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THE EQUIVALENT CIRCUIT OF THE DRIFT TRANSISTOR

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THE EQUIVALENT CIRCUIT OF THE DRIFT TRANSISTOR

Approved

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The four-terminal admittances for the drift transistor, derived from the admittance equations worked out by Kroemer[1], have been examined as functions of frequency and approximated by an equivalent circuit. The equivalent circuit is in two forms, one suitable for the common-base connection and the second suitable for the common-emitter connection. The frequency response of each equivalent circuit admittance is plotted and compared with that of the device admittance function which it represents.

The differences in the behavior of the phase of α at high frequencies are discussed qualitatively for the drift and diffusion transistors.

1. Introduction

Much has been written about equivalent circuits for junction transistors[1]. Most of the development to date, however, has been concerned with equivalent circuits for the ordinary diffusion transistor. Since the drift transistor employs a new principle not utilized in the diffusion transistor, it is to be expected that equivalent circuits for the diffusion transistor will not be applicable in all respects to the drift transistor. Kroemer[2] in 1954 derived expressions for the four-terminal admittance parameters for the drift transistor and gave a low-frequency equivalent circuit for the common-base connection. His admittance expressions are re-examined with the object of deriving equivalent circuits for both the common-base and the common-emitter connection. The results of this investigation are reported in the following sections.

The physical principles and methods of construction of the drift transistor are not discussed in detail in this bulletin. A thorough treatment of these aspects may be found in references 2 to 6.

In section 2, the four-terminal admittances as functions of frequency are examined and approximations necessary to reduce them to usable forms are introduced. The functions are then plotted for typical values of device parameters. A π configuration is assumed for the equivalent circuit and the elements of the π network are written in terms of the admittance functions. In section 3 the elements of the π network are approximated by equivalent two-terminal electrical networks. The procedure in arriving at the equivalent networks is to expand the combinations of the admittance functions, which make up a given π element, into a power series in a normalized frequency parameter x, where the frequency corresponding to $x = 1$ is approximately the α-cutoff frequency. The admittance of a suitably chosen two-terminal electrical network is also expanded in powers of a normalized frequency and its elements then evaluated by equating coefficients. Values of the circuit elements are calculated for typical values of the device parameters. In section 4, the diffusion and the drift transistor are discussed qualitatively in an attempt to explain the difference in behavior of the phase of α at high frequencies for the drift transistor.

2. The Four-Terminal Admittance Functions

The circuit equations for the three-terminal box shown in Fig. 1 may be written:

$$\begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \end{vmatrix} \tag{1}$$

When the equations are written in the form (1) terminal 3 is taken to be at the reference or ground potential. It is often more convenient to leave the reference potential unspecified. This can be done by forming the indefinite admittance matrix[7]:

$$\begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} & -Y_{11} - Y_{12} \\ Y_{21} & Y_{22} & -Y_{21} - Y_{22} \\ -Y_{11}-Y_{21} & -Y_{12}-Y_{22} & Y_{11} + Y_{21} + Y_{12} + Y_{22} \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \tag{2}$$

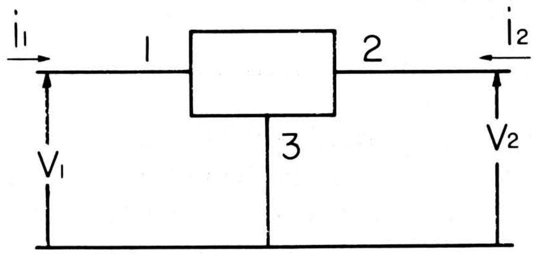

Fig. 1 – Three terminal box representation.

To form the admittance equations with any of the three terminals as the reference, the appropriate row and column of the indefinite matrix are crossed out. Then, provided the admittance matrix is known for one terminal common, the admittance matrix can easily be obtained if either of the other two terminals are considered to be common.

The four admittance functions for the drift transistor with the base lead as the common terminal have been worked out by Kroemer[2] for a p-n-p transistor and are repeated here for convenience.

$$(Y_{11})_b = (i_e^{(p)} - i_s)\frac{q}{kT}\,\frac{1+\frac{f}{\delta}\coth\frac{w}{\delta}}{1+\frac{f}{\delta_o}\coth\frac{w}{\delta_o}} \qquad (3)$$

$$(Y_{12})_b = i_c^{(p)}\frac{dw}{dV_c}\,\frac{e^{-w/f}}{\delta \sinh\frac{w}{\delta}} \qquad (4)$$

$$(Y_{21})_b = -(i_e^{(p)} - i_s)\frac{q}{kT}\,\frac{e^{w/f}}{\delta \sinh\frac{w}{\delta}\left(\frac{1}{f}+\frac{1}{\delta_o}\coth\frac{w}{\delta_o}\right)} \qquad (5)$$

$$(Y_{22})_b = -\left[i_c^{(p)}\left(\frac{1}{\delta \tanh\frac{w}{\delta}} - \frac{1}{f}\right) - i_d\frac{1}{L}\right]\frac{dw}{dV_c} \qquad (6)$$

where
$$i_s = \frac{qAD_p p_o}{\delta_o}\left(\frac{1}{\tanh\frac{w}{\delta_o}} + \frac{\delta_o}{f} - \frac{e^{w/f}}{\sinh\frac{w}{\delta_o}}\right) \qquad (7)$$

= the emitter saturation current which flows when a reverse voltage $\gg \frac{kT}{q}$ is applied at the emitter.

$$i_d = \frac{qAD_p}{L}\,p_o(w) \qquad (8)$$

= the hole saturation current of a p-n diode where the impurity concentration on the n-side is constant and equal to the impurity concentration at the end of the base region.

