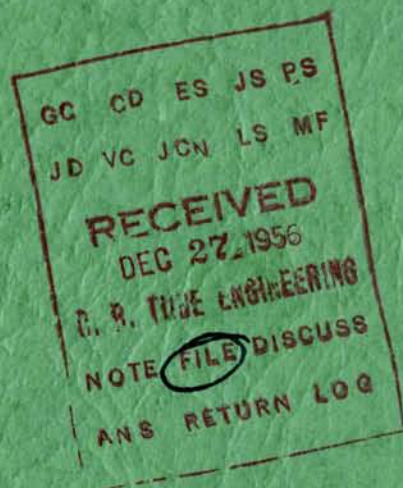




LB-1056

**THERMAL STABILITY OF
JUNCTION TRANSISTORS AND
ITS EFFECT ON MAXIMUM
POWER DISSIPATION**



**RADIO CORPORATION OF AMERICA
RCA LABORATORIES
INDUSTRY SERVICE LABORATORY**

DECEMBER 12, 1956

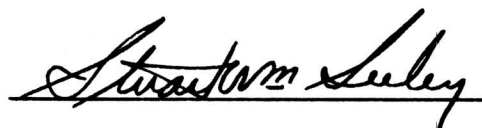
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The maximum permissible power dissipation of a transistor is limited by thermal stability considerations in most cases. Thermal instability occurs when internal temperature and collector current increase in a regenerative and uncontrollable fashion. The limit depends on factors both within and external to the transistor. The internal factors are the thermal resistance, the current amplification factor and the base lead resistance. The external factors are ambient temperature, collector voltage, circuit resistances, and the thermal coupling between the transistor and temperature compensation elements, if any. Circuits with greater temperature stability permit a higher maximum power dissipation. This bulletin discusses the factors which determine thermal stability and consequently, the maximum dissipation of transistors. The criteria for stability are derived mathematically and some thermal compensation techniques are treated. Examples are given to illustrate the use of these criteria.

General Discussion

The collector current of a transistor increases with internal temperature to an extent which depends on the electrical parameters of the transistor and the circuit. When power is dissipated, the internal temperature rises. This usually results in increased collector current and dissipation which further raise the temperature. If the heat can be removed fast enough, stable thermal equilibrium may be reached. If heat is not removed fast enough, thermal regeneration may continue and "runaway" occurs.

The dependence of collector current on temperature was discussed in LB-979, *Temperature Effects in Circuits Using Junction Transistors*. Two factors which determine the collector-current increase with temperature are: (1) the collector junction saturation current, and (2) the d-c transfer conductance which relates collector current to emitter-base voltage. Saturation current flowing through any resistance in series with the base will create a forward base-to-emitter bias and increase the collector current, often by a large factor. The increase in d-c transfer conductance with temperature will also increase the collector current. These effects are embodied in Shockley's equations for transfer characteristics¹ and the phenomenon of runaway may be treated analytically. This bulletin considers the critical conditions for thermal stability and shows what factors limit the maximum allowable dissipation.

When the collector of a transistor is biased more than a few tenths of a volt in the reverse direction, the d-c collector current, I_C , can be expressed approximately as

$$I_C = I_{CE} e^{qV_{B'E}/kT} \quad (1)$$

where I_{CE} is the d-c current coefficient, q is the electronic charge, k is Boltzmann's constant, T is the absolute temperature, and $V_{B'E}$ is the forward bias on the junction. As was pointed out in LB-979, I_{CE} varies nearly exponentially with increase in temperature. Also, $V_{B'E}$ is a function of the collector current and saturation current, which also varies nearly exponentially with temperature. Thus, Eq. (1) can be written as

$$I_C = I_{CE1} e^{\phi \delta T} e^{(q/kT_1)} / (I_S e^{\phi \delta T}) \quad (2)$$

where I_{CE1} and I_S are the d-c current coefficient and the saturation current at an ambient temperature T_1 , respectively, ϕ is the mean fractional increase in d-c current coefficient or saturation current per degree centigrade, and δT is the rise of junction temperature above ambient

¹W. Shockley, "P-N Junctions," *Phys. Rev.*, Vol. 83, p. 151, July, 1951.

temperature. For germanium transistors, $\phi \cong 0.1$ per degree C.

