows.

By definition $$R_d = \frac{dV}{dI}$$

and $$R_o = \frac{V}{I}$$

$$\frac{d \log V}{d \log I} = I \frac{d \log V}{dI} = \frac{I}{V}\frac{dV}{dI}$$

$$= R_d/R_o.$$

It can be seen from curve a of Fig. 2 that a simple ohmic resistance is inadequate for the heater since the curve exceeds the power limits when it is extended to the line-voltage limits. Similarly curve b shows that even a single element with infinite dynamic resistance would not suffice. It is apparent then that if an element with positive dynamic resistance is used for the heater, it must be connected in series with a control unit having a higher ratio of dynamic-to-static resistance so that a greater fraction of the line voltage changes appears across the control than across the heater.

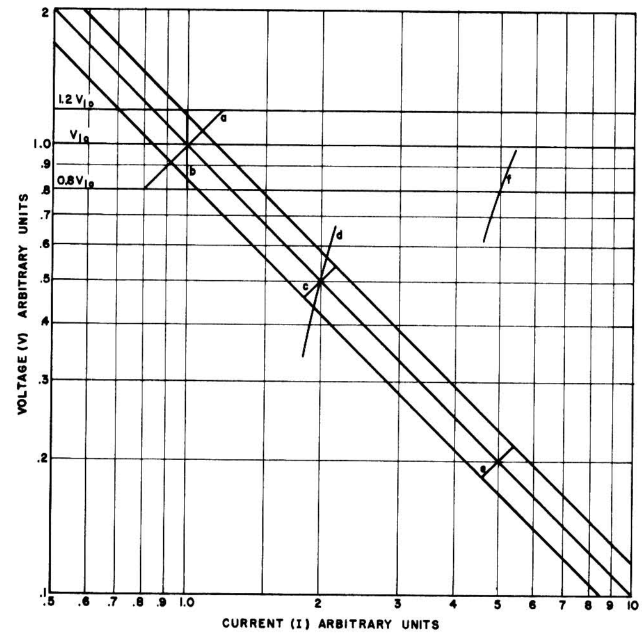

Fig. 2 - Graphical stabilization of positive resistance.

Curve c of Fig. 2 is the characteristic of an ohmic-resistance heater chosen so that half the line voltage or 0.5 appears across it at the operating point and the curve is drawn between the allowable power extremes. Curve d is the characteristic that the series control element must have if the line voltage varies between its allowable extremes. The curve is determined graphically as follows: The three points on curve c corresponding to minimum, normal, and maximum line voltage are known. At these three currents the heater voltage is subtracted from the line voltage to give the control element voltage and a smooth curve is drawn through the three points so obtained. It is seen that the dynamic-to-static resistance ratio is greater for the control element than for the heater. Curves e and f are similar characteristics where the normal voltage drop (and hence the dissipation) is greater for the control element. By way of compensation, the required slope of the control element has been reduced.

If the heater characteristic is redrawn with a greater slope, it is found that the control characteristic must also be steeper. This will be verified in later analysis, but before judging this apparent anomaly, it should be noted that required current extremes are reduced for the greater heater slope. This is an important consideration if an iron-hydrogen ballast tube is used for the control resistance. It may also be remarked that, although a heater characteristic with a steeper slope is more nearly self-stabilizing, its final stabilization may be more difficult since the control element must have a steeper logarithmic characteristic than the heater. For example, a heater with infinite dynamic resistance cannot be stabilized at all!

The question of whether to use a negative dynamic resistance heater can be investigated with the help of these curves. Such a heater may not need a series control element to limit the power variation, but will need it to achieve stable operation. The total series resistance must be positive, which means that the current will increase with the line voltage and that the voltage drop across the negative dynamic resistance of the heater will decrease. In Fig. 3, curve a represents such a possible negative resistance and the characteristic must be traversed from left to right as the line voltage increases. Curve b in the same figure represents an ohmic (positive) resistance drawn through the same operating point while curves d and e are, respectively, the characteristics of the control elements required to stabilize the heaters and are determined as discussed earlier in this section. It is seen that curve d is steeper than curve e, but curve e covers a wider current range. This appears to be generally true; the limiting case is the convergence of curves a and b to a horizontal line c, which requires the intermediate stabilizer characteristic f. As one departs from the horizontal curve c, the stabilizer for a positive heater resistance becomes steeper and shorter, and conversely for a negative heater resistance.

