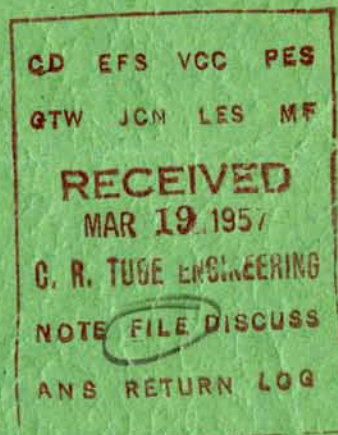




LB-1063

ENVELOPE DISTORTION IN

TRANSISTOR-TUNED AMPLIFIERS



RADIO CORPORATION OF AMERICA
RCA LABORATORIES
INDUSTRY SERVICE LABORATORY

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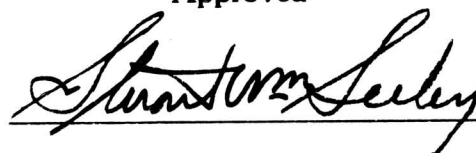
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Approved

A handwritten signature in cursive script, reading "Stuart M. Seely", is written over a horizontal line.

When modulated signals exceeding a few millivolts in magnitude are encountered at the input terminals of a tuned transistor-amplifier, envelope distortion of the output signal may ensue. This bulletin discusses the parameters that influence envelope distortion, and quantitative relations are developed which are supported by experimental evidence. Circuit means for accommodating relatively large signals for certain operating conditions are described.

Introduction

Envelope distortion is a term applied to that distortion of a modulated carrier which produces distortion in the recovered modulation. Thus, for example, a square-law amplifier which distorts the modulated carrier does not produce envelope distortion¹. Envelope distortion in radio receivers ordinarily occurs only at relatively strong received signals. High-level signals may be encountered at the input stage and at subsequent stages, depending on the antenna employed and the manner in which the gain of the stages is controlled. In a properly designed receiver, envelope distortion becomes appreciable only at some upper-limiting field strength of the on-tune signal. At this field strength the receiver is said to overload².

In transistor amplifiers the normal input voltages are several orders of magnitude smaller than in tube amplifiers at the same signal power level. By virtue of this, the onset of envelope distortion in transistor amplifiers at input signals of a few millivolts is not incompatible with achieving performance in transistor receivers comparable to that of their tube counterparts.

Transistor Amplifiers

Tuned amplifiers in receivers may be broadly separated into two types: those to which little or no gain control is applied, but which may operate at high signal levels with full gain, and those to which gain control is applied. The latter type may operate at a nearly constant

¹See RB-7, *Heterodyne Mixer Cross Modulation Tests*, for a discussion of transfer characteristic curvature.

²For a discussion of strong off-tune signals, see LB-1008, *Cross Modulation in Transistor RF Amplifiers*.

output-signal level for several orders of magnitude change in input-signal level. The last i-f stage and the r-f stage of an auto radioreceiver are examples of these two types, respectively.

The transfer characteristic of an ideal transistor follows an exponential law³:

$$I_c = I_{ce} \epsilon^{\Lambda V_{be}} + I_{cs} \quad (1)$$

where I_c is collector current, I_{ce} and I_{cs} are transistor current coefficients, V_{be} is the applied base-to-emitter voltage, and $1/\Lambda = 0.026$ volt at 25 degrees C. Thus for some signal voltage, v , added to a base-to-emitter bias voltage, there is a corresponding output signal current, i , added to the quiescent collector current⁴, I :

$$i = (I - I_{cs}) (\epsilon^{\Lambda v} - 1) \quad (2)$$

In an actual transistor, signal current flowing in the base-lead resistance produces a voltage which subtracts from the source voltage. In addition, if the source is represented as a generator in series with a source impedance, an additional voltage proportional to signal current is developed in the source impedance. The effect of these impedances is to linearize the transfer characteristics at higher operating currents. The envelope characteristic of an ideal transistor is of interest, nonetheless, since at the low operating currents that may be encountered in gain-controlled stages, the ideal transistor is a good approximation of the actual transistor. At higher currents the envelope distortion may be related to that of the ideal transistor.