From this point on, δT will be defined as the rise of junction temperature above ambient temperature. If the active area of the transistor were connected to a perfect heat sink, then regardless of dissipation, the internal temperature would never vary from ambient. This, unfortunately, is never the case.

Eq. (2) gives one relationship between the collector current and the internal junction temperature. These quantities can also be related by computing the dissipation (from the collector current and voltage) and the resulting temperature rise. The two relationships may then be combined to determine I and δT .

The heat developed by dissipation is generally removed by a combination of conduction, convection, and radiation. Conduction is a linear process, that is, the rate of heat flow between two points by conduction is proportional to the first power of the temperature difference between the points. The rate of heat flow by convection increases slightly more rapidly than the first power of the temperature difference, usually as the $5/4$ power. Although the power radiated by a body varies with the fourth power of its temperature, the rate of power transfer between two bodies at slightly different temperatures may be approximated as a linear function.

For small temperature differences near room temperature, conduction can be the most effective method of heat removal. This fact is used in designing transistors which are to handle any appreciable amount of power. For example, the collector is often soldered directly to a copper bolt which is fastened through a metal chassis. Thus, conduction usually plays the dominant role and is a linear process, convection is usually second in importance and is only somewhat nonlinear, and radiation is also an approximately linear process for small temperature differences. One would expect, then, that a relation of the form

$$\delta T = \theta W_c \quad (3)$$

would be reasonably accurate. Here W_c is the internal dissipation and θ is the "thermal resistance" of the transistor and mounting. Measurements made on a number of transistor types confirm this expectation. Methods for measuring the thermal resistance are described in RB-30, *Equipment for Measuring Junction Temperature of an Operating Transistor*.

In applying Eq. (3) the problem of defining collector dissipation in terms of collector voltage and current requires some consideration. The instantaneous dissipation is $V_{CE} I_C$ where V_{CE} is the collector-to-emitter voltage. If V_{CE} and I_C are a-c quantities, however, one

must decide whether to set W_c equal to the peak dissipation or to the average dissipation. The decision depends on the relationship between the a-c frequency and the "thermal time constant" of the transistor or, in other words, on whether the junction temperature can follow the variation of instantaneous dissipation. It would be desirable to relate W_c to the supply voltage, nature of the load, class of operation (A, B, A-B etc.) but the problem becomes far too complex to be handled analytically. Since the stability criteria are most often used to compute maximum ratings and since a safety factor is desirable, first consider the least stable case, that is, where both V_{CE} and I_C are d-c quantities and are independent of each other (i.e., no load in the collector circuit), then

$$\delta T = \theta V_{CE} I_C \quad (4)$$

as the second relation ship between δT and I_C .

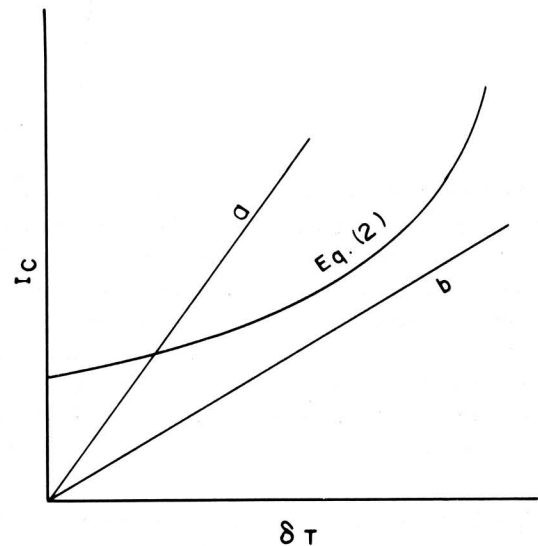


Fig. 1 - Plot of Equation 2 and 4.