All the control elements shown in Fig. 3 dissipate the same power at the operating point and so the question is, which characteristic is easier to realize? The answer cannot be given categorically, but it may be said that the problem deals with devices of the ballast type--that is, large dynamic-to-static resistance ratios. In general, these are suitable for operation over a restricted current range and do not sustain their desired characteristics over large current ranges. For example, even a tungsten heater, for which there is less current variation than an ohmic resistance, exceeds the current variation limits of existing ballast tubes when the line voltage varies ±20 per cent. It would thus appear that the effective use of a negative dynamic heater resistance would be difficult in practice although not out of the question in principle.

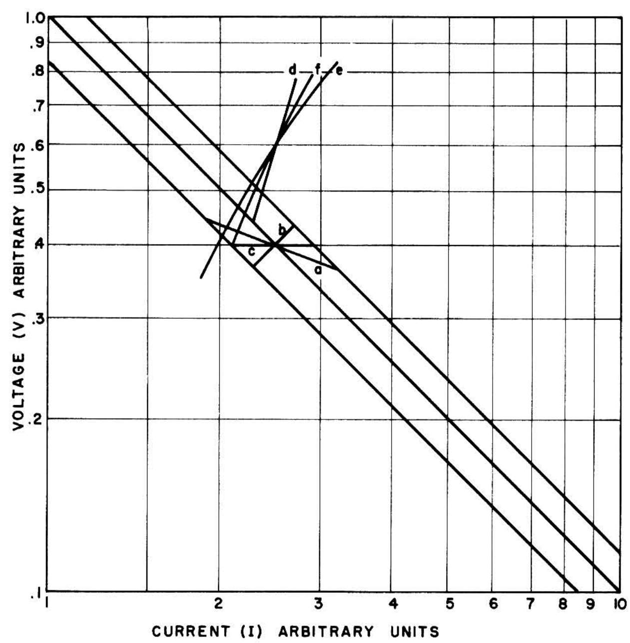

Fig. 3 - Relation between heater and control at constant power.

A special case which is interesting enough to warrant further discussion is that of a negative heater resistance that is approximately linear in Cartesian coordinates. Its power dissipation will go through a maximum with increasing line voltage. This can readily be appreciated since in Cartesian coordinate the loci of constant power are equilateral hyperbolas, V·I = const. A straight line of negative slope will be tangent to one of the hyperbolas at some point and there the power, V·I, will be a maximum for that negative resistance characteristic. For maximum power stability, it is natural to have the normal operating point at this power maximum. In most cases, then, where the average line voltage is the most probable one, the negative dynamic resistance heater would be operating at the undesirable condition of maximum power when the line voltage is normal. This heater characteristic may, however, be desirable for some applications--for example, where the normal value of line voltage occurs nearer to one of the extremes than to the other.

The most generally useful type of heater characteristic still appears to be a positive dynamic resistance and a general analysis can be made for the case where this is in series with a control element, also with positive dynamic resistance. The parameters which will enter into the analysis are: ratios of dynamic-to-static resistance for each element, the tolerances on line voltage and heater power, and the ratio of control power to heater power. This analysis is carried out in Appendix II and yields the following results; the fractional deviation in current from the normal value is

$$\Delta I / I_o = t_p \,/\, (1 + r_h)$$

where t_p is the fractional deviation in power of the heater and r_h is the dynamic-to-static resistance ratio for the heater.

The ratio of control element power to heater power at normal line voltage is

$$p_o = \frac{(t_{vl}/t_p)\,(1 + r_h) - r_h}{r_c - (t_{vl}/t_p)\,(1 + r_h)}$$

where t_{vl} is the fractional line voltage tolerance and r_c is the dynamic-to-static resistance ratio of the control element.

