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LB-1061

ANALYSIS OF TRANSISTOR OSCILLATORS FOR BROADCAST RECEIVERS

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Approved

Stuart Wm. Seeley

The dependence of the amplitude of oscillations upon feedback is described for a transistor oscillator. Graphs are included which show the relationship between the oscillator input voltage and an *effective transconductance* which is related to the feedback network. Therefore these graphs may be used to predict the amplitude of the oscillations for a given circuit.

Large-signal equivalent circuits are presented for a transistor which allows calculation of the input impedance under varying operating conditions. It is shown that the input impedance of the transistor does not change with biasing for the type of circuit considered here, provided the base-lead resistance is negligibly small. Therefore, frequency deviations accompanying amplitude control (caused by changing input impedance) may be ascribed to the presence of the base-lead resistance.

Introduction

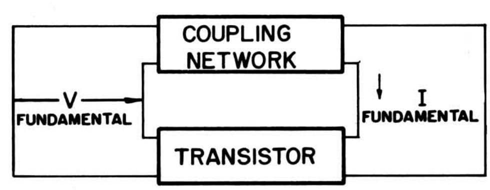
This bulletin presents an analysis of transistor oscillators and shows some design considerations which result. First, an explanation is given of the manner in which the oscillations stabilize at a certain value. The amplitude of oscillations is, of course, dependent upon the characteristics of the transistor. A transistor with an idealized emitter junction is considered and conclusions are drawn concerning the amplitude of oscillations and the control of this amplitude. Next, a transistor with an exponential voltage-current relationship within the emitter junction is considered. The frequency stability of the transistor oscillator is then discussed.

General Comments Concerning the Oscillator

A type of circuit often used at broadcast frequencies is one in which the input and output impedances of the transistor are high with respect to the driving and load impedances. Both resistive and reactive loading of the circuit by the transistor are small in this type of circuit. A large power gain must be available because of the inefficiency of this type of coupling.

With these stipulations the transistor appears to be driven from a constant voltage source and itself appears to be a constant current source (looking back from the load). Transconductance is a useful consideration in this case because it is a current to voltage transfer ratio. The effective transconductance, $g_{m\,eff}$, of the transistor is the ratio of the fundamental component of current at the collector to the sinusoidal voltage applied between emitter and base, which is causing the current to flow. See table of definitions (p. 8).

The transfer admittance, y_{12}, of the coupling (feedback) network is also a useful parameter to consider. The order of the subscripts was chosen to identify the current with the input terminals of the network (output terminals of the transistor) and the voltage with the output terminals of the network (transistor input terminals). See Fig. 1.



$$g_{m\,eff} = \frac{I_{FUNDAMENTAL}}{V_{FUNDAMENTAL}}$$

Fig. 1 – Transistor oscillator.

When the $g_{m\,eff}$ is considered rather than the gain of the transistor, the criterion of oscillation, which is usually stated as being unity loop gain, may be restated as follows: $g_{m\,eff} = -y_{12}$. The assumption is made (and is considered further below) that $r_{bb'}$ may be neglected. In this special case $g_{m\,eff}$ has no phase shift associated with it; therefore, only g_{12}, the real part of y_{12}, need be considered and the criterion for oscillation may be restated: $g_{m\,eff} = -g_{12}$, where g_{12} is the transfer conductance of the coupling network. This relationship must be satisfied because, as is shown in Fig. 1, the coupling network and transistor terminals are coincident (input with output).

Oscillator Amplitude vs. Feedback

The $g_{m\,eff}$ of a transistor changes with the amplitude of a-c input voltage. In an oscillator the amplitude of oscillations adjusts to a value which produces the correct $g_{m\,eff}$ for the oscillations to be sustained; $g_{m\,eff} = -g_{12}$. The mechanism which causes $g_{m\,eff}$ to vary is discussed because it influences the amplitude of oscillations, and therefore the amplitude stability and controllability.

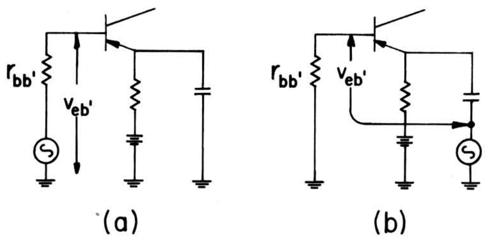


Fig. 2 – A biasing method for base and emitter signal injection.