If there are values of δT and I_C that can satisfy both the transistor characteristic of Eq. (2) and the heat conduction of Eq. (4), a thermal equilibrium can be reached. If there is no solution of δT and I_C that can satisfy Eqs. (2) and (4), the system is unstable. When plotted, Eq. (4) is a straight line as in Fig. 1(a) or 1(b). If this straight line intersects a plot of Eq. (2) as line (a) does, the circuit will reach equilibrium at the point of intersection. If there is no intersection (as in the case of line b), the transistor will run away. To find the criteria for thermal stability, solve for the condition of tangency of the curves for Eq. (2) and Eq. (4). As Eq. (2) implicitly involves circuit parameters, the stability criteria for different circuits are necessarily different.

Circuit Considerations

In LB-979, the variation of operating currents with temperature in different circuit configurations was discussed. In this bulletin, the effect of these configurations on maximum permissible dissipation are considered.

Case (a) $R_B = 0, R_E = 0, R_C = 0$

Fig. 2 shows the general d-c connection of a transistor circuit. When the external resistances are zero, the internal base-to-emitter bias is less than the external bias, V_{BE} , by an amount equal to the voltage drop due to base current, I_B , flowing in the base lead resistance, $r_{BB'}$.

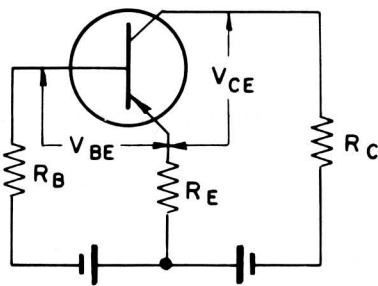


Fig. 2 - Transistor d-c connection.

$$V_{B'E} = V_{BE} - I_B r_{BB'}$$

The relationship between the collector current and the base current, as mentioned previously, is governed by the collector-to-base current amplification factor, α_{CB} . As

$$\alpha_{CB} \cong I_C / [I_B + (I_S) \delta T] \quad (5)$$

$$V_{B'E} = V_{BE} - I_B r_{BB'} = V_{BE} - [I_C / \alpha_{CB} - (I_S) \delta T] r_{BB'} \quad (6)$$

where $(I_S) \delta T$ is the base saturation current corresponding to internal temperature. Substituting Eq. (5) into Eq. (2) and solving for the condition of tangency of Eqs. (2) and (4), (see Appendix 7), the stability criterion becomes

$$P_m = \frac{1}{\phi \theta} \ln \left[\frac{1}{\phi \theta \alpha_{CB} V_{CE} I_S} + \frac{1}{\Lambda r_{BB'} I_S} \left(\frac{1}{\phi \theta P_m} - 1 \right) \right] \quad (7)$$

where P_m = maximum stable dissipation of the transistor in watts, I_S = saturation current at temperature of heat sink (ambient), and $\Lambda = q/kT \cong 40$.

Eq. (7) indicates that maximum dissipation decreases as I_S increases. The reason is that the forward base-to-emitter bias created by I_S flowing through any resistance in series with the base increases with internal-temperature rise of the transistor. As the rate of change of I_S with respect to temperature is proportional to the absolute value of I_S , the rate of increase of the forward bias is also proportional to I_S . A greater increase in forward bias causes a greater change in the collector current, hence the tendency to run away is also greater. The fact that P_m decreases with increase in I_S also means that P_m decreases with increase in the ambient temperature.

In Eq. (7), if the term $(1/\phi \theta P_m - 1)/\Lambda r_{BB'} I_S$ is negligible, as in certain cases to be discussed later, then P_m is proportional to the logarithm of I_S . Since I_S varies nearly exponentially with temperature, the reduction in P_m varies nearly linearly with increase in the ambient temperature.