In this analysis $t_{vl} = 0.2$ and $t_p = 0.16$ and so

$$t_{vl}/t_p = 1.25$$

In the expression for p_o the numerator will always be positive; thus there is meaning only when r_c is sufficiently larger than r_h for the denominator to be positive. In this range p_o will be a monotonically decreasing function of r_c. This is shown in Fig. 4 for two types of heater, an ohmic resistance ($r_h = 1$) and tungsten ($r_h = 2$). Note that for the same p_o, a steeper control characteristic (greater r_c) is needed with a steeper heater characteristic (greater r_h) as already mentioned. This is true for other values of r_h, too, since

$$\frac{\partial r_c}{\partial r_h} = \frac{(t_{vl} - t_p) + t_{vl}\, p_o}{t_p\, p_o}$$

which is certainly positive for $t_{vl} > t_p$.

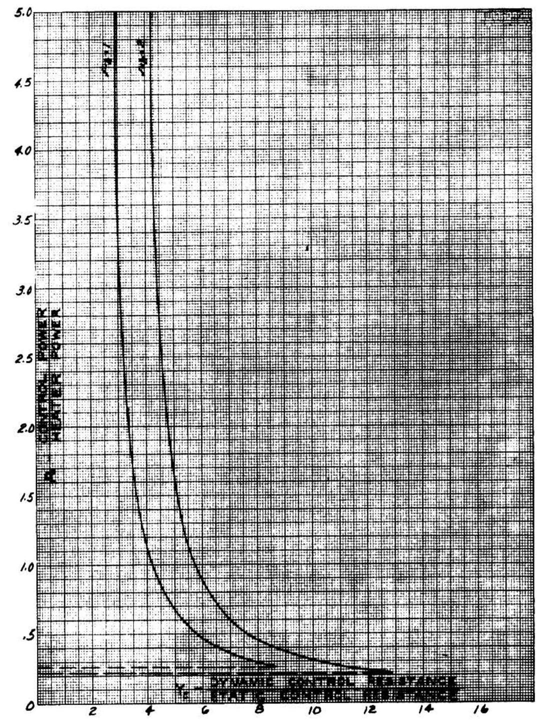

Fig. 4 – Average power in control element.

Although the analytic expressions for p_o as a function of r_c shows $p_o \rightarrow 0$ as $r_c \rightarrow \infty$, there is actually a non-zero lower limit to p_o. The discrepancy arises because the mathematical expressions used do not distinguish the physically significant point where the heater voltage becomes equal to the line voltage. Thus, a lower limit on V_{co} is set by the condition that V_{co} must be at least as large as the change in V_{co} required when V decreases by the maximum amount. A minimum V_{co} means a minimum for $p_o = V_{co}/V_{ho}$ which we may call P_{om}. It is shown in Appendix II that

$$P_{om} = \frac{t_{vl} - t_{vh}}{1 - t_{vl}}$$

where t_{vh} is the fractional voltage change allowed for the heater and is given by

$$t_{vh} = t_p - (\Delta I / I_o)$$

$$= t_p\, r_h \,/\, (1 + r_h).$$

For a constant resistance heater, $t_{vh} = t_{ph} / 2 = 0.08$ with the tolerances already specified and $P_{om} = 0.15$. A tungsten heater, with $r_h = 2$, has $t_{vh} = 0.15$ and $P_{om} = 0.06$. Thus with present heaters, the power loss in the control will be at least 6 per cent of that in the heater at the normal operating point even if the control has the ideal characteristic of zero voltage drop at minimum line voltage. This may be compared with the value of 38 per cent found in Part II for existing ballast tubes.

At this point, one may question the desirability of a high slope (high-temperature exponent) heater material since it seems to require a higher slope control element. The crux lies in the reduced current variation possible when r_h is larger. For example, it has already been mentioned that even with a tungsten heater and a present-day ballast tube, these equations would call for the operation of the ballast tube beyond its linear region, the result being a rapid decrease in the effective r_c of the ballast.

Brief mention will now be made of bridge circuits with non-linear elements. These have been reviewed by Kallman.[15] Any material with a decreasing resistance characteristic can be used; for example, thermistors, thyrite and tube and contact rectifiers. In a rectifier power supply with a 0.1-megohm load, the overall regulation was reported as $\Delta V_h = \pm 0.5$ per cent for an alternating-current line voltage change of 22 per cent, which is very good. The efficiency, however, is poor with a minimum wasted power of three times the useful power. Moreover the normal operating point is at maximum voltage, not average voltage.