$i_e^{(p)}$ = the emitter hole current

$i_c^{(p)}$ = the collector hole current

D_p = Diffusion constant for holes

τ_p = recombination lifetime for holes

q = charge on the electron

A = junction area

p_o = equilibrium hole concentration at the base side of the emitter-base junction

$\frac{dw}{dV_c}$ = change of base-width with collector voltage

w = the average base-width

ΔV = potential energy difference in the base region due to the impurity distribution = qFw

F = drift field in the base region

$p_o(w)$ = equilibrium hole concentration at the collector end of the effective base

and the following notations and relations are used:

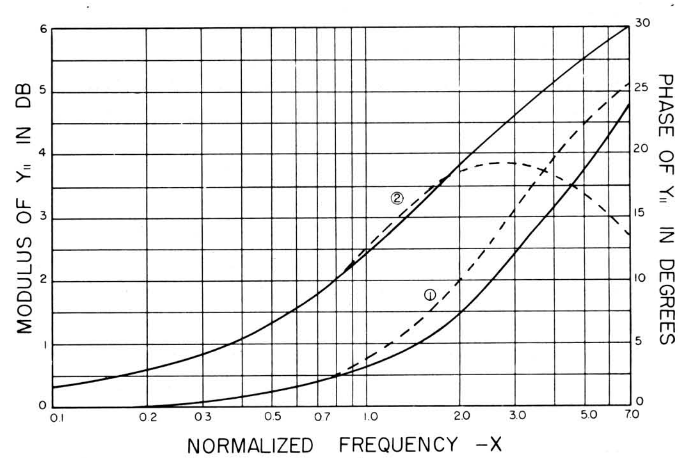

Fig. 2 – The variation of $\frac{Y_{11}}{Y_{11}(o)}$ with frequency. Curve 1 represents the modulus and curve 2 the phase. The dotted curves show the equivalent circuit approximation.

$$L = \sqrt{D_p \tau_p}\,, \qquad f = \frac{2kT}{qF}\,, \qquad \frac{1}{\delta_o^2} = \frac{1}{f^2} + \frac{1}{L^2} \tag{9}$$

$$\frac{1}{\lambda^2} = \frac{1 + j\omega\tau_p}{D_p \tau_p} = \frac{1}{L^2} + \frac{j\omega}{D_p}\,, \qquad \frac{1}{\delta^2} = \frac{1}{f^2} + \frac{1}{\lambda^2}$$

Eqs. (3) and (7) consider only the effect of the hole current (minority carriers). Kroemer points out[2] that the electron currents may be neglected in all of the parameters except Y_{11}. The effect of the electron current can be approximated in the admittance Y_{11} if the current gain a, the emitter efficiency γ, and the transport factor β are included.

$$a = -\frac{Y_{21}}{Y_{11}} = -\frac{Y_{21}^{(p)}}{Y_{11}^{(p)} + Y_{11}^{(n)}} = -\frac{Y_{21}^{(p)}}{Y_{11}^{(p)}}\;\frac{Y_{11}^{(p)}}{Y_{11}^{(p)} + Y_{11}^{(n)}} = \beta\gamma \tag{10}$$

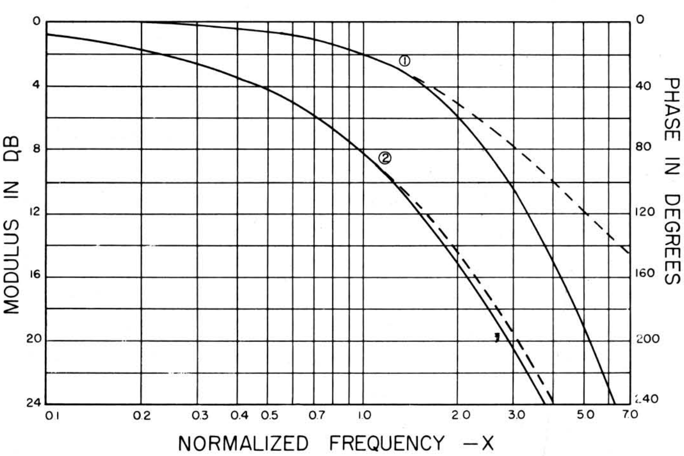

Fig. 3 – The variation of $\frac{Y_{12}}{Y_{12}(o)}$ and $\frac{Y_{21}}{Y_{21}(o)}$ with frequency. Curve 1 represents the modulus and curve 2 the phase. The dotted curves show the equivalent circuit approximations.

The emitter efficiency γ may be written

$$\gamma = \frac{1}{1 + \frac{Y_{11}^{(n)}}{Y_{11}^{(p)}}} \approx 1 - \frac{Y_{11}^{(n)}}{Y_{11}^{(p)}} \tag{11}$$

This effect will be of interest only when it is necessary to subtract from the term Y_{11} another admittance Y_{21}. These admittances are of the same order of magnitude. The important term under these conditions is $(1 - a)$ which may be written $(1 - a) = 1 - \beta\gamma$. At low frequencies the value of this term is much less than unity as both β and γ are close to unity. Consequently, the low-frequency value of γ becomes quite critical. At higher frequencies

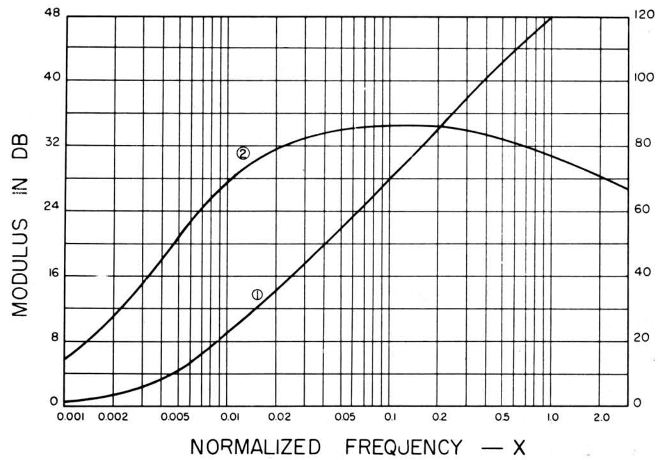

Fig. 4 – The variation of $\frac{Y_{22}}{Y_{22}(o)}$ with frequency. Curve 1 represents the modulus and curve 2 the phase.

both β and γ decrease with frequency, but β is a much stronger function of frequency than γ, since the only frequency dependent term in γ is the ratio $Y_{11}^{(n)}/Y_{11}^{(p)}$ which is small compared to unity (Eq. 11). Therefore, the frequency variation of γ can be neglected and only the low-frequency value γ_o be used throughout. Thus for the complete admittance $(Y_{11})_b$, $i_e^{(p)}/\gamma_o$ can be substituted for $i_e^{(p)}$. This discussion has neglected the effect of the emitter and collector transition capacitances which arise from the fact that with a change in voltage there is a change in depletion layer thickness and thus in the total amount of charge in the depletion layer. These capacitances could be added to the admittance functions[2] at this point and an equivalent circuit found which included them. However, another approach is to obtain an equivalent circuit for the intrinsic transistor and then to add the transition capacitances as external elements.

A few new terms are now defined and Eqs. (3) to (7) rewritten. Also, the saturation currents i_s and i_d compared to i_e and i_c are neglected.

$$\frac{w}{f} = \epsilon\,, \quad \frac{w}{\delta_o} = \epsilon' \quad \text{and} \quad \frac{f}{\delta_o} = \frac{\epsilon'}{\epsilon} = \sqrt{1 + \frac{w^2}{L^2\epsilon^2}} \tag{12}$$

$\epsilon = \frac{\Delta V}{2kT}$ and is a measure of the strength of the drift field.