ϕ is the percentage increase in base saturation current or d-c conductance coefficient with respect to temperature. ϕ is a constant of the base material of the transistor. θ is the thermal resistance. When heat can be removed readily θ is small, and the temperature rise inside the transistor is small. Maximum dissipation increases as the product $\phi \theta$ is reduced.

When $r_{BB'}$ is zero, the effect of temperature is controlled by the change in d-c transfer conductance. In Eq. (7) when $r_{BB'} = 0$, $P_m = 1/\phi \theta$.

The presence of base lead resistance, $r_{BB'}$, has two effects. One effect is that the flow of saturation current in $r_{BB'}$ creates a forward bias which increases the collector current. The other effect is that this resistance connected between a fixed bias voltage and a variable d-c base input conductance tends to hold the base current constant. If the first effect dominates, a larger value of $r_{BB'}$ makes the collector current more sensitive to temperature, reducing the maximum power dissipation. If the second effect dominates, a larger value of $r_{BB'}$ helps reduce the variation of collector current with temperature, hence increasing P_m . Whether the first effect or the second is dominant depends on the value of I_S . A higher value of I_S creates a greater forward bias, and, therefore, tends to make the first effect more dominant. From Eq. (7), it can be seen that for $I_S < 1/2.718 \phi \theta \alpha_{CB} V_{CE}$, the addition of $r_{BB'}$ increases the maximum dissipation.

When the value of $r_{BB'}$ is high enough to hold the base current substantially constant, a further increase in $r_{BB'}$ does not increase the maximum dissipation appreciably. In Eq. (7), when $r_{BB'}$ is high, the second term in the numerator is negligible. P_m becomes

$$P_m = \frac{1}{\phi\theta} \ln(1/\phi\theta a_{CB} V_{CE} I_S) \quad (8)$$

An increase in current amplification factor, a_{CB} , decreases the maximum power dissipation. The higher a_{CB} , the greater is the variation of collector current with respect to variations in the forward base-to-emitter bias, hence the smaller is P_m .

An increase in collector-to-emitter voltage, V_{CE} , reduces the maximum power dissipation. As the voltage is increased, a greater variation in power dissipation is experienced for a given variation in collector current. Hence there is a greater change in temperature inside the transistor. In Fig. 1, the effect of an increase in collector voltage is to reduce the slope of the dissipation curve, thus increasing the tendency to run away. In Eq. (7) an increase in V_{CE} has the same weight as an increase in a_{CB} .

Eq. (7) is plotted in Fig. 3. With this graph, it is possible to determine the maximum power dissipation of a transistor, operating in a circuit with no external resistance in the circuit. (See Example (1), Appendix 1).

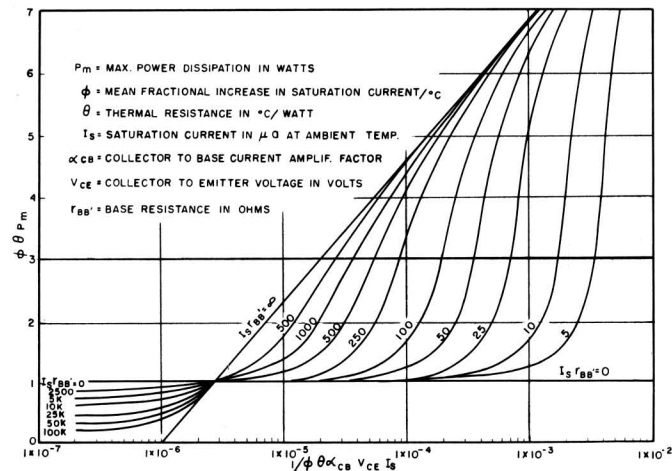


Fig. 3 - Graph for determining the maximum power dissipation of a transistor operating in a circuit with no external resistance.