V. Alternate Heating Methods

It is possible that a different method of heating the cathode may be more readily controlled than a resistance heater. For example, the tube shown in Fig. 5 can be made inherently self-stabilizing. K and P are the oxide-coated-cathode and the plate of a conventional tube which may also have some grids in it. F is a filament heated by the voltage E_f which is derived from the same ultimate power source as the voltage E_o and hence the two voltages fluctuate in proportion.

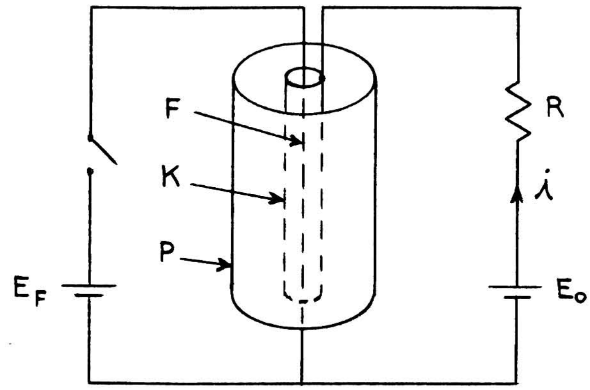

Fig. 5 – Stabilized bombardment heater.

If the electron emission from F is temperature-limited, the temperature of K will be stabilized because of the very great dependence of such emission on temperature. To understand this qualitatively, suppose E_o were to increase, this by itself would tend to increase the power input to cathode K. At the same time however, E_f increases proportionately, raising the temperature of filament F and resulting in a larger emission current. Thus the plate resistance of the diode formed by F and K would drop relative to R and reduce the voltage appearing between F and K.

Analysis shows that if F is a tungsten filament and if R is chosen to have a voltage drop 20 per cent greater than the diode, stabilization will be perfect. That is, power input to the cathode will have a maximum or stationary value with respect to changes in supply voltage E_o. Unfortunately, perfect stabilization at one point does not necessarily mean wide operating range and for change in E_o greater than ± 3 per cent this device is no better than a conventional tungsten heater.

VI. Relay Regulators

The most efficient way of controlling the cathode temperature is by keeping the average heater current constant without wasting power in an external load. This might be achieved by interrupting the current regularly and varying the duty cycle as the supply voltage changes so that the average heater current is kept at the correct value. The heat capacity of the cathode assembly would keep its temperature substantially constant if the interruptions were rapid enough.

An excellent way of controlling the average heater current (or voltage), as suggested, is to have the circuit closed through a relay which can be energized by the plate current in an emission-limited diode. The diode filament is connected in parallel with the other heaters and the relay is normally closed. If the diode filament gets too hot, its plate current increases and the relay opens the heater circuit long enough for the filament to cool slightly.

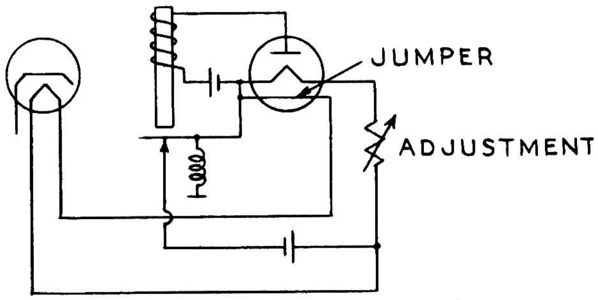

Fig. 6 - Emission operated relay.

The circuit is given in Fig. 6. The diode envelope should contain a jumper as shown which is connected in series with the controlled heaters. This is important so that heater voltage may not be applied to the tubes without the diode being in place. A resistor in series with the filament enables the heater current to be stabilized at any desired value. This method of control has the disadvantage of high initial surge current although the cathodes will not reach an excessive temperature because the diode filament will heat faster. As a matter of fact, by increasing the thermal lag of the diode filament, the circuit provides a means for shortening the initial warm-up period by supplying overvoltage when the power is first turned on. In the event of diode or relay failure, the controlled tubes will be overheated. There does not seem to be any way of eliminating this hazard although the tubes could easily be protected from burnout by a slow-blowing fuse in the heater circuit.