The emitter junction has the characteristics of a switch in that it has a high back impedance and a low forward impedance. It is this characteristic which causes the $g_{e\,eff}$ and hence the $g_{m\,eff}$ to be a function of the a-c input voltage, and therefore allows the $g_{m\,eff}$ to adjust itself to the value required for oscillation. The manner in which this characteristic affects the $g_{e\,eff}$ depends upon the biasing method. Fig. 2 illustrates a type of biasing used for both common base and common emitter connections. Many other biasing arrangements are equivalent to this type of biasing. The base-lead resistance, $r_{bb'}$, is shown external to the transistor. For transistors with good high-frequency characteristics, such as the 2N140, the effect of $r_{bb'}$ upon $g_{e\,eff}$ is small. Therefore, $r_{bb'}$ is neglected as shown in Fig. 3. The diode in this figure represents the base-to-emitter junction. From the standpoint of biasing, this circuit is equivalent to those shown in Fig. 2. The $g_{e\,eff}$ vs $V_{eb'}$ characteristic is derived for the biasing method represented by these figures.

The Diode With Fixed Forward Resistance

Consider the operation of a circuit in which the diode

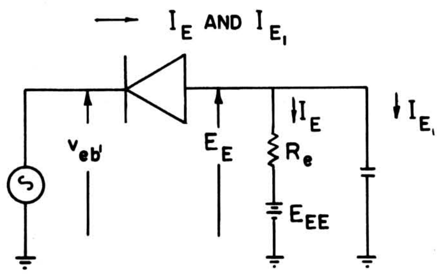


Fig. 3 – Equivalent circuit for biasing.

is a switch with infinite impedance when open and a fixed resistance, r_e, when closed. This operation is an approximation to the actual operation of a transistor because the emitter has neither an infinite back impedance nor a fixed forward resistance; however, the operation of a transistor is so similar to this simplified case that it is useful to examine this case and to draw some general conclusions. The case for which the emitter junction is more accurately described as having an exponential characteristic is considered later.

Fig. 4 shows the circuit to be analyzed. The bypass capacitor is large enough so that the emitter may be assumed to be at a constant d-c voltage. With no signal applied the d-c potential will be $E_E = E_{EE} \dfrac{r_e}{R_e + r_e}$. Current will flow through the diode as long as the base remains negative with respect to the emitter. With the application of a signal a sine wave of current will flow through the diode, provided the peak voltage, $V_{eb'}$, is not greater than $E_{EE} \dfrac{r_e}{R_e + r_e}$. Whenever $V_{eb'}$ exceeds this value (which will be small if the diode has a low r_e and a moderate bias current), the current will flow for less than 360 degrees. This nonsinusoidal current will have a d-c component which tends to bias the emitter more negative than in the no-signal case. As the signal strength increases, the negative bias increases and the flow angle will decrease. Fig. 5 shows waveforms for flow angle $2\,\theta_1$.

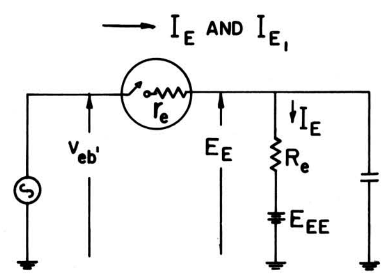


Fig. 4 – Equivalent circuit for biasing -- idealized emitter junction.

A mathematical analysis of the action is given in Appendix A. The d-c component of the current (I_e) is found by Fourier analysis. Fig. 6 shows the results of this analysis. The discussion is temporarily limited to those curves shown by solid lines. The direct current and flow angle vary as previously stated. The fundamental component of emitter current (I_{E1}) can also be determined by Fourier analysis and is likewise shown. The ratio of I_{E1} to $V_{eb'}$, $g_{e\,eff}$, is also plotted.