By replacing P_m in Eq. (7) with the product, $V_{CE} I_C$, the thermal stability criterion can also be expressed in terms of normalized voltage, $(\phi\theta a_{CB} I_S) V_{CE}$, and normalized current, $I_C/(a_{CB} I_S)$. They are plotted in Fig. 4 with $I_S r_{BB'}$ as a running parameter. If the instantaneous dissipation of a transistor amplifier is not to exceed P_m at any time, the load line should lie below the curve in Fig. 4 corresponding to the value of $I_S r_{BB'}$ of the transistor. (See Example (2), Appendix 1)

This procedure is based on operating the transistors with the least stable signal of a very long duration. In an ordinary audio amplifier, the lowest operating frequency has a period much shorter than the thermal time constant

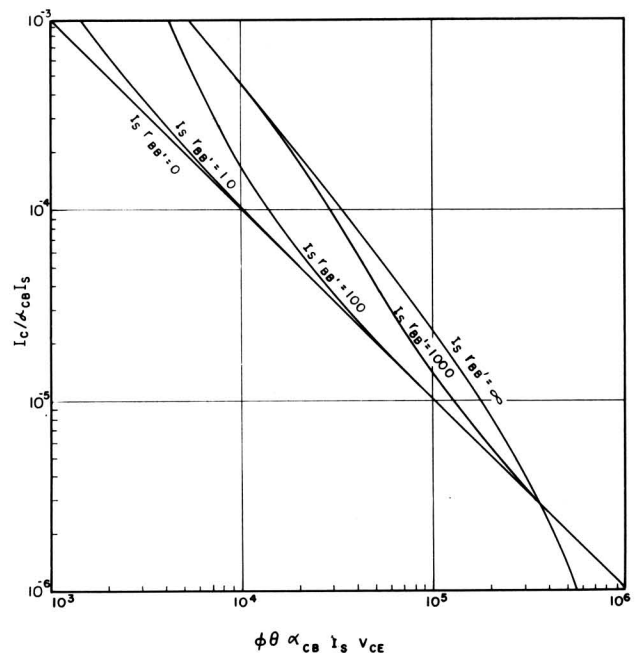


Fig. 4 - Plot of the thermal stability criterion.

of the transistor, so that even if the dissipation limit is exceeded momentarily the conducting transistor does not have enough time to run away before dissipation is reduced. In class B operation each transistor conducts only half of the time, and hence the maximum average dissipation is only half of the maximum dissipation. Thus, for ordinary audio amplifiers, the foregoing procedure would result in gross overdesign. A more realistic approach is to use the average voltage in Eq. (7) or in Fig. 3 for finding the maximum average permissible dissipation.

Case (b) $R_B \neq 0, R_C = 0, R_E = 0$

An external resistance, R_B , connected in series with the base has the same effect as though the base lead resistance, $r_{BB'}$, were increased by an amount equal to R_B . In moderate temperature operation where I_S is small, the addition of R_B usually helps increase the maximum power dissipation. The rate of increase, however, diminishes as $r_{BB'}$ is increased above a sufficiently high value. In high ambient temperature operation, where I_S is large, the introduction of R_B reduces the maximum power dissipation. These effects have been discussed in Case (a).

Case (c) $R_B = 0, R_C \neq 0, R_E = 0$

When a d-c resistance, R_C , is connected in series

with the collector, the collector-to-emitter voltage, V_{CE} , is equal to the difference between the supply voltage, V_{CC} , and the voltage drop in R_C . When collector current increases, V_{CE} is reduced. The tendency to run away is, therefore, decreased.

Analytically, the stability criterion can be found by a method similar to *Case (a)*, (See Appendix 2).

$$P_m = \frac{1}{\phi\theta} \ln \left\{ \frac{1}{\phi\theta a_{CB} V_{CE} I_S \sqrt{1 - 4P_m R_C / V_{CC}^2}} + \frac{1}{\Lambda r_{BB'} I_S} \left[\frac{1}{2\phi\theta P_m} \left(\frac{1}{\sqrt{1 - 4P_m R_C / V_{CC}^2}} + 1 \right) - 1 \right] \right\} \quad (9)$$