Variations of this control circuit are possible, but are less desirable. For example, the diode filament may be connected in series with the heaters, but then the number of tubes controlled could not be readily changed. To avoid switching the full heater current, the relay could be arranged to short a series resistor intermittently. This would result in extra power being wasted in this resistor although the device would still be more efficient than a ballast tube. To reclaim this power, the resistor might be replaced by a section of the heater which could be intermittently shorted by the relay. In this case, however, there would be no suitable place to connect the diode filament since the heater power would not be reflected by the voltage across either section (or all) of the heater because the effective heater resistance would be changing.

A relay may also be used in a simple device to permit stepwise control of the heater voltage. The relay coil is connected in parallel with the supply voltage. The relay contacts are normally closed and short a series resistor. If the supply voltage rises beyond a predetermined value, the series resistance will be thrown into the circuit. Consequently, the range of line voltages that the tube will stand will be effectively almost doubled.

VII. Miscellaneous Devices

A few other devices[18] will be considered only briefly because their advantages and disadvantages are obvious.

A thermostatic switch may be incorporated in the tube either to open the heater circuit when the cathode is too hot or to short-out part of the heater when the cathode is cool. This scheme has the merit of not wasting power at any voltage level, but is limited by the reliability of the contact and introduces a possible noise source into the tube. It would be necessary to protect the switch from the influence of anode heating in the tube. This device emphasizes the fact that it is only average heater current that need be stabilized and suggests the possibility of devising a relay oscillator or thyratron circuit external to the tube which has the property of passing a constant average current independent of line voltage. The advantage would again be one of a saving in power compared with a voltage-dropping device.

A cathode coating that increases its emissivity with temperature would reduce temperature changes by requiring a disproportionately large power increase to raise its temperature. Since, however, the power already varies as a large exponent of the temperature, the emissivity change would have to be relatively large. Although the process is essentially one of reducing the cathode efficiency, the over-all efficiency might be better than that obtained by using an external stabilizing device which absorbs extra power even on low-voltage operation.

An even more attractive-sounding solution would be a cathode coating capable of operation over a wider range of temperature. For example, a pure SrO coating could be run hotter than the usual BaO-SrO mixture. The net result of this and the preceding developments, however, might well be a cathode inherently very inefficient under all operating conditions.

Either alternating-current or direct-current voltages at high currents can readily be stabilized to a high degree, if the source is alternating current, by using a diode-controlled regulator[17] employing a saturable core reactor. The unit necessarily employs heavy transformers and could be justified only where weight and space were secondary factors. More pertinent is the simpler constant-voltage transformer which is now available in a form providing a-c plate and heater voltages directly. Although normally intended for ±15 per cent voltage variation (with ±1 per cent output regulation), this is not a basic limitation. The units require an extra weight and space allowance as well as special precautions against frequency variations such as normally occur in aircraft generators.

VIII. Conclusions

Although vacuum-tube heaters may be operated with a voltage variation of ±10 per cent, a range of twice this would be desirable. The series ballast tube exists as an attractive solution but has a short life compared to an electron tube and on the average would dissipate a power at least equal to 38 per cent of that in the tube heater. It is possible that better ballast resistor materials may be discovered which would decrease this extra power dissipation. The divers new magnetic alloys deserve further investigation.

The discovery of a new heater material with a much larger temperature exponent of resistivity than tungsten would not be a complete solution, but would extend the voltage range of operation. The platinum group and other metals have some desirable heater properties but insufficient data are available.

A graphical representation shows that negative-resistance heaters are undesirable because of the excessive power dissipated in the series stabilizing element. A general analysis of two positive resistances in series explains their performance in terms of the ratio of dynamic-to-static resistance. This ratio for the stabilizer must always be greater than for the heater. With a tungsten heater, having a dynamic-to-static resistance ratio of 2, the average control power theoretically can be as low as 6 per cent of the average heater power for the tolerances already specified.