³See W. Shockley "P-N Junctions," *Phys. Review*, Vol. 83, p. 151, July, 1951, for the basic equations from which this expression is derived. Note that I_{cs} is the collector saturation current, that flows when both collector and emitter are reverse-biased.

⁴See appendix.

Envelope Distortion in an Ideal Transistor

The major mechanism producing gain reduction with reduction in operating current is apparent from Eq. (2), where the magnitude of the output signal current, i , for an input voltage, v , is directly proportional to the "active" portion of the collector bias current, $(I - I_{cs})$. The non-linearity of the signal current arises from the factor $(\epsilon^{\Lambda v} - 1)$, and thus depends only on the magnitude of v , and not upon the magnitude of the operating current.

It has been shown⁵ that the amplification of a modulated signal by a non-linear device produces harmonic distortion as follows: for an input signal, $v = V_1(1 + M \cos mt) \cos pt$, applied to an amplifier with the transfer characteristic, $i = K_1 v + K_2 v^2 + K_3 v^3 \dots$, (neglecting terms in v^4 and above), the second harmonic distortion at an envelope frequency of $2m$ in the output signal is:

$$\text{Distortion} = \frac{9}{8} \frac{K_3}{K_1} M V_1^2 \frac{1}{1 + \frac{9}{4} \frac{K_3}{K_1} V_1^2 (1 + \frac{M^2}{4})} \quad (3)$$

Higher-order harmonics are relatively insignificant.

A power series representation of the transfer characteristic of the ideal transistor yields:

$$i = (I - I_{cs}) (\Lambda v + \frac{1}{2} \Lambda^2 v^2 + \frac{1}{6} \Lambda^3 v^3 \dots) \quad (4)$$

The second harmonic distortion for this case is:

$$\text{Distortion} = \frac{3}{16} M \Lambda^2 V_1^2 \frac{1}{1 + \frac{3}{8} \Lambda^2 V_1^2 (1 + \frac{M^2}{4})} \quad (5)$$

Approximately, then, envelope distortion increases directly with the per cent modulation, and as the square of the signal carrier-voltage. Distortion as a function of signal voltage for this theoretical case, with $M = 50$ per cent, is plotted in Fig. 1, together with experimental data taken on an RCA 2N140 transistor at 1 mc, for various emitter operating-currents. Experimental data, for an experimental RCA drift transistor, of distortion-vs-per cent modulation at 50 microamperes operating current and various signal voltages is shown in Fig. 2.

Operation at Higher Currents

The reduction in distortion at higher currents for a constant terminal signal voltage arises from two causes.

First, the terminal voltage is shared between the base-lead resistance, $r_{bb'}$, and the actual base-to-emitter junction, which may be represented by the impedance $z_{b'e}$, whose magnitude is inversely proportional to current. The appropriate V_1 for Eq. (5) is therefore smaller than the terminal signal voltage by the factor $1/(1 + r_{bb'}/z_{b'e})$. Second, the total impedance in series with the generator exerts a degenerative influence. A modest degree of degeneration can cause the transfer characteristic to have essentially square-law curvature, resulting in the elimination of envelope distortion altogether, at some operating current. The phase-difference between the base and collector signal-currents complicates the picture, but minima in envelope distortion at specific operating currents and circuit impedances have been observed. The theoretical dependence of envelope distortion on operating current and circuit impedances is developed in the appendix; the experimentally observed dependence may be seen in Figs. 1, 3, and 4. A second curve is plotted on Fig. 4, in which the generator voltage has been increased with current so as to maintain a linear relationship between the emitter current and the output signal. This presumably corresponds to a constant signal voltage at the base-to-emitter junction, permitting the degenerative action of the base-lead resistance to be observed alone.

A significant reduction in envelope distortion is observed in an amplifier which is degenerative at audio frequencies. If the bias arrangement of the amplifier is of the "constant-emitter current" type, bypassed only for r-f, so as to permit negligible audio or direct-current variation from the quiescent value, this means that the total current, $I + i$, derived from Eq. (2):

$$I + i = I_o \epsilon^{\Lambda v} + I_{cs} \quad (6)$$

may have no signal-dependent audio or direct components. Since the factor $\epsilon^{\Lambda v}$ experiences audio and d-c variation, the factor I_o must exhibit an inverse variation. When this is taken into account, the equation for envelope distortion becomes, approximately⁴:

$$\text{Distortion} = - \frac{3}{16} M \Lambda^2 V_1^2 \quad (7)$$

the negative sign indicating a reversal in sense from that of Eq. (5). A more complete analysis, taking into account less-than-total audio degeneration should be expected to yield the experimentally observed distortion reduction.