If $\epsilon > 1$, $\frac{w^2}{L^2\epsilon^2}$ may be neglected compared to unity in most cases and $\epsilon' = \epsilon$. However, there is one occasion when this slight difference is important and this difference is preserved for that reason. One further change is made in notation. Let $\Omega = \sqrt{1 + j\omega\frac{\delta_o^2}{D_p}}$

Eq. (3) then becomes*

$$Y_{11} = \frac{i_e^{(p)}}{Y_o}\,\frac{q}{kT}\;\frac{1 + \dfrac{\epsilon'}{\epsilon}\,\Omega\,\dfrac{1 + e^{-2\epsilon'\Omega}}{1 - e^{-2\epsilon'\Omega}}}{1 + \dfrac{\epsilon'}{\epsilon}\,\dfrac{1 + e^{-2\epsilon'}}{1 - e^{-2\epsilon'}}} \tag{13}$$

The hyperbolic cotangents have been written in their exponential form. For $\epsilon' >> 1$ the negative exponents may be neglected since for $\epsilon' = 2$ the error in neglecting the exponents would be of the order of 5 percent, while for $\epsilon' = 3$ the error would be less than 1 percent. It is assumed that the drift field is strong enough to neglect $e^{-\epsilon'}$ compared to unity. It is also assumed for this purpose that $\epsilon/\epsilon' = 1$.

Eq. (13) then reduces to:

$$Y_{11} = \frac{i_e^{(p)}}{Y_o}\,\frac{q}{kT}\,\frac{1+\Omega}{2} \tag{14}$$

Making the same approximations as before, Eq. (4) reduces to

$$Y_{12} = 2\,i_c^{(p)}\,\frac{\epsilon'}{w}\,\frac{dw}{dV_c}\,\Omega\, e^{-(\epsilon + \epsilon'\Omega)} \tag{15}$$

Eq. (5) reduces to

$$Y_{21} = -\,i_e^{(p)}\,\frac{q}{kT}\,\Omega\, e^{(\epsilon - \epsilon'\Omega)} \tag{16}$$

and Eq. (6) reduces to

$$Y_{22} = -\,\frac{i_c^{(p)}}{w}\,\frac{dw}{dV_c}\left[2\epsilon'\Omega\, e^{-2\epsilon'\Omega} + \epsilon'\Omega - \epsilon\right] \tag{17}$$

In this last equation one more term has to be used in the expansion of the hyperbolic tangent, since the earlier terms almost cancel. For convenience in calculation it is simpler to put the admittance functions in terms of their low frequency values. Then,

$$\frac{Y_{11}}{Y_{11(o)}} = \frac{1+\Omega}{2}, \qquad \frac{Y_{12}}{Y_{12(o)}} = \Omega\, e^{\epsilon'(1-\Omega)}$$

$$\frac{Y_{21}}{Y_{21(o)}} = \Omega\, e^{\epsilon'(1-\Omega)}, \qquad \frac{Y_{22}}{Y_{22(o)}} = \frac{\Omega\, e^{2\epsilon'(1-\Omega)} + \frac{1}{2} e^{2\epsilon'}\left(\Omega - \frac{\epsilon}{\epsilon'}\right)}{1 + \frac{1}{2}\left(1 - \frac{\epsilon}{\epsilon'}\right) e^{2\epsilon'}} \tag{18}$$

These functions have been plotted for a value of $\epsilon' = 4$, and in the case of Y_{22} it has been assumed that $\epsilon/\epsilon' = 0.999688$, which is equivalent to assuming $w/L = 0.1$. The functions are plotted in terms of x, where $x = \omega\delta_o^2/D_p$ and $\Omega = \sqrt{1 + jx}$. These functions are shown in Figs. 2 to 4. The shape of the curve for Y_{22} (Fig. 4) depends quite critically on the assumed value of ϵ/ϵ'. The shape of the curve at low frequency depends on the relative low frequency values of the two terms in Eq. (17). At higher frequencies the second term dominates and the phase asymptotically approaches 45 degrees.

When considering the representation of the transistor by means of an equivalent circuit, the equivalent circuit parameters will be combinations of the four admittance parameters discussed above. To obtain the desired accuracy in some of these combinations, it may be necessary to obtain more accurate expressions for the admittance parameters than those used to plot the functions in Figs. 2 to 4. It is useful at this point to consider just what combinations of the original admittance functions will arise in the equivalent circuit representation. The form of equivalent circuit chosen in a π network with a current generator in the output leg. The form of the circuit is shown in Fig. 5. Because the admittance Y_{12} is much less than Y_{11} or Y_{21} it may be neglected when added to these terms. However, Y_{22} and Y_{12} are of the same order of magnitude over part of the frequency range of interest and, as they are opposite in sign, the term $Y_{22} + Y_{12}$ should be considered. An equivalent circuit of the form shown in Fig. 5 is quite useful for the common-base con-

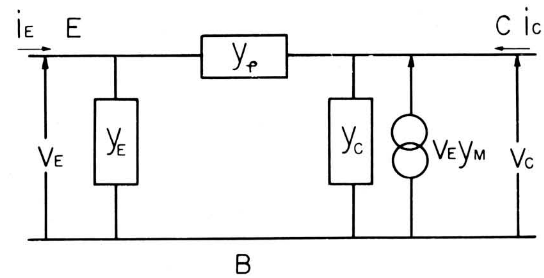

Fig. 5 — π network for common-base connection, where $Y_p = -Y_{12}$, $Y_e = Y_{11} + Y_{12}$, $Y_c = Y_{22} + Y_{12}$ and $Y_m = Y_{21} + Y_{12}$.

*Note: The subscript (b) on the four-terminal admittances has been dropped for convenience. When these admittances are used in the remainder of the bulletin, it is to be understood that they refer to the common-base connection.

nection; however, for the common-emitter, it would be more suitable in the form shown in Fig. 6.