Applications

The $g_{e\,eff}$ vs $V_{eb'}$ characteristic can be used to

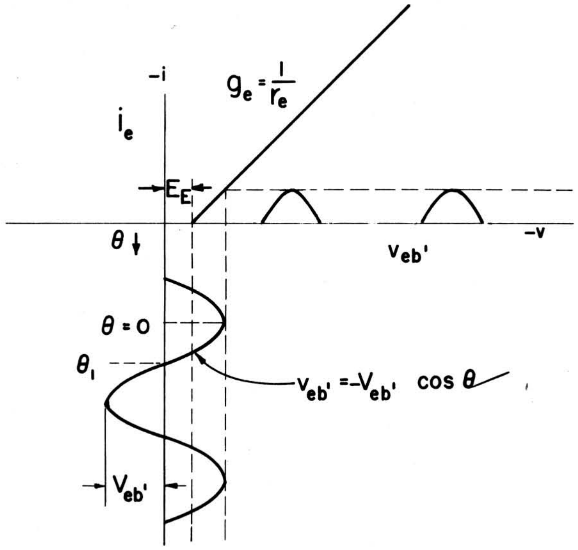


Fig. 5 – Emitter junction waveforms -- idealized junction.

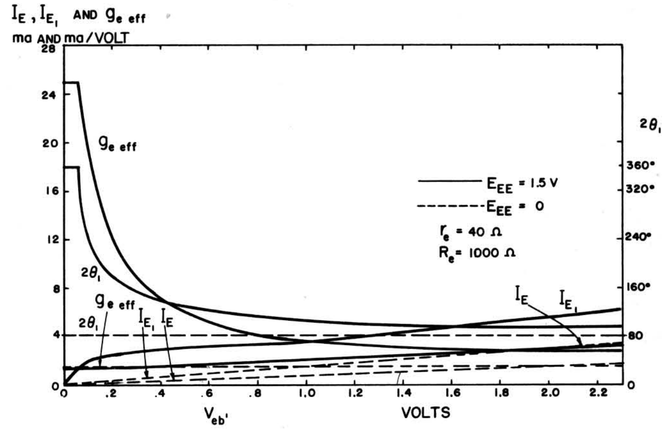


Fig. 6 – Diode characteristics (infinite back impedance linear forward resistance).

determine the a-c voltage between base and emitter for an oscillator (and from this the amplitude of oscillations at any point within the circuit) once the g_{12} of the feedback network is specified. To do this a horizontal line which represents the g_{12} of the feedback network may be drawn on the characteristic. The point of intersection of this line and the $g_{m\ eff}$ curve[1] is the point where the oscillations will stabilize. The a-c base-to-emitter voltage may be read directly on the abscissa.

The small signal g_m of the transistor is the largest value of $g_{m\ eff}$ which the transistor can furnish. There is also a lower limit to the $g_{m\ eff}$ which is determined by circuitry considerations as well as by the transistor characteristics. Note in Fig. 6 that the $g_{e\ eff}$ curve asymptotically approaches the zero bias $g_{e\ eff}$. This zero bias $g_{e\ eff}$ is determined only by the ratio of the forward resistance of the diode, r_e, to the emitter biasing resistance, R_e. Everitt[2] has a graph from which the reciprocal of $g_{e\ eff}$ (effective input resistance) may be determined for any ratio of r_e/R_e. The g_{12} of the feedback network must be larger than the zero bias $g_{m\ eff}$ if the mode of operation described in this bulletin is to be obtained; otherwise the amplitude of oscillations will tend to rise unbounded and will stabilize because of some nonlinear action such as collector bottoming rather than because of the nonlinear characteristic of the emitter junction.

Once the g_{12} of the coupling network has been specified, the impedance levels and coupling may be selected. For example, if transformer coupling between collector and base is employed, the g_{12} of this network would be the collector-to-base turns ratio divided by the total shunt resistance referred to the collector. Either high resistance and high collector-to-base turns ratio or low resistance and low collector-to-base turns ratio may be chosen to give the same g_{12}. Therefore, different values of voltage swing at the collector may be obtained for a certain g_{12} to allow operation within the desired portion of the collector characteristic. The range in the values of turns ratios and resistances which will give a desired g_{12} is ultimately determined by the terminal impedances of the transistor.