An amplifier in a receiver may experience envelope distortion by this same modulation-mechanism due to the presence of audio voltages in the bias network, originating, for example, in the a-g-c network or on the supply bus. Voltage limiting in the collector circuit may also introduce envelope distortion. These effects may produce some cancellation of distortion over a limited range of input signals.

⁵See Phillips Technical Library, Book IV, p. 315, and the appendix of this bulletin.

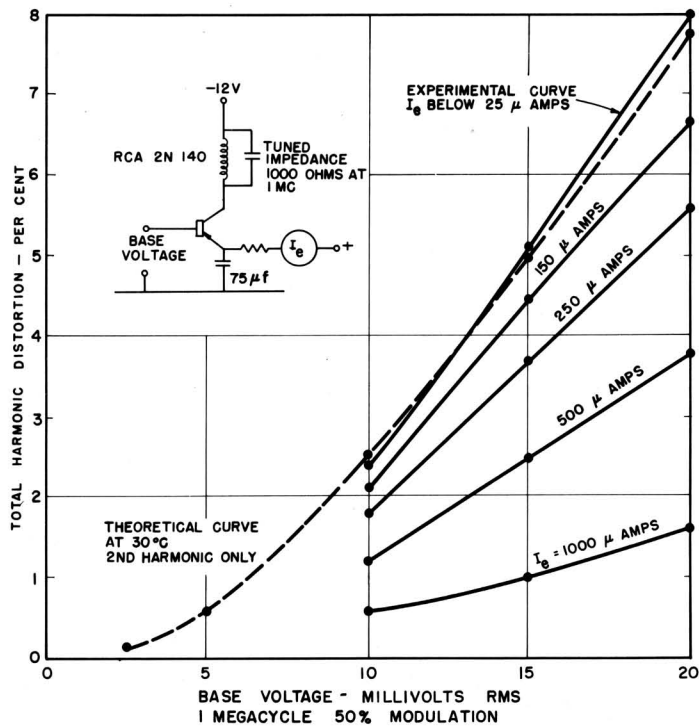


Fig. 1 - Envelope distortion as a function input-signal voltage.

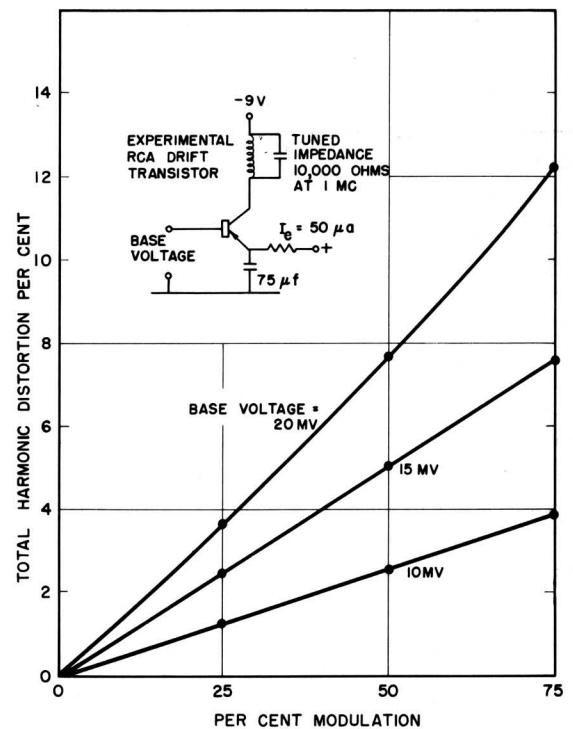


Fig. 2 - Envelope distortion vs per cent modulation.