There may be a solution to the problem by some different method of heating the cathode. The scheme suggested using electron bombardment is self-stabilizing, but has a very limited operating voltage range.

A relay-operated switch in series with the heater supply and controlled by the temperature-limited emission from an auxiliary diode is a simple device that would control the average heater current without any power loss in series or shunt circuit elements. Step-wise voltage control, via a relay which introduces series resistance at high supply voltage, would double the existing voltage range of tube heaters. These devices, however, do not provide "fail-safe" operation.

Improvements in the cathode coating would be desirable but may be considered a separate problem whose solution does not seem to be at hand. A thermal switch inside the tube is less desirable than a non-mechanical solution to the problem but may be the only simple, low-cost, built-in means of solving the problem. Finally, standard electronic regulator circuits are likely either to be bulky or to consume excessive power, while standard constant-voltage transformers are frequency sensitive as well as bulky.

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Appendix I

Theory of the Ballast Resistor

The mechanism of the gas-filled ballast tube's action is treated in two papers by Jones.[18,19] Some further discussion can be found in an earlier treatment by Busch[20] which is well summarized in Ollendorf's book.[21] A brief explanation of the ballast action, based on these references, will be given here.

Consider a voltage E to be applied to the terminals of a resistance R which increases with temperature: $dR/dT>0$. Regardless of the exact mechanism of heat loss from R, the power dissipation W may also be assumed to increase with temperature: $dW/dT>0$. In the steady state, the input power is equal to W or $E^2/R = W$. It then follows that $dE/dT>0$. Thus as E increases so does the temperature and hence R does. If R were to increase rapidly enough, it might keep the current I from increasing--or even cause it to decrease, resulting in a characteristic like that of Fig. 7 which is distinguished by I having a relative maximum at X. The region from X to Y represents a voltage-controlled negative resistance: $dE/dI<0$.

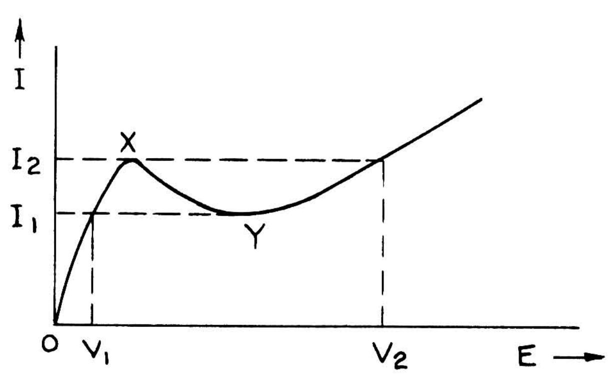

Fig. 7 – Voltage controlled negative resistance.

Qualitatively it is apparent that to achieve the negative slope on the I-E curve, the increase in R should be large for a given rise in temperature, but that this temperature change must result from a sufficiently small increase in W since the power increment is reduced due to the current decrement; that is, dR/dT should be large and dW/dT should be small. The exact condition for negative resistance is quite simple to derive as follows:

$$\log (E/I) = \log E - \log I = \log R, \quad \text{(A-1)}$$

$$\log E\cdot I = \log E + \log I = \log W. \quad \text{(A-2)}$$

Subtracting (A-1) from (A-2) and differentiating gives

$$\frac{2}{I}\frac{dI}{dT} = \frac{1}{W}\frac{dW}{dT} - \frac{1}{R}\frac{dR}{dT}. \quad \text{(A-3)}$$

However, since $dE/dT>0$ and for negative resistance $dI/dE<0$.

$$\frac{dI}{dT} = \frac{dI}{dE}\frac{dE}{dT} < 0$$

and so (A-3) yields the condition

$$\frac{1}{R}\frac{dR}{dT} > \frac{1}{W}\frac{dW}{dT} \quad \text{(A-4)}$$

for the negative resistance characteristic.