All of the curves described in previous paragraphs are also shown (dotted lines) for the case of zero bias voltage. The flow angle and $g_{e\ eff}$ are independent of input voltage for this case. Note that $g_{e\ eff}$, I_E, I_{E1}, and $2\theta_1$ for the case with bias voltage tend to converge to the respective curves for the case of no bias voltage when $V_{eb'}$ is large. The reason for this is that the injection voltage, $V_{eb'}$, when large, effectively swamps out the effects of the relatively small bias voltage. Therefore, one may intuitively describe the dependence of the transistor $g_{e\ eff}$ on the input voltage by noting that, at small signal levels, the bias voltage causes a larger flow angle (and hence larger $g_{e\ eff}$) than would be the case if no bias

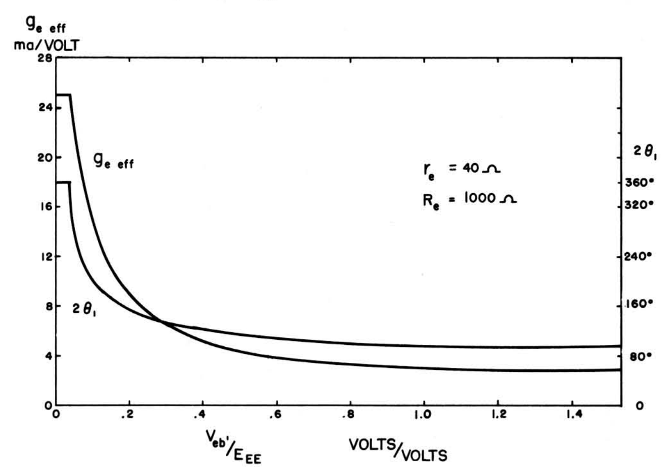


Fig. 7 – Diode characteristics (normalized abscissa).

[1] $g_{m\ eff} \cong |g_{e\ eff}|$ for typical values of d-c alpha. Therefore the the $g_{e\ eff}$ curve is used as if it were the $g_{m\ eff}$ curve.

[2] See COMMUNICATION ENGINEERING, W. L. Everitt, 2nd Edition, pp. 427 to 433 for additional explanation of the detector operation (zero bias voltage).

battery were included; however, at large signal levels, the bias battery is ineffective and so the $g_{e\,eff}$ is smaller.

The convergence of the flow angle curves and the $g_{e\,eff}$ curves for the two cases illustrates the interrelationship of the signal voltage and the bias voltage. As might be expected, it is the ratio of the two which determines the flow angle and the $g_{e\,eff}$. Fig. 7 illustrates this, for the abscissa in this figure in $V_{eb'}/E_{EE}$. $V_{eb'}$ (hence the amplitude of oscillations) is directly proportional to E_{EE} which may be varied to provide linear modulation, a-g-c., or other desired control.

The Diode With An Exponential Characteristic

Appendix B is an analysis for a diode having an exponential voltage-current relationship. The waveforms for a circuit with this type of diode are shown in Fig. 8. The $g_{e\,eff}$ vs $V_{eb'}$ characteristic for this analysis is shown in Fig. 9. It is interesting to note that a $g_{e\,eff}$ characteristic which decreases with increasing a-c base-to-emitter voltage results from a device with a rising exponential characteristic. This occurs because the transistor is biased back rapidly enough that $g_{e\,eff}$ does decrease. The influence of E_{EE} differs in this case from the case of a diode with fixed forward resistance in that the amplitude of oscillations can no longer be expected to be directly proportional to E_{EE} because the forward resistance of the emitter junction is not constant. The flow angle will not remain constant with changes in E_{EE} for the same reason. The operation for both cases is quite similar except for the small quantitative differences pointed out.

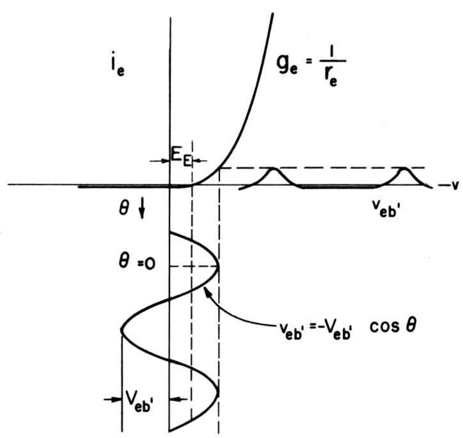


Fig. 8 – Emitter junction waveforms -- exponential junction.

When the base-to-emitter voltage swing becomes moderately large (100 mv for example), the resistance of the junction may be comparable with $r_{bb'}$ plus the driving resistance for much of the current flow period; thus, the operation will be intermediary between the case where the emitter junction is considered to have an exponential characteristic and the case where the forward resistance is a fixed value $r_{bb'}$ plus the driving resistance. It therefore is not useful to consider large values of a-c input voltage for the ideal exponential characteristic of Fig. 9 because this ideal operation is limited to low levels of a-c input voltage.