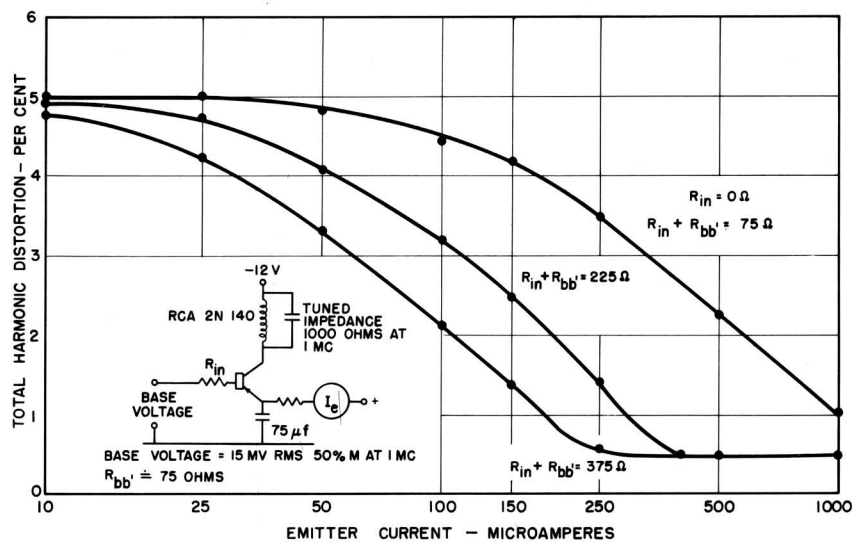


Fig. 3 - Influence of base resistance on envelope distortion.

Operation Beyond Cutoff

In a broadcast receiver, the i-f stages may be operated, by suitable design, below the signal-input levels indicated in the foregoing discussion. In a receiver employing a capacitive antenna and, typically, an r-f first stage, input signal levels on the order of tenths of a volt may be encountered. One means by which these signals can be accommodated is the incorporation of an a-g-c circuit capable of biasing the emitter junction of the r-f transistor beyond cutoff.

When the r-f amplifier transistor is operated beyond cutoff, signal is transferred from the antenna circuit to the interstage circuit by the transition capacitance of the collector-to-base junction of the r-f transistor⁶. This capacitance may be neutralized by a suitable network; and a fictitious "feed-through" capacitance may be assigned a value (positive or negative) representing the error in neutralization. (Variation in the transition capacitance

⁶See LB-915, *A P-N-P Triode Alloy Junction Transistor for Radio-Frequency Amplification*.

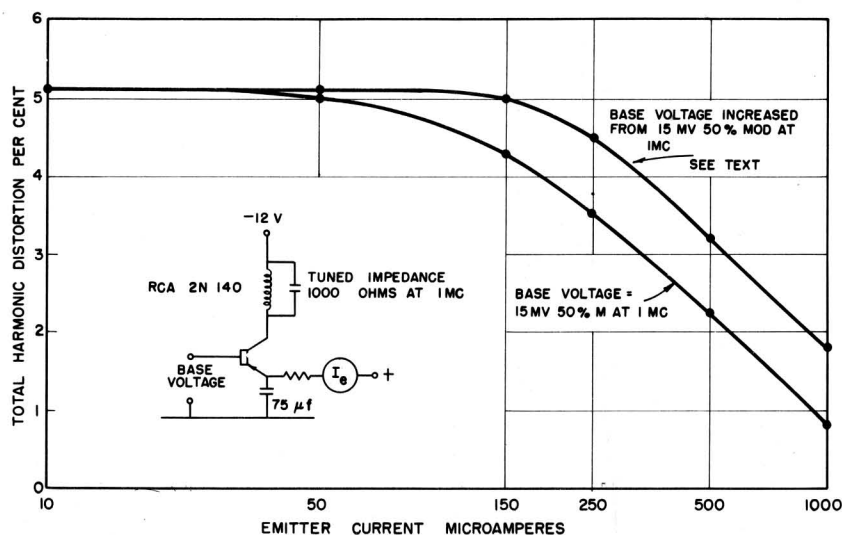


Fig. 4 – Separation of attenuating and degenerative effects of base resistance.