This equivalent circuit can easily be formed from Fig. 5 by using the indefinite admittance matrix previously described. The only element which appears different in this equivalent circuit is the input branch $Y_{11} + Y_{21} - Y_{12}$. Again, Y_{12} may be neglected compared to Y_{11} and Y_{21}. The two combinations of admittances which are of interest in the equivalent circuits of Figs. 5 and 6* are $Y_{11} + Y_{21} = Y_s$ and $Y_{22} + Y_{12} = Y_c$. The expressions for these two combined admittances can be developed from Eqs. (4) to (7) and the results are given below.

$$Y_c = Y_{22} + Y_{12} = -\frac{i_c}{w}\frac{dw}{dV_c}\left[\epsilon'\left(\Omega - \frac{\epsilon}{\epsilon'}\right)\left(1 - 2\epsilon'\Omega e^{-2\epsilon\Omega}\right)\right] \qquad (19)$$

$$Y_s = Y_{11} + Y_{21}$$

$$= i_e \frac{q}{kT}\frac{1}{\gamma_o\left(1+\frac{\epsilon'}{\epsilon}\right)}\left[\left(1+\frac{\epsilon'}{\epsilon}\Omega\right) - 2\gamma_o\Omega e^{-\epsilon'\left(\Omega - \frac{\epsilon}{\epsilon'}\right)}\right] \qquad (20)$$

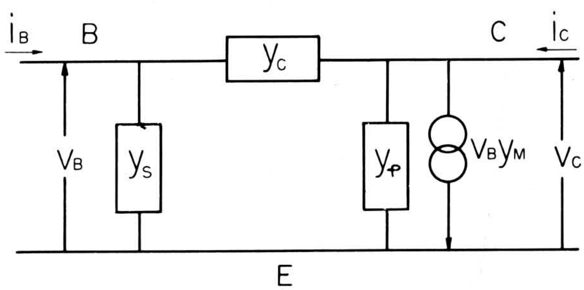

Fig. 6 — π network for common-emitter connection, where $Y_s = Y_{11} + Y_{21} - Y_{12}$. The other elements are the same as in Figure 5.

The effect of the exponential term in Eq. (19) can be neglected apart from the slight modification it will make at low frequencies. At high frequencies the expression for Y_c will be of the same form as the expression for Y_{22}. The low frequency value of Y_c will depend quite critically on the ratio of ϵ/ϵ'. For an assumed value of $\epsilon/\epsilon' = 0.999688$ and $\epsilon' = 4$ the function is plotted in Fig. 9. The value of ϵ/ϵ' will also affect the value of $Y_{11} + Y_{21}$, but this effect is by no means as strong and can largely be accounted for in the low-frequency value of γ_o. The function is plotted in Fig. 10 for $\epsilon' = \epsilon$ and $\gamma_o = 0.975$.

*Note: "V_C" in these figures refers to the output voltage in each case. Distinguishing subscripts have been omitted as unnecessary.

3. Equivalent Circuit Representation

In this section equivalent circuit representations of the admittance functions Y_e, Y_c, Y_s, Y_p, and Y_m, described in the previous section, are derived. The method of derivation is essentially the same as that used by Kroemer[3]. The admittance functions are expanded to the third term in a power series in a normalized frequency parameter x, where $x = \omega\,\delta_o^2/D_p$, and $x < 1$. The frequency corresponding to $x = 1$ is of the order of the α-cutoff frequency of the intrinsic transistor. Three element networks are used to represent the functions. The values of the elements are derived by writing the admittance of a network in powers of normalized frequency and equating coefficients to the coefficients in the expansion of the corresponding admittance function.

In this derivation the transition capacitances have been omitted as well as the external base lead resistance. These elements will be added to the equivalent circuit of the intrinsic transistor in a later section.

The admittance Y_e:

Taking Eq. (14) and writing $\Omega = \sqrt{1 + jx}$,

$$Y_e = \frac{i_e^{(p)}}{\gamma_o}\frac{q}{kT}\frac{1 + \sqrt{1 + jx}}{2} \qquad (21)$$

For $x < 1$, $\sqrt{1 + jx}$ may be written as $1 + j\frac{x}{2} + \frac{x^2}{8}$ - - - -

Keeping only the first three terms in the expansion and substituting in Eq. (21),

$$Y_e \approx \frac{i_e^{(p)}}{\gamma_o}\frac{q}{kT}\left(1 + j\frac{x}{4} + \frac{x^2}{16}\right) \quad \text{for } x^2 \ll 1 \qquad (22)$$

The admittance of the circuit shown in Fig. 7 may be written

$$Y = g_1 + \frac{j\omega C}{1 + \frac{j\omega C}{g_2}} \approx g_1\left[1 + \frac{j\omega C}{g_1} + \frac{\omega^2 C^2}{g_2 g_1}\right] \text{ for } \frac{\omega C}{g_2} < 1 \qquad (23)$$

If $x = \omega a$, where $a = \delta_o^2/D_p$, coefficients of ω in Eqs. (22) and (23) may be equated and solved for C, g_1 and g_2. Then,

$$g_1 = \frac{i_e^{(p)}}{\gamma_o}\frac{q}{kT}, \quad C = \frac{a}{4}g_1 \text{ and } g_1 = g_2 \qquad (24)$$

The variation of the admittance of the circuit shown in Fig. 7 is plotted in Fig. 2 (dotted) for the ratios of the elements given by Eq. (24). It can be seen from this curve where the approximations break down above $x = 1$.

The admittance Y_p:

This term may be dealt with in the same manner as that used for Y_{11}; the most suitable three-element circuit is the one shown in Fig. 8. This circuit equivalent has however two disadvantages. The first is that the capacitance C turns out to be negative. The second is that at high frequencies the circuit admittance begins to increase with frequency whereas the admittance function Y_p keeps on decreasing with frequency. A more useful approximation would be simply the g and L elements in series, as in most circuit applications it is not necessary to know the exact variation of Y_p with frequency, but only to know that its admittance decreases with increasing frequency.*

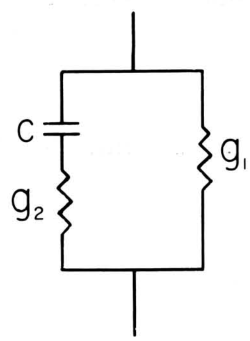

Fig. 7 – Circuit representation of Y_e.

The admittance Y_m:

In the equivalent circuits of Figs. (5) and (6), Y_m appears as the multiplier of the input voltage to make up the current generator in the output leg. Although an equivalent circuit may be formed to represent its frequency variation, it would need to be somewhat complex and its usefulness would be limited. It was thought preferable to obtain a simplified expression for the frequency variations of this function that could be used simply as a multiplier for the current generator.