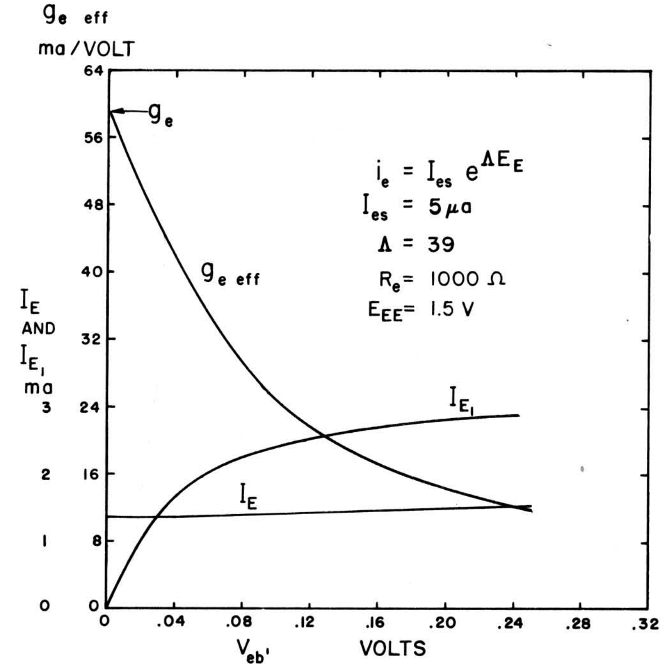


Fig. 9 – Diode characteristics (exponential current -- voltage relationship).

Equivalent Circuits for Large-Signal Operation

If operation in the region where the emitter junction has an exponential characteristic is considered again, the results of the analysis may be used to obtain equivalent circuits for the transistor for the fundamental frequency.

As an example of this, an equivalent circuit for the common-emitter connection is extended to apply to the large-signal case considered here by modifying the components as shown in Fig. 10. The current generator in the small-signal case, Fig. 10(a), relates the collector current to $V_{eb'}$. Since the relationship between the fundamental current component and $V_{eb'}$ for the large-signal case has been determined by the preceding analysis, a current generator may be incorporated into the fundamental component equivalent circuit which shows this relationship. The collector current will then be as shown in Fig. 10(b). The time constant of $r_{b'e}$ and $C_{b'e}$ is independent of bias current in the small-signal case ($r_{b'e} \propto 1/Id.c.$,

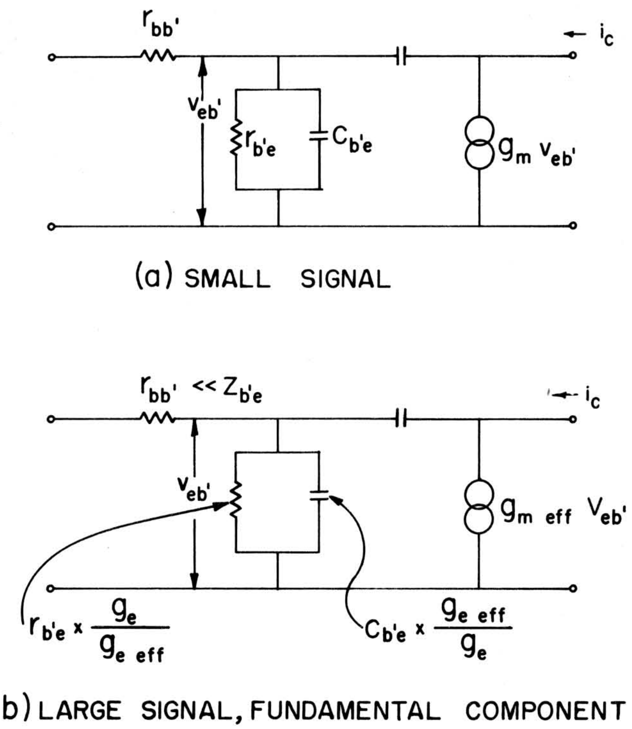


Fig. 10 – Common emitter equivalent circuits.

$C_{b'e} \propto Id.c.$); the time constant in the large-signal case will also be expected to be invarient even though there will be current components above the fundamental frequency. Therefore, $r_{b'e}$ and $C_{b'e}$ are modified so that the large signal time constant is the same as in the small-signal circuit. The internal feedback capacitor is unchanged. The large-signal circuit values approach the small-signal circuit values as $g_{m\ eff}$ approaches g_m.