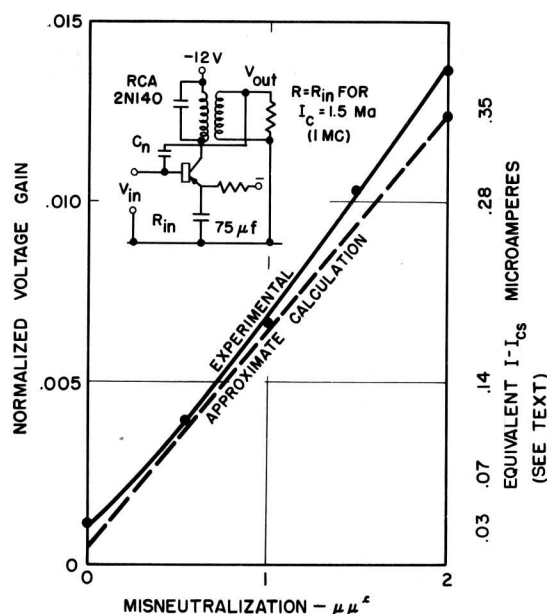


Fig. 5 – Feedthrough gain vs misneutralization.

produces increasing proportional error, of course, as the neutralization becomes more nearly perfect.) The voltage gain, from the antenna circuit to the interstage circuit, is then directly proportional to the magnitude of this feedthrough capacitance. Although the collector transition capacitance is, in fact, non-linear, the tenth-volt signal levels introduce negligible distortion. Under reverse-bias conditions, the r-f stage is also essentially immune to cross modulation².

As the received signal diminishes, the a-g-c circuit may be presumed to relax the reverse bias. From Eqs. (1) and (2), it is apparent that the transition from reverse to forward bias does not introduce any discontinuity in the transfer characteristic. The component of signal

transferred from the antenna circuit to the interstage circuit by the "active" transistor increases from some negligible value until it is comparable in magnitude to (and ordinarily nearly in quadrature with) the fed-through signal. The active component of signal is susceptible to envelope distortion; the transition from a fed-through signal to an active signal must not occur until the r-f stage input-voltage level is down to the order of 5 to 15 millivolts if distortion is to be avoided. Many additional considerations influence the signal-level chosen for this transition. Among them are medium-signal noise performance, crosstalk, tuning characteristic, and overload of subsequent stages.

The normalized voltage gain of a representative r-f stage as a function of feedthrough capacitance is shown in Fig. 5. Also noted on the ordinate is the magnitude of $(I - I_{cs})$ at which transition from a fed-through to an active signal would take place. The departure from zero-gain for capacitive cancellation is due to a conductive component of feedthrough.

Conclusion

Tuned transistor amplifiers operated at currents on the order of 1 ma can accommodate input signals of tens of millivolts without appreciable envelope distortion, and higher signals when carrier or audio degeneration is effected. Under reverse-bias conditions, signal levels of tenths of a volt are permissible. At reduced currents, for forward-bias conditions, envelope distortion becomes independent of operating current, and varies approximately as the per cent modulation and as the square of the carrier voltage, becoming appreciable at 5 to 15 mv, rms.

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List of Symbols

I_c	Total collector current
I_{ce}	D.C. collector-current coefficient
I_{cs}	Collector saturation current
I	Quiescent collector current
i	Signal-dependent component of collector current
I_o	Signal-dependent collector-current coefficient (see Appendix E)
Λ	q/KT , where q is the electron charge, K is Boltzmann's constant, and T is absolute temperature. Numerically, $\Lambda = 38.5 \text{ volts}^{-1}$ at 25 degrees C.
V_{be}	Total base-to-emitter voltage
v	Base-to-emitter signal voltage
V_1	Carrier-amplitude of applied signal
M	Per cent modulation of applied signal
m	Angular modulation frequency
p	Angular carrier frequency
K_1, K_2, K_3	Power series coefficients
v'	Portion of applied signal, v , which is linear with collector current, i .
v''	Portion of applied signal, v , which effects an exponential transfer to collector current, i .
$r_{bb'}$	Base-lead resistance
$z_{b'e}$	Input impedance, excluding base lead resistance
Z_b	Base-circuit impedance
Z_e	Emitter-circuit impedance
g_m	Intrinsic transconductance. Approximately, $g_m = \Lambda I_c$
x	See Appendix 4-D
a	See Appendix 5-D
Z	Impedance relating v' , above, to i . $Z = v'/i$