If this were the whole story, a resistance which met condition (A-4), (e.g., an iron wire in hydrogen), would not have a constant current characteristic, but over an operating range of voltages, V_1 to V_2 of Fig. 1, the current would vary between the limits I_1 and I_2. However, although this negative resistance characteristic can be predicted from the relations $E = \sqrt{W(T)\cdot R(T)}$ and $I = \sqrt{W(T)/R(T)}$, using experimental values of W and R for iron in hydrogen, it is not observed experimentally for the system. The explanation[18] is that (A-4) is also the condition that the system be isothermally unstable: that is, the equilibrium which has a resistance wire at a uniform temperature between points X and Y of Fig. 7 is an unstable one. Part of the wire will cool to the temperature corresponding to point X and the rest will heat to the temperature of point Y. The sharpness of the demarcation is limited only by the thermal conductance of the wire. If the voltage across the wire increases, the current will stay constant but the hot length of the wire will increase. When the hot section comprises the whole wire, the current will again increase in a more conventional fashion.

The unstable nature of the negative resistance condition, (A-4), is inherent in the theory of voltage-controlled negative resistances.[22] This theory shows that the operating point on the negatively sloping region is unstable whenever the external series resistance is greater than the absolute magnitude of the negative resistance. It is better in the present instance to restate this to say that the point is unstable whenever the external conductance is less than the absolute magnitude of the negative conductance at the point in question. Thus, another voltage-controlled negative resistance in series would represent a *negative* external conductance which is certainly small enough for instability. Each elemental section of the ballast resistor is of this type and each acts as the external resistance which causes the ultimate isothermal instability of the entire group.

It is convenient to make the quantities in (A-4) dimensionless by multiplying through by the absolute temperature T and to define

$$w \equiv \frac{T}{W}\frac{dW}{dT} = \frac{d\ (\log W)}{d\ (\log T)} \qquad (A-5)$$

and

$$n \equiv \frac{T}{R}\frac{dR}{dT} = \frac{d\ (\log R)}{d\ (\log T)}. \qquad (A-6)$$

Relation (A-4) may now be written

$$n > w, \qquad (A-7)$$

which gives the condition for ballast action to occur.

Appendix II

General Analysis of Positive Resistance Heater and Series Control Resistance

In the following relations the subscript "o" will denote values when the line voltage is normal; "2" will denote values for maximum line voltage

V_l = line voltage

V_h = heater voltage

V_c = control element voltage

I = current

Both resistances will be assumed linear over the restricted operating range, that is,

$$V_h = R_{ho} I_o + R_{hd} (I - I_o)$$

and

$$V_c = R_{co} I_o + R_{cd} (I - I_o).$$

Thus there can be defined what may be called static resistance (R_{ho} and R_{co}) and dynamic resistance (R_{hd} and R_{cd}) for the heater and control element respectively. Symbols for their ratios are:

$$r_h = R_{hd}/R_{ho}$$

$$r_c = R_{cd}/R_{co}$$

It was demonstrated in Part IV of the text that r_h and r_c are the slopes of log-log plots of the heater and control characteristics at the operating point.

Since the two elements are in series, they have the same current, I, and their powers P_h and P_c are proportional to their voltage drops. The power ratio at the operating point can thus be defined as p_o where $p_o = V_{co}/V_{ho} = R_{co}/R_{ho}$.

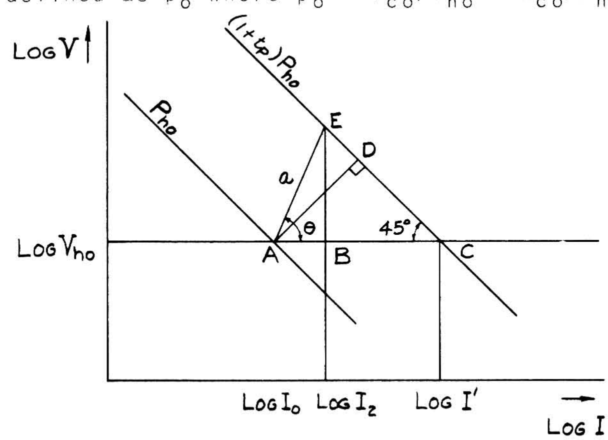

Fig. 8 - Construction for $\Delta I/I_o$.