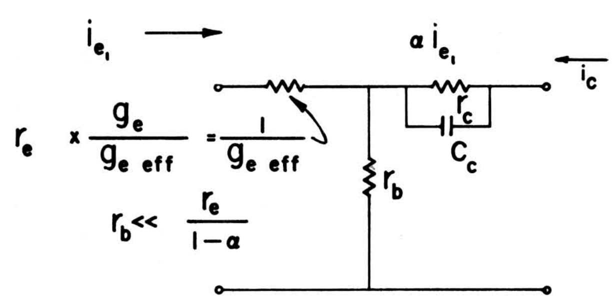
A common-base equivalent circuit may be similarly derived and is shown in Fig. 11. As in the common-emitter case, the circuit has been altered to show the change in the currents which flow due to $V_{eb'}$ in going from the small signal to the large-signal case. These currents still maintain the same relationship to each other (in either small or large-signal operation); therefore, α is unchanged. Both of these circuits apply only to the case where $r_{bb'}$ may be considered negligible. If the voltage dropped across $r_{bb'}$ is appreciable, then the voltage across the base-to-emitter junction may no longer be assumed to be sinusoidal, and $g_{m\ eff}$ will differ from the values obtained herein.

Frequency Stability

These equivalent circuits may be used to show the loading effects by the transistor upon the oscillator circuit. It was originally stated that in the type of circuit being considered here, the input and output impedances of the transistor are considered high with respect to the impedances of the circuit so that the loading effects of the transistor on the circuit are small. However, even a small percentage of reactive loading is important, as a small percentage change in frequency is significant.

For example, the g_{12} of the coupling network will change with tuning as a result of impedance and coupling changes. The $g_{m\ eff}$ of the transistor is at all times the same as the magnitude of the g_{12} of the coupling network; therefore, the value of g_{12} of the coupling network at any particular frequency may be substituted into the equivalent circuit of the transistor to determine what the reactive loading of the transistor upon the circuit is. The effect upon tracking due to changes of g_{12} may thus be determined.

It is sometimes desirable to vary the magnitude of oscillations, hence the conversion gain. Therefore, a.g.c. may be provided by varying the d-c bias voltage of the oscillator. This method of gain control is not desirable if the oscillator frequency changes excessively as a result of changes in the biasing of the transistor. To see what effect changing the biasing would be expected to produce upon the frequency of oscillation, one may look at the equivalent circuit (Fig. 10 for this example). Whereas the small-signal equivalent circuit of the transistor has many elements whose value depends upon the bias voltage (E_{EE}), it should be noted that the corresponding large-signal equivalent circuit elements are not dependent upon E_{EE}, but only upon $g_{m\ eff}$. Changing E_{EE} should not affect the g_{12} of the external circuit; therefore, the $g_{m\ eff}$ and hence the reactive loading of the transistor should remain constant. The physical explanation for this is that the $g_{m\ eff}$ must always remain equal to $-g_{12}$ of the coupling network for oscillations to be sustained; therefore, $g_{m\ eff}$ will be independent of E_{EE}. When E_{EE} is changed, the flow angle of current changes in such a manner that both the input impedance (to the fundamental frequency) and the $g_{m\ eff}$ remain the same (they are related



LARGE SIGNAL, FUNDAMENTAL COMPONENT

Fig. 11 – Common base equivalent circuits.

by a constant so the input impedance can not change while $g_{m\,eff}$ remains constant). These conclusions are, of course, contingent upon the previously made assumptions that the base-lead resistance and the nonlinearities in the transistor other than the base-to-emitter junction may be neglected. The presence of the base-lead resistance probably has the most significant effect in causing frequency variations due to bias changes. At any rate, the conclusion may be drawn that frequency changes of a transistor oscillator caused by the transistor input reactance result only indirectly from changes in bias.

T. G. Marshall

Table of Definitions

g_{12} = the real part of y_{12} of the coupling network. In this analysis it is the ratio of the fundamental component of current going into the coupling network to the voltage component of the fundamental frequency at the output.

g_e = the slope of the diode-current voltage characteristic of the emitter junction. Therefore it is the small-signal conductance.