Appendix

A. Derivation of signal current from total current.

The transfer characteristic of an ideal transistor may be written, as in the text:

$$I_c = I_{ce} \epsilon^{\Lambda V_{be}} + I_{cs} \quad (1-A)$$

$$I_c = I + i = I_{ce} \epsilon^{\Lambda(V+v)} + I_{cs} \quad (2-A)$$

For $v = 0$, $i = 0$, by definition, so

$$I = I_{ce} \epsilon^{\Lambda V} + I_{cs} \quad (3-A)$$

$$I_{ce} \epsilon^{\Lambda V} = I - I_{cs} \quad (4-A)$$

$$I + i = (I - I_{cs}) \epsilon^{\Lambda v} + I_{cs} \quad (5-A)$$

$$i = (I - I_{cs}) (\epsilon^{\Lambda v} - 1) \quad (6-A)$$

B. Envelope distortion, general transfer characteristic.

$$i = K_1 v + K_2 v^2 + K_3 v^3 \quad (1-B)$$

$$v = V_1 (1 + M \cos mt) \cos pt \quad (2-B)$$

Terms arising from $K_1 v$:

$$i(K_1) = K_1 V_1 (1 + M \cos mt) \cos pt. \quad (3-B)$$

Terms arising from $K_2 v^2$:

$$i(K_2) = K_2 V_1^2 (1 + \frac{1}{2} M^2 + 2M \cos mt + \frac{1}{2} M^2 \cos 2mt) \times (\frac{1}{2} + \frac{1}{2} \cos 2pt). \quad (4-B)$$

Terms arising from $K_3 v^3$:

$$i(K_3) = K_3 V_1^3 (1 + \frac{3}{2} M^2 + (3M + \frac{3}{4} M^3) \cos mt + \frac{3}{2} M^2 \cos 2mt + \frac{1}{4} M^3 \cos 3mt) \times (\frac{3}{4} \cos pt + \frac{1}{4} \cos 3pt) \quad (5-B)$$

Considering only the carrier and sidebands, i.e., terms containing $\cos pt$ as a factor:

Carrier output:

$$i(\cos pt) = K_1 V_1 (1 + \frac{3}{4} \frac{K_3}{K_1} V_1^2 (1 + \frac{3}{2} M^2)) (\cos pt) \quad (6-B)$$

Fundamental modulated output: (7-B)

$$i(\cos pt, \cos mt) = K_1 V_1 M (1 + \frac{9}{4} \frac{K_3}{K_1} V_1^2 (1 + \frac{1}{4} M^2)) \cos mt \cos pt.$$

Second harmonic modulated output: (8-B)

$$i(\cos pt, \cos 2mt) = K_1 V_1 M (\frac{9}{8} \frac{K_3}{K_1} V_1^2 M) \cos 2mt \cos pt$$

Third Harmonic modulated output: (9-B)

$$i(\cos pt, \cos 3mt) = K_1 V_1 M (\frac{3}{16} \frac{K_3}{K_1} V_1^2 M^2) \cos 3mt \cos pt$$

The second harmonic distortion in the demodulated output is then the ratio of (8-B) to (7-B):

$$\text{Distortion}(2m) = \frac{9}{8} \frac{K_3}{K_1} V_1^2 M \frac{1}{1 + \frac{9}{4} \frac{K_3}{K_1} V_1^2 (1 + \frac{1}{4} M^2)} \quad (10-B)$$

C. Envelope distortion, exponential transfer characteristic:

An exponential transfer characteristic:

$$i = (I - I_{cs}) (\epsilon^{\Lambda v} - 1) \quad (1-C)$$

may be represented as the power series;

$$= (I - I_{cs}) (\Lambda v + \frac{1}{2} \Lambda^2 v^2 + \frac{1}{6} \Lambda^3 v^3 + \dots) \quad (2-C)$$

Identifying, in (1-A),

$$K_1 = (I - I_{cs}) \Lambda, K_2 = (I - I_{cs}) \frac{1}{2} \Lambda^2, K_3 = (I - I_{cs}) \frac{1}{6} \Lambda^3;$$

and substituting in (10-B)

$$\text{Distortion}(2m) = \frac{3}{16} M \Lambda^2 V_1^2 \frac{1}{1 + \frac{3}{8} \Lambda^2 V_1^2 (1 + \frac{1}{4} M^2)} \quad (3-C)$$