There will be a certain fractional allowed variation in line voltage, t_{vl}, and in heater power, t_p. As these tolerances are utilized the current in the system will vary. One can relate r_h to the required operating range of current, $\Delta I = I_2 - I_o$, by the geometric construction of Fig. 8. Curve $\underline{a}$ is a selected linear portion of the heater characteristic near the normal operating point. Note that

$$\tan \theta = r_h$$

and

$$I' - I_o = t_p I_o.$$

From Fig. 8

$$\overline{AD} = \overline{AC} \cos 45^o = \overline{AC} / \sqrt{2}$$

$$\overline{AE} = \overline{AD} / \cos (\theta - 45^o)$$

$$= \overline{AC} / \sqrt{2} \cos (\theta - 45^o)$$

$$= \overline{AC} / (\cos \theta + \sin \theta)$$

$$\overline{AB} = \overline{AE} \cos\theta = \overline{AC} \cos \theta / (\cos\theta + \sin\theta)$$

$$= \overline{AC} / (1 + \tan \theta)$$

$$= \overline{AC} / (1 + r_h)$$

$$1 / (1 + r_h) = \overline{AB} / \overline{AC}$$

$$= \frac{\log I_2 - \log I_o}{\log I' - \log I_o} = \frac{\log (I_2 / I_o)}{\log (I' / I_o)}$$

$$= \frac{\log \left(1 + \frac{I_2 - I_o}{I_o}\right)}{\log \left(1 + \frac{I' - I_o}{I_o}\right)}$$

To simplify the last expression, use the relation

$$\log (1 + x) \approx x, \qquad x << 1.$$

The maximum value corresponding to x is t_p which is 0.23 in this study and corresponds to an error of 11 per cent in the logarithm. However, the error in the *ratio* of the logs will be somewhat less. Therefore

$$(I_2 - I_o) / (I' - I_o) = 1 / (1 + r_h)$$

or

$$\Delta I / I_o = t_p / (1 + r_h) \qquad \text{(A-8)}$$

The line-voltage tolerance may be expressed as

$$t_{vl} = (V_{l2} - V_{lo}) / V_{lo}$$

$$= \frac{\Delta I \; (R_{hd} + R_{cd})}{I_o \; (R_{ho} + R_{co})}$$

Using (A-8), $t_{vl} (1 + r_h) / t_p = (R_{hd} + R_{cd}) / (R_{ho} + R_{co})$.

This equation involves four resistance coefficients, but since it is linearly homogeneous in them, it can be reduced to three ratios of resistances. These ratios will be expressed in terms of $r_h = R_{hd}/R_{ho}$, $r_c = R_{cd}/R_{co}$ and $p_o = R_{co}/R_{ho}$. This gives

$$t_{vl} (1 + r_h) / t_p = (r_h + p_o r_c) / (1 + p_o).$$

This can be solved for p_o:

$$p_o = \frac{(t_{vl}/t_p)(1 + r_h) - r_h}{r_c - (t_{vl}/t_p)(1 + r_h)}.$$

The solution for r_c is:

$$r_c = (t_{vl}/p_o t_p)(1 + p_o)(1 + r_h) - (r_h/p_o);$$

that for r_h is:

$$r_h = \frac{p_o r_c - (t_{vl}/t_p)(1 + p_o)}{(t_{vl}/t_p)(1 + p_o) - 1}.$$

There is a minimum allowed value of p_o which corresponds to the case where $V_c = 0$ at minimum line voltage. Call this minimum p_{om}. It occurs when $\Delta V_c = -V_{co}$.

$$\Delta V_l = \Delta V_h + \Delta V_c = \Delta V_h - V_{co}$$

Dividing through by

$$V_{lo} = V_{ho} + V_{co}$$

gives

$$t_{vl} = \frac{-\Delta V_l}{V_{lo}} = \frac{-\Delta V_h + V_{co}}{V_{ho} + V_{co}}$$

$$= \frac{(-\Delta V_h/V_{ho}) + (V_{co}/V_{ho})}{1 + (V_{co}/V_{ho})}$$

$$= \frac{t_{vh} + p_{om}}{1 + p_{om}}$$

where $t_{vh} = -\Delta V_h/V_{ho}$

is the heater voltage tolerance.

Solving for P_{om},

$$p_{om} = \frac{t_{vl} - t_{vh}}{1 - t_{vl}}.$$

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