Using Eq. (16),

$$Y_m = i_c^{(p)} \frac{q}{kT} \Omega e^{(\epsilon - \epsilon'\Omega)} \tag{25}$$

Letting g_m equal the low-frequency value of Y_m, Eq. (25) may be written in the form,

$$Y_m = g_m \Omega e^{\epsilon'(1-\Omega)} \tag{26}$$

If Ω is expanded in powers of x and only terms up to x^2 retained, Y_m may be written in the form

$$Y_m \approx g_m \frac{e^{-j\omega A}}{1 + j\omega B} \tag{27}$$

where

$$A = \frac{a}{2}\left[\epsilon' - 1 - (\epsilon' - 2)^{1/2}\right]$$

$$B = \frac{a}{2}(\epsilon' - 2)^{1/2}$$

for $\epsilon' = 4$, $A \approx 0.8a$ and $B \approx 0.7a$. A good approximation to the original function can be obtained if A and B are made equal. Eq. (27) is plotted on Fig. 3 (dotted) as a function of the frequency parameter $x = \omega a$, for $A = B = 0.74a$. Comparing these curves with the original curves it can be seen that the modulus curve is a good approximation up to $1.5x$ while the phase curve gives a good approximation somewhat higher

The admittance Y_c:

The next element to consider is Y_c. Using Eq. (19) and neglecting the last term,

$$Y_c = Y_{22} + Y_{12} \approx -\frac{i_c}{w}\frac{dw}{dV_c}\epsilon'\left[1 - \frac{\epsilon}{\epsilon'} + j\frac{x}{2} + \frac{x^2}{8}\right]$$

$$= -\frac{i_c}{w}\frac{dw}{dV_c}(\epsilon' - \epsilon)\left[1 + j\frac{x}{2}\frac{\epsilon'}{\epsilon' - \epsilon} + \frac{x^2}{8}\frac{\epsilon'}{\epsilon' - \epsilon}\right] \tag{28}$$

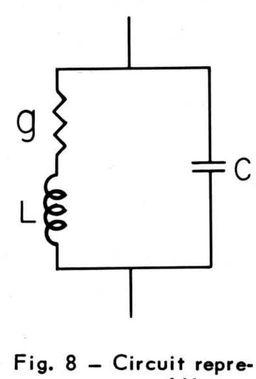

Fig. 8 – Circuit representation of Y_p.

*Note: A slightly better approximation may be obtained for Y_p at the lower frequencies by placing a small capacitance across the conductance in Fig. 8.

An equivalent circuit may be derived for this admittance function similar to the one used for Y_e and is illustrated in Fig. 7. The two conductances and the capacitance will then be given by,

$$g_1 = -\frac{i_c}{w}\frac{dw}{dV_c}(\epsilon' - \epsilon), \quad C = g_1 \frac{a}{2}\frac{\epsilon'}{\epsilon' - \epsilon} \quad g_2 = 2\left(\frac{\epsilon'}{\epsilon' - \epsilon}\right) g_1 \qquad (29)$$

The admittance of this equivalent circuit is plotted in Fig. 9 (dotted) and compared to the admittance function. It can be seen from this curve that the approximation is very good up to $x = 1$. At frequencies much above this point, however, the approximations break down and there is a large difference between the two curves, particularly in the phase curves.

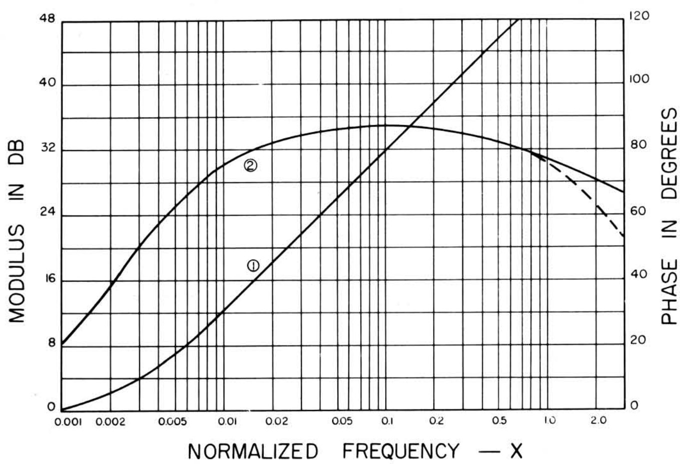

Fig. 9 – The variation of $\frac{Y_{22} + Y_{12}}{(Y_{22} + Y_{12})_{(o)}} = \frac{Y_c}{Y_{c(o)}}$ with frequency. Curve 1 represents the modulus and curve 2 the phase. The dotted curves show the equivalent circuit approximation.

The admittance Y_s:

The last admittance function that is considered is $Y_s = Y_{11} + Y_{21}$. For this case assume $\epsilon/\epsilon' = 1$ and use Eq. (20). Again expanding in powers of x and keeping only the first three terms,

$$Y_s = Y_{11} + Y_{21} \approx i_e^{(p)} \frac{q}{kT}\frac{(1-\gamma_o)}{\gamma_o}\left[1 + j\frac{x}{4}\left(\frac{1 + 2\gamma_o(\epsilon - 1)}{1 - \gamma_o}\right) + \frac{x^2}{16}\frac{(1 + 2\gamma_o(\epsilon^2 - \epsilon - 1))}{1 - \gamma_o}\right] \qquad (30)$$

If the same type of circuit is used to approximate this function as the one shown in Fig. 7, equating the coefficients gives,

$$g_1 = i_e^{(p)} \frac{q}{kT}\frac{(1-\gamma_o)}{\gamma_o} \qquad C = \frac{a}{4}\left(\frac{1 + 2\gamma_o(\epsilon - 1)}{1 - \gamma_o}\right) g_1 \qquad (31)$$

$$g_2 = \frac{[1 + 2\gamma_o(\epsilon - 1)]^2 g_1}{[1 + 2\gamma_o(\epsilon^2 - \epsilon - 1)][1 - \gamma_o]}$$

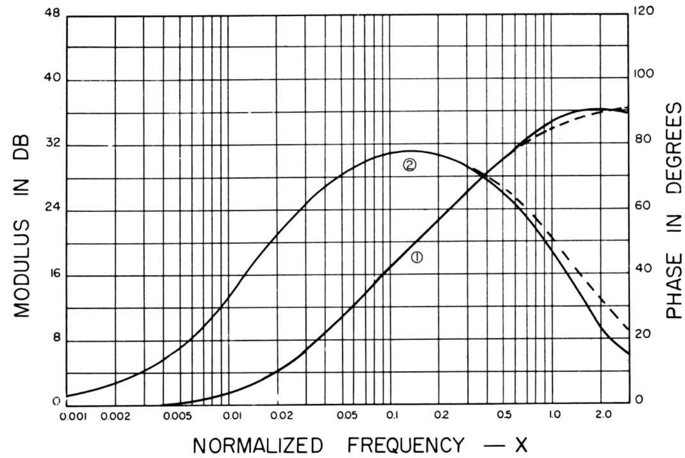

Fig. 10 – The variation of $\frac{Y_{11} + Y_{21}}{(Y_{11} + Y_{21})_{(o)}} = \frac{Y_s}{Y_{s(o)}}$ with frequency. Curve 1 represents the modulus and curve 2 the phase. The dotted curves show the equivalent circuit approximation.