D. Envelope distortion, exponential-linear transfer characteristic.

Consider a transfer characteristic in which the applied voltage, v , is divided into a portion v' , which is linear with the output current, i , and a portion v'' , which effects an exponential transfer to i :

$$v = v' + v'' = i Z + v'' \quad (1-D)$$

$$i = (I - I_{cs}) (\epsilon^{\Lambda v''} - 1) \quad (2-D)$$

Then, to eliminate v' and v'' explicitly,

$$\Lambda v = \frac{i}{I - I_{cs}} \Lambda(I - I_{cs}) Z + \ln\left(1 + \frac{i}{I - I_{cs}}\right) \quad (3-D)$$

Defining, for convenience,

$$x = \frac{i}{I - I_{cs}} \quad (4-D)$$

$$a = (I - I_{cs}) \Lambda Z. \quad \text{Then,} \quad (5-D)$$

$$\Lambda v = ax + \ln(1 + x) \quad (6-D)$$

To represent x as a power series in Λv ,

$$x = \frac{1}{0!} x(\Lambda v = 0) + \frac{\Lambda v}{1!} \frac{dx}{d\Lambda v} (\Lambda v = 0) + \frac{\Lambda^2 v^2}{2!} \frac{d^2 x}{d(\Lambda v)^2} (\Lambda v = 0) + \frac{\Lambda^3 v^3}{3!} \frac{d^3 x}{d(\Lambda v)^3} (\Lambda v = 0) \dots \quad (7-D)$$

Then, for $\Lambda v = 0$, $x = 0$.

To find $\frac{dx}{d\Lambda v}$, from (19-A),

$$1 = a \frac{dx}{d\Lambda v} + \frac{1}{1+x} \frac{dx}{d\Lambda v}$$

$$\frac{dx}{d\Lambda v} = \frac{1+x}{1+a+ax} = \frac{1}{a} - \frac{1}{a} \frac{1}{1+a+ax} \quad (8-D)$$

$$\text{at } x = 0, \frac{dx}{d\Lambda v} = \frac{1}{1+a} \quad (9-D)$$

From (21-A)

$$\frac{d^2 x}{d(\Lambda v)^2} = \frac{1}{(1+a+ax)^2} \frac{dx}{d\Lambda v} \quad (10-D)$$

$$\text{At } x = 0 \quad \frac{d^2 x}{d(\Lambda v)^2} = \frac{1}{(1+a)^3} \quad (11-D)$$

From (23-A)

$$\frac{d^3 x}{d(\Lambda v)^3} = \frac{-2a}{1+a+ax^3} \left(\frac{dx}{d\Lambda v} \right)^2 + \frac{1}{1+a+ax^2} \frac{d^2 x}{d(\Lambda v)^2} \quad (12-D)$$

$$\text{At } x = 0, \quad \frac{d^3 x}{d(\Lambda v)^3} = \frac{1-2a}{(1+a)^5} \quad (13-D)$$

The power series representation of x is then:

$$x = \frac{\Lambda v}{1+a} + \frac{(\Lambda v)^2}{2(1+a)^3} + \frac{(1-2a)(\Lambda v)^3}{6(1+a)^5} \dots \quad (14-D)$$

The factor $\Lambda(I - I_{cs})$ may be recognized as the small-signal transconductance, g_m , of the ideal transistor of Eq. (1-C). Making this substitution in the definition of (a) , Eq. (5-D), the power series representation of the exponential-linear transfer characteristic is:

$$i = (I - I_{cs}) \left(\frac{\Lambda v}{1+g_m Z} + \frac{\Lambda^2 v^2}{2(1+g_m Z)^3} + \frac{(1-2g_m Z)\Lambda^3 v^3}{6(1+g_m Z)^5} \dots \right) \quad (15-D)$$

The second harmonic distortion in the demodulated signal is, approximately:

$$\text{Distortion } (2m) = \frac{3}{16} M \frac{(1-2g_m Z)}{(1+g_m Z)^4} \Lambda^2 V_1^2 \quad (16-D)$$

Note that the distortion decreases as $g_m Z$ becomes large, and vanishes (for this approximate calculation) for $g_m Z = 1/2$. The impedance Z was defined in terms of collector signal current to represent a counter-e.m.f. at the amplifier input terminals that added to that attributable to the ideal transistor. Thus, for series base and emitter impedances; Z_b and Z_e for example, Z may be written:

$$Z = Z_e + (Z_b + Z_e) \frac{\partial i_b}{\partial i_c} \quad (17-D)$$

The factor $\partial i_b / \partial i_c$ is ordinarily largely reactive at radio and intermediate frequencies. The quantitative agreement between experiment and calculations based on the foregoing has not been striking.