$g_{e\,eff}$ = the effective conductance of the emitter junction, and is the ratio of the fundamental component of the emitter current to the sinusoidal voltage across the junction which causes the current to flow. This term refers to the large-signal condition where the emitter current is not necessarily sinusoidal as opposed to the small-signal case where it is. The voltage across the junction is always assumed to be sinusoidal.

g_m = the small signal transconductance of the transistor.

$g_{m\,eff}$ = the effective transconductance of the transistor, and is the $|g_{e\,eff}|$ times the d-c collector to emitter alpha.

I_E = direct current flowing through the diode.

I_{E1} = fundamental component of diode current.

Λ = $\frac{q}{KT}$ Dimensions are volts^{-1}. q is the charge of the minority carrier, K is Boltzmann's constant, and T is the absolute temperature (at $300^\circ K$, $\Lambda = 38.6$ volts^{-1}).

r_e = $1/g_e$

θ_1 = one-half the current flow angle (Fig. 5).

$V_{eb'}$ = the peak, a-c, input voltage between the emitter and internal base. This voltage is obtained from the selective feedback network and so is always assumed to be sinusoidal.

Fig. 4 illustrates the circuit to be analyzed. With the application of the sinusoidal input voltage, $v_{eb'}$, current pulses will flow as shown in Fig. 5. $v_{eb'}$ is made equal to *minus* $V_{eb'} \cos\theta$ in order that the period of current flow straddles $\theta = 0$. This facilitates the calculations.

The instantaneous current during conduction is then

$$i_e = g_e (v_{eb'} - E_E) = g_e (-V_{eb'} \cos\theta - E_E)$$

Fig. 5 shows that

$$v_{eb'} \Big|_{\theta_1} = - V_{eb'} \cos\theta_1 = E_E \qquad (1)$$

By Fourier analysis, the dc component of current is

$$I_E = \frac{g_e}{\pi} \int_0^{\theta_1} (- V_{eb'} \cos\theta - E_E)\, d\theta$$

$$= \frac{g_e}{\pi} (- V_{eb'} \sin\theta_1 - E_E\, \theta_1)$$

$$= \frac{g_e}{\pi} (- V_{eb'} \sin\theta_1 + \theta_1 V_{eb'} \cos\theta_1)$$

$$I_E = \frac{V_{eb'} g_e}{\pi} (\theta_1 \cos\theta_1 - \sin\theta_1) \qquad (2)$$

Also $$I_E = \frac{E_E - E_{EE}}{R_e} = \frac{-(V_{eb'} \cos\theta_1 + E_{EE})}{R_e} \qquad (3)$$

Knowing $V_{eb'}$, the three equations (1), (2), and (3) are sufficient to enable the unknown quantities I_E, E_E, and θ_1 to be found. The most convenient method of obtaining a solution is by solving for I_E and $V_{eb'}$ in terms of θ_1 as follows:

Equating equations (2) and (3) and solving one obtains:

$$V_{eb'} = \frac{E_{EE}/\cos\theta_1}{\frac{R_e g_e (\tan\theta_1 - \theta_1)}{\pi} - 1} = f(\theta_1) \qquad (4)$$

Equation (3) may be restated

$$V_{eb'} = \frac{E_{EE} - I_E R_e}{\cos\theta_1}$$

Substituting this equation (2) and solving, one obtains:

$$I_E = \frac{E_{EE} g_e (\tan\theta_1 - \theta_1) \frac{1}{\pi}}{\frac{R_e g_e (\tan\theta_1 - \theta_1)}{\pi} - 1} = f(\theta_1) \qquad (5)$$

Values of θ_1, may be selected and the corresponding values of $V_{eb'}$ and I_E may then be determined from equations (4) and (5). The results of this are shown in Fig. 6, where I_E and $2\theta_1$ are ordinates, and $V_{eb'}$ is the abscissa.

The Fourier analysis may also be used to determine the fundamental component of the current pulses.

$$I_{E1} = \frac{2g_e}{\pi} \int_0^{\theta_1} (- V_{eb'} \cos\theta - E_E) \cos\theta\, d\theta =$$

$$\frac{2g_e}{\pi} \left[\frac{-V_{eb'}(\theta_1 + \sin\theta_1 \cos\theta_1)}{2} - E_E \sin\theta_1 \right]$$

Simplifying:

$$I_{E1} = \frac{-V_{eb'} g_e}{\pi} (\theta_1 - \cos\theta_1 \sin\theta_1) = f(\theta_1) \qquad (6)$$

Values of θ_1 may be selected and the corresponding I_{E1} may be determined. Figure 6 shows these results also.