The admittance of this network is plotted in Fig. 10 (dotted) and may be compared to the function it is to represent for $\gamma_o = 0.975$ and $\epsilon = 4$. It can be seen that the equivalent circuit representation only differs from the admittance function appreciably for $x > 1$.

The admittance functions that have been represented in this section by two terminal networks can be combined to give the complete equivalent circuits of the transistor in the form of the circuits in Figs. 5 and 6. This is done in Figs. 11 and 12 for the common-base and common-emitter circuits respectively. In these circuits the collector and emitter transition capacitances as well as the external base resistance have been included.

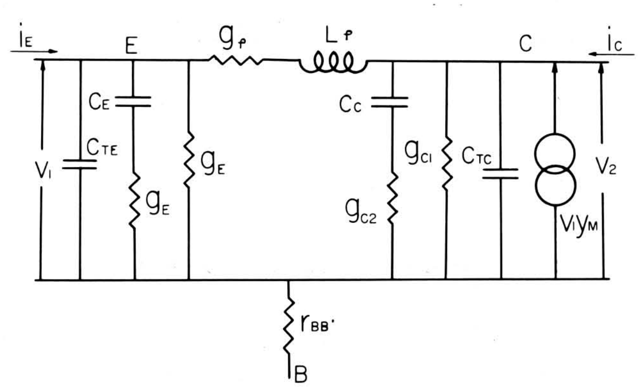

Fig. 11 – Equivalent circuit for the common-base connection.

In Table II the expressions, in terms of the device parameters, are summarized together with the calculated values using the device constants shown in Table I.

Table I

Estimated typical values for the device parameters of a drift transistor

$w = 10^{-3}\ cm$

$\frac{dw}{dV_c} = 1.2 \times 10^{-5}$ cm/volt

$w/L = 0.1$

$D_p = 44$ cm²/sec

$i_e = 1$ ma

$q/kT = 40$ volt^{-1}

$\epsilon' = 4$

$\gamma_o = 0.975$

$a = \frac{w^2}{D_p}\frac{1}{\epsilon^2} = 1.4 \times 10^{-9}$ sec

(implying an alpha-cutoff frequency of 100 mc/s.)

$\epsilon/\epsilon' = 0.999688$

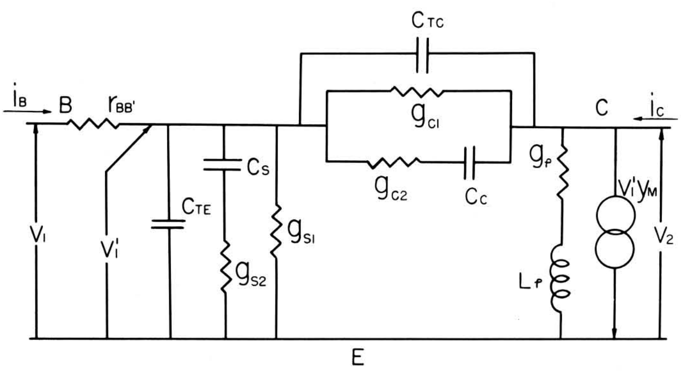

Fig. 12 – Equivalent circuit for the common-emitter connection.

Table II

Equivalent Circuit Element	Corresponding Term in Approximate Device Equations	Calculated Values Using Parameter Values Given in Table I
C_e	$\frac{a}{4} g_e$	14 $\mu\mu$f
g_e	$\frac{i_e q}{\gamma_o kT}$	41,000 μmho
g_p	$-2\frac{i_c \epsilon e^{-2\epsilon}}{w}\frac{dw}{dV_c}$	0.032 μmho
L_p	$\frac{a}{2}\frac{(\epsilon-1)}{g_p}$	0.066 henry
g_m	$i_e \frac{q}{kT} e^{(\epsilon'-\epsilon)}$	40,000 μmho
g_{c1}	$-\frac{i_c}{w}\frac{dw}{dV_c}(\epsilon'-\epsilon)$	0.015 μmho
g_{c2}	$2\left(\frac{\epsilon'}{\epsilon'-\epsilon}\right) g_{c1}$	96 μmho
C_c	$g_{c1}\frac{a}{4}\frac{\epsilon'}{\epsilon'-\epsilon}$	0.017 $\mu\mu$f
g_{s1}	$i_e \frac{q}{kT}\frac{(1-\gamma_o)}{\gamma_o}$	1,025 μmho
g_{s2}	$\frac{[1+2\gamma_o(\epsilon-1)]^2 g_{s1}}{[1+2\gamma_o(\epsilon^2-\epsilon-1)][1-\gamma_o]}$	86,000 μmho
C_s	$\frac{a}{4}\frac{1+2\gamma_o(\epsilon-1)}{1-\gamma_o} g_{s1}$	100 $\mu\mu$f

It should be emphasized that the calculated values given above are only representative. However, they should correspond with the actual values of a drift transistor having the device parameter values given in Table I.

4. The Current Amplification Factor

An important transistor parameter which so far has not been considered explicitly is the current amplification factor α for the grounded base connection. One of the interesting features of the drift transistor is the manner in which the phase of α increases with increasing frequency. As was pointed out in section 2, the controlling factor for the intrinsic transistor is the transport factor β, the injection efficiency γ of the intrinsic transistor being relatively insensitive to frequency ($\alpha = \beta\gamma$). Therefore, only β is considered here. Curve No. 1 of Fig. 13 is a plot of the modulus and phase of β for the drift transistor. Curve 2 represents β for the diffusion transistor and for comparison curve 3 represents the modulus and phase of the impedance of a simple parallel R-C network. Thus, at the 3-db point, the phase of β for a typical drift transistor is 105 degrees, that for the diffusion transistor is 57 degrees, and that of the R-C network is 45 degrees.

This does not mean, of course, that at any given frequency the phase of β is larger for the drift transistor than for a diffusion transistor having the same device parameters. On the contrary, the drift field considerably reduces the transit time, and therefore the phase of β. The drift field has an even stronger effect on the modulus, however, extending its cutoff frequency sufficiently so that the phase of β at that frequency at which the modulus is reduced by 3-db has grown comparatively large.