E. Reduction of envelope distortion by audio degeneration.

An expression for the total current output for the case of the exponential transfer characteristic, (1-B), may be written:

$$I + i = I_o e^{\Lambda v} + I_{cs} \quad (1-E)$$

when ordinarily, $I_o = I - I_{cs}$ and v represents the voltage applied to the transistor as well as the generator voltage. If the circuit is arranged to be degenerative at low frequencies, so that $(I + i)$ contains no audio or signal-

dependent d-c terms, then the circuit gives rise to a direct-and-audio voltage, v' , which subtracts from the generator voltage, v , so that the net applied voltage, $(v-v')$ results in no direct-and-audio components in the signal current, i . Then:

$$I + i = (I - I_{cs}) \epsilon^{\Lambda(v-v')} + I_{cs} \quad (2-E)$$

and I_o may be identified from (2-E),

$$I_o = (I - I_{cs}) \epsilon^{-\Lambda v'} \quad (3-E)$$

and calculated from Eq. (1-E),

$$I_o = (I - I_{cs}) \div (\text{audio and signal-dependent terms of } \epsilon^{\Lambda v}). \quad (4-E)$$

For a signal of the form of Eq. (2-B) and the approximation,

$$\epsilon^{\Lambda v} = 1 + \frac{\Lambda v}{1} + \frac{\Lambda^2 v^2}{2} + \frac{\Lambda^3 v^3}{6} \quad (5-E)$$

$$I_o = (I - I_{cs}) \div \left(1 + \frac{1}{4} \Lambda^2 V_1^2 \left(1 + \frac{M^2}{2} \right) + \frac{1}{2} \Lambda^2 V_1^2 M \cos mt + \frac{1}{8} \Lambda^2 V_1^2 M^2 \cos 2mt \right) \quad (6-E)$$

and approximately,

$$I_o = (I - I_{cs}) \left(1 - \frac{1}{4} \Lambda^2 V_1^2 \left(1 + \frac{M^2}{2} \right) - \frac{1}{2} \Lambda^2 V_1^2 M \cos mt - \frac{1}{8} \Lambda^2 V_1^2 M^2 \cos 2mt \right). \quad (7-E)$$

Multiplying the above by the expressions in (6-B), (7-B), and (8-B), in which the appropriate substitutions have been made for K_1 , K_2 , and K_3 ; collecting terms including the factor, pt , and neglecting terms in higher-order harmonics and terms in which $\Lambda^4 V_1^4$, or higher-order appear, yields:

Carrier output:

$$i(\cos pt) = (I - I_{cs}) \Lambda V_1 \left(1 - \frac{1}{8} \Lambda^2 V_1^2 \left(1 + \frac{3}{2} M^2 \right) \right) \cos pt \quad (8-E)$$

Fundamental modulated output:

$$i(\cos pt \cos mt) = (I - I_{cs}) \Lambda V_1 M \left(1 - \frac{3}{8} \Lambda^2 V_1^2 \left(1 + \frac{1}{4} M^2 \right) \right) \cos mt \cos pt \quad (9-E)$$

Second harmonic modulated output:

$$i(\cos pt \cos 2mt) = (I - I_{cs}) \Lambda V_1 M \left(-\frac{3}{16} \Lambda^2 V_1^2 M \right) \cos 2mt \cos pt \quad (10-E)$$

The second harmonic distortion in the demodulated output is then the ratio of (10-E) to (9-E)

$$\text{Distortion (2m)} = \frac{-\frac{3}{16} \Lambda^2 V_1^2 M}{1 - \frac{3}{8} \Lambda^2 V_1^2 \left(1 + \frac{1}{4} M^2 \right)} \quad (11-E)$$

Note the reversals in algebraic sign from Eq. (3-C).