The effective g_e, $g_{e\,eff}$, of the transistor is the ratio of the fundamental component of current which flows to the input voltage which causes it, and is:

$$g_{e\,eff} = \left| \frac{I_{E1}}{V_{eb'}} \right| = \frac{g_e}{\pi} (\theta_1 - \cos\theta_1 \sin\theta_1) \qquad (7)$$

This is also shown in Fig. 6.

Eq. (4) may be restated as below,

$$\frac{V_{eb'}}{E_{EE}} = \frac{1/\cos\theta_1}{\frac{R_e g_e (\tan\theta_1 - \theta_1)}{\pi} - 1} \qquad (8)$$

from which Fig. 7 is obtained.

This analysis is for a diode having an exponential current-voltage relationship such as an emitter junction. Fig. 3 illustrates the circuit, and Fig. 8 shows the resulting current pulses which will flow as a result of $v_{eb'}$. The instantaneous current, i_e, is,

$$i_e = -I_{es}\, e^{-\Lambda(v_{eb'} - E_E)} = -I_{es}\, e^{\Lambda E_E}\, e^{\Lambda V_{eb'} \cos\theta}$$

By Fourier analysis, the dc component of current is,

$$I_E = \frac{-I_{es}}{2\pi}\, e^{\Lambda E_E} \int_0^{2\pi} e^{\Lambda (V_{eb'} \cos\theta)}\, d\theta$$

Integrating

$$I_E = -I_{es} e^{\Lambda E_E}\, I_0 (\Lambda V_{eb'}) \cong \qquad (1)$$

$$I_{es}\, e^{\Lambda E_E} \frac{e^{\Lambda V_{eb'}}}{\sqrt{2\pi \Lambda_{eb'}}}$$ (for $\Lambda V_{eb'} \geqq 10$, accuracy is 2% – accuracy is better for higher $V_{eb'}$)

$I_0 (\Lambda V_{eb'})$ is a modified Bessel function of the first kind, values of which are given in the TABLE OF FUNCTIONS of Jahnke-Emde, pp. 226-229 for arguments up to 10. Note that $I_0 (x) = J_0 (ix)$. The approximation[3] may be used for $\Lambda V_{eb'} \geqq 10$.

[3]TABLES OF INTEGRALS AND OTHER MATHEMATIC DATA, H. B. Dwight, p. 182.

Also,

$$I_E = \frac{E_E - E_{EE}}{R_e} \qquad (2)$$

In order to obtain a solution of the dc current, I_E, which flows as a result of $V_{eb'}$, one may assume values of I_E, and by using equations (1) and (2) and the aforementioned reference, find in turn

$$E_E,\ e^{\Lambda E_E},\ \frac{I_E}{-I_{es}\, e^{\Lambda E_E}} = I_0 (\Lambda V_{eb'}), (\Lambda V_{eb'}), \text{and } V_{eb'}.$$

The results of this are shown in Fig. 9 which is a graphical solution for the direct current corresponding to the values of $V_{eb'}$ shown.

The Fourier analysis may also be used to find the fundamental component of current.

$$I_{E1} = \frac{-I_{es}}{\pi}\, e^{\Lambda E_E} \int_0^{2\pi} e^{\Lambda (V_{eb'} \cos\theta)} \cos\theta\, d\theta$$

$$I_{E1} = -2\, I_{es} e^{\Lambda E_E}\, I_1 (\Lambda V_{eb'}) \cong \qquad (3)$$

$$-2\, I_{es} e^{\Lambda E_E} \frac{e^{\Lambda V_{eb'}}}{\sqrt{2\pi \Lambda V_{eb'}}} \left(1 - \frac{3}{8 \Lambda V_{eb'}} + ...\right)$$

(for $\Lambda V_{eb'} \geqq 10$, accuracy is 2% – accuracy is better for higher $V_{eb'}$).

$I_1 (\Lambda V_{eb'})$ is also modified Bessel function of the first kind, and values are given in the previously mentioned reference. The approximation[3] may be used for $\Lambda V_{eb'} \geqq 10$. Values of I_{E1} were obtained for the values of E_E and $(\Lambda V_{eb'})$ used above are shown in Fig. 9. $g_{e\,eff} = |I_{E_1}/V_{eb'}|$ is also shown.