In this section the roles that the diffusion and the drift field play in the mechanisms of carrier transit and of β fall-off are examined with the hope of reaching a better understanding of just why the drift field improves the β fall-off situation more than it does the phase of β. It is found that contrary to the case of the diffusion transistor, where both processes depend on diffusion in an inseparable fashion, the carrier transit in the drift transistor depends primarily on the drift field, while β fall-off depends only on diffusion. If one could eliminate diffusion while keeping the drift field, the signal would be propagated across the base with a velocity proportional to the drift field and with no decrease in amplitude. Then the phase of β at the cutoff frequency would be infinite, a meaningless statement since the cutoff frequency itself would be infinite (neglecting other limiting effects). With diffusion, fall-off in amplitude is roughly proportional to the product of the square of the average concentration gradient in the base and the length of time the signal remains in the base (transit time). Thus, if the drift field is doubled, the transit time is approximately halved, the cutoff frequency is increased by approximately $2^{3/2}$, and the phase of β at the cutoff frequency is increased by approximately $2^{1/2}$

To understand properly the physical processes which relate the phase and modulus of β, it is helpful to examine (1) the concentration of minority carriers in the base and (2) the current at any point in the base as functions of frequency. For a p-n-p transistor, if p (s, t) is the concentration of excess minority carriers (holes) in the base, then, from Eq. (19) of Kroemer[2]

$$p(s,t) = \frac{qU_e}{kT} e^{j\omega t} \quad p_o e^{\frac{qV_e}{kT}} \quad e^{s/f} \frac{\sinh \frac{w-s}{\delta}}{\sinh \frac{w}{\delta}} \tag{34}$$

where $U_e \, e^{j\omega t}$ is the applied signal at the emitter and p_o is the equilibrium hole concentration at the emitter. "s" represents distance, measured from the emitter end of the base.

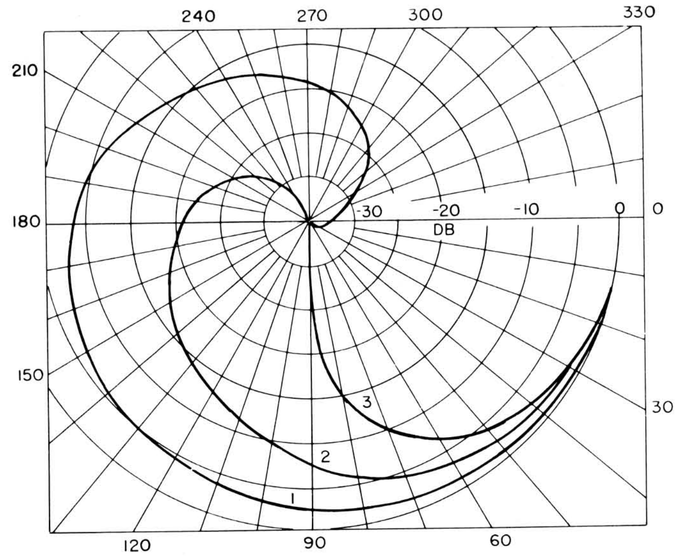

Fig. 13 – Polar diagram showing the phase change of the current transfer ratio with its reduction in modulus. Curve 1 is for the drift transistor. Curve 2 is for the diffusion transistor. Curve 3 is for a simple R-C representation.

Thus

$$p(s,t) = p(s,\omega)\, e^{j\omega t} = p(o)\, e^{s/f} \frac{\sinh \frac{w-s}{\delta}}{\sinh \frac{w}{\delta}} \, e^{j\omega t} \tag{35}$$

The alternating current $i(s,t)$ is given by

$$i(s,t) = A \mu_p \, kT \left(\frac{2}{f}\, p(s,t) - \frac{dp(s,t)}{ds} \right) \tag{36}$$

So that

$$\beta(s,\omega) = \frac{i(s,t)}{i(o,t)} = \frac{e^{s/f}\left[\sinh \frac{w-s}{\delta} + \frac{f}{\delta} \cosh \frac{w-s}{\delta}\right]}{\sinh \frac{w}{\delta} + \frac{f}{\delta} \cosh \frac{w}{\delta}} \tag{37}$$

Here the normal definition of β is intended to have meaning at any point in the base.

a. The Diffusion Transistor

For the diffusion transistor, Eq. (35) reduces to

$$p(s,\omega) = p(o) \frac{\sinh \frac{w-s}{\delta}}{\sinh \frac{w}{\delta}} \approx p(o) \frac{\sinh \sigma z(1+j)}{\sinh \sigma(1+j)}$$

where

$$\sigma^2 = \frac{\omega w^2}{2D_p} = \frac{\omega}{\omega_o}, \qquad z = \frac{w-s}{w}$$

The approximation is the neglect of recombination in the base. Written in terms of modulus and phase, Eq. (35) becomes

$$p(z,\omega) = p(o)\left[\frac{\sinh^2\sigma z + \sin^2\sigma z}{\sinh^2\sigma + \sin^2\sigma}\right]^{1/2} \tag{35a}$$

$$\times\, exp - j\left[\tan^{-1}\frac{\tan\sigma}{\tanh\sigma} - \tan^{-1}\frac{\tan\sigma z}{\tanh\sigma z}\right]$$

Similarly the hole current ratio (transport factor) becomes

$$\beta(z,\omega) = \frac{\cosh z\sigma(1+j)}{\cosh\sigma(1+j)} = \left[\frac{\cosh^2\sigma z - \sin^2\sigma z}{\cosh^2\sigma - \sin^2\sigma}\right]^{1/2} \tag{37a}$$

$$\times\, exp - j[\tan^{-1}\tan\sigma\tanh\sigma - \tan^{-1}\tan\sigma z\tanh\sigma z]$$

At low frequencies, $(\sigma \to 0)$

$$p(z,\omega) = p(o)z\left[1 - \frac{\sigma^4}{45}(1 - z^4)\right]e^{-j\frac{\sigma^2}{3}(1-z^2)} \tag{35a.1}$$

$$\beta(z,\omega) = \left[1 - \frac{\sigma^4}{3}(1 - z^4)\right]e^{-j\sigma^2(1-z^2)} \tag{37a.1}$$

b. The Drift Transistor

For the drift transistor, Eqs. (35) and (36) may be written

$$p(z,\omega) = p(o)e^{\epsilon(1-z)}\frac{\sinh z\epsilon\Omega}{\sinh\epsilon\Omega} \tag{35b}$$

$$= p(o)e^{\epsilon(1-z)}\left[\frac{\sinh^2 z\epsilon c + \sin^2 z\epsilon d}{\sinh^2\epsilon c + \sin^2\epsilon d}\right]^{1/2}$$

$$\times\, exp - j\left[\tan^{-1}\frac{\tan\epsilon d}{\tanh\epsilon d} - \tan^{-1}\frac{\tan z\epsilon d}{\tanh z\epsilon d}\right]$$

where

$$\Omega = \left[1 + \frac{j\omega\delta_o^{\,2}}{D_p}\right]^{1/2} = c + jd$$

and

$$\beta(z,\omega) = e^{\epsilon(1-z)}\left[\frac{\sinh\epsilon z\Omega + \cosh\epsilon z\Omega}{\sinh\epsilon\Omega + \cosh\epsilon\Omega}\right] \tag{37b}$$

c. Discussion

The moduli of the hole concentrations $p(z,\omega)$, as given in Eqs. (35a) and (36a) are plotted in Figs. 14 and 15 for the diffusion transistor and for the drift transistor respectively. The solid lines AO and BO represent the envelope of the concentration at very low frequencies. At higher frequencies the modulus of the concentration is less than the low frequency limit at every point in the base except for the emitter. The broken lines ACO and BDO represent the envelope of the concentration for that frequency for which the phase of $\beta(w,\omega)$ is minus 180 degrees. For the diffusion transistor this occurs (see Eq. (37a)) at $\sigma = \pi$, i.e., at approximately 10 times the cutoff frequency. For the drift transistor, the phase of β is minus 180 degrees at approximately twice the cutoff frequency.

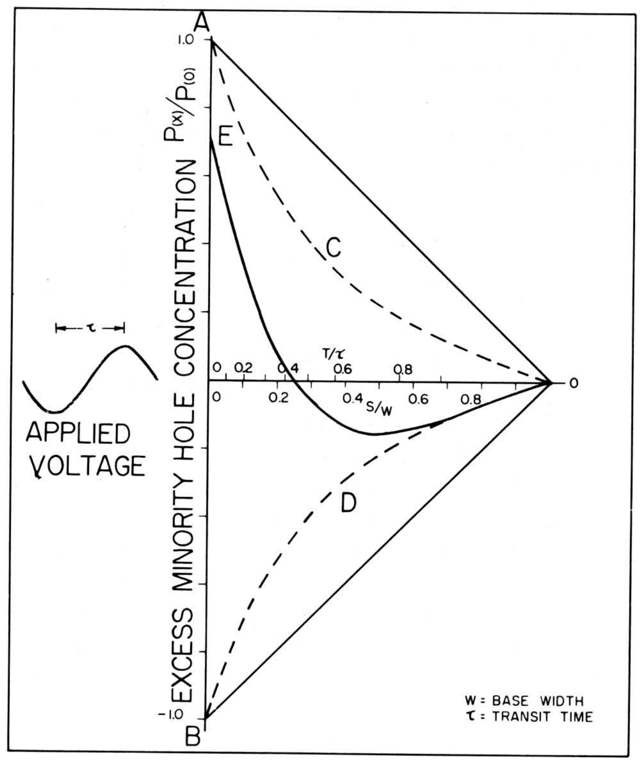

Fig. 14 — Excess minority hole concentration and time delay in the base of a diffusion transistor.

The solid line EO depicts the actual concentration at one instant in time at the frequency described above. The modulus of β may be obtained from this curve by inspection. The instant in time chosen is that at which the emitter and collector currents are at maximum and separated in phase by 180 degrees. Remembering that the current in a diffusion transistor is proportional to the concentration gradient, the modulus of β is merelv the negative of the ratio of the concentration gradient at the collector to that at the emitter. For the drift transistor, from Eq. (36)

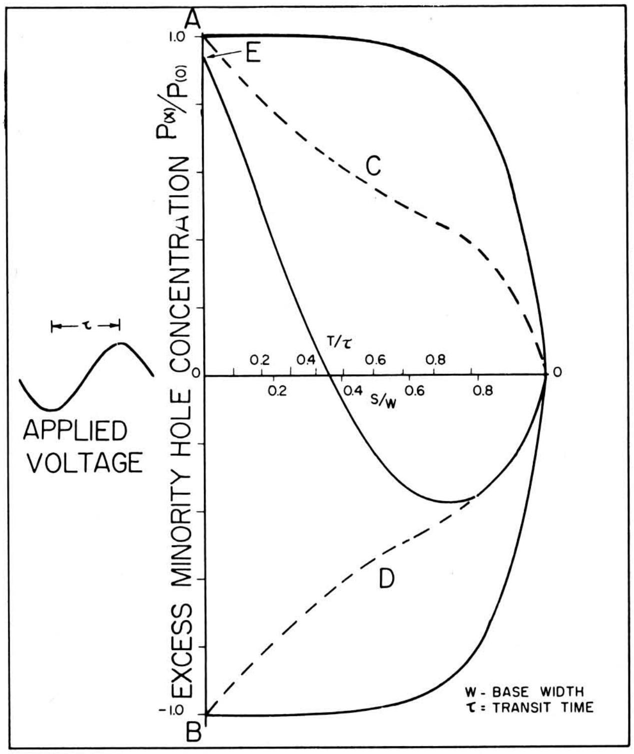

Fig. 15 – Excess minority hole concentration and time delay in the base of a drift transistor.

$$\beta(w,\omega) = \frac{-p'(w,\omega)}{\frac{2}{f}\,p(o,\omega) - p'(w,\omega)}$$

It is interesting to compare the emitter and collector currents of the two transistors at the frequencies giving a change in phase of 180 degrees for β. For the diffusion transistor, β has fallen off 21.3 db, the emitter current increasing by 13 db and the collector current decreasing by 8.3 db. In the case of the drift transistor, β has fallen off only 8.5 db, the emitter current increasing 1.7 db and the collector current decreasing 6.8 db. It thus appears that one effect of the drift field is to keep the emitter current almost constant as a function of frequency.

The reason for the large phase change of β of the drift transistor at the 3-db point, compared to that of the diffusion transistor, now becomes clear. In the diffusion transistor, diffusion is the only mechanism working. The same concentration gradients on which hole transport depends are also responsible for the fall-off of β, since diffusion tries to make a non-uniform hole concentration uniform. In the drift transistor, on the other hand, diffusion is only a hindrance. It does not come into play until the frequency is high enough so that there are appreciable concentration gradients in the base. If the diffusion constant were small, one could picture a signal in the base as an exponentially damped sine wave, the "damping" being due to the diffusion of the concentration "hills" into the concentration "valleys". This mechanism decreases the collector current and thus the modulus of β.

5. Conclusion

The four-terminal admittance equations derived by Kroemer [2] have been used to form an equivalent circuit for the drift transistor. The equivalent circuit is in two forms, one suitable for the common-base connection and the second suitable for the common-emitter connection. The equivalent circuit is rather complex for circuit analysis, but it can probably be simplified for a particular type of drift transistor.

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