In this expression $\sqrt{1 - 4P_m R_C / V_{CC}^2}$ is always less than unity. Compared with *Case (a)*, the term $1/\phi\theta a_{CB} V_{CE} I_S$ in Eq. (7) can be considered as being increased by $1/\sqrt{1 - 4P_m R_C / V_{CC}^2}$ times, and the term $(1/\phi\theta P_m - 1)/\Lambda r_{BB'} I_S$ in Eq. (7) is increased by an amount equal to $(1/\sqrt{1 - 4P_m R_C / V_{CC}^2} - 1)/2\phi\theta \Lambda r_{BB'} I_S P_m$. P_m , as expressed in Eq. (9), can be found graphically as shown in Example (3), Appendix 1.

The maximum dissipation thus obtained can be realized only if its value is less than $V_{CC}^2/4R_C$, because this is the maximum power that can be delivered from the power supply.

Case (d) $R_B = 0, R_C = 0, R_E \neq 0$

When a resistance R_E , much greater than the internal emitter resistance of the transistor, is connected in series with the emitter, it tends to hold the emitter current constant. As the collector current, I_C , is nearly equal to the emitter current, the variation of I_C with temperature can be substantially reduced by R_E . This results in increased maximum power dissipation.

Another reason for increased power dissipation is that R_E has an effect similar to R_C in *Case (c)* in reducing the collector-to-emitter voltage. Together with the first-mentioned effect this method is more effective than *Case (c)* in increasing the maximum power dissipation when the same supply and operating voltages are used.

When R_E is present, there exists in the external loop between the base and the emitter a voltage drop

caused by emitter current flowing in R_E . When I_E is expressed in terms of I_S and I_C , and the voltage drop in $r_{BB'}$ is taken into account, the internal base-to-emitter voltage becomes

$$V_{B'E} = V_{BE} + (I_S)\delta T (r_{BB'} + R_E) - \frac{I_C}{a_{CB}} - (r_{BB'} + R_E + a_{CB} R_E) \quad (10)$$

In comparison with the corresponding bias in *Case (a)* as given in Eq. (6), the effect of R_E is the same as though the base lead resistance were increased to $r_{BB'} + R_E$, and current amplification factor*, reduced to $(\frac{r_{BB'} + R_E}{r_{BB'} + a_{CB} R_E}) a_{CB}$. In using Eq. (9) for finding P_m the values of a_{CB} and $r_{BB'}$ should be transformed accordingly. When $R_E \gg r_{BB'}/a_{CB}$, the effective current amplification factor is approximately equal to $(r_{BB'} + R_E)/R_E$ and the maximum power dissipation becomes independent of a_{CB} . The increase in P_m over *Case (a)* is $\frac{1}{\phi\theta} \ln \left(\frac{a_{CB} R_E}{r_{BB'} + R_E} \right)$.

If R_B is present, the effect is the same as though $r_{BB'}$ were increased by an amount equal to R_B .

To increase the maximum power dissipation of a base-input transistor stage, it is more advantageous from the standpoint of power gain and interchangeability to use external emitter resistance than to use a lower collector voltage or lower a_{CB} transistor even if R_E is unbypassed for signal current, because the former provides greater power gain and better interchangeability.

Case (e) Collector-to-base Feedback Circuit

When d-c collector current is fed back to the base in a direction to reduce the variation in base and collector currents, the maximum power dissipation can be increased. One method for accomplishing this negative d-c feedback is by the circuit² shown in Fig. 5. In this circuit, a feedback resistance is connected between the collector and the base. When the collector current increases the voltage drop in R_C increases, lowering the collector voltage. This change in collector voltage is

*The effective values of a_{CB} , derived here and afterwards are equal in magnitude to the stability factors derived by Shea etc. in FUNDAMENTALS OF TRANSISTOR ENGINEERING. They are, however, not equal to the stability factors given in LB-979 which are more precise expressions for dI_C/dI_S .

²Lo, Endres, Jawals, Waldhauer and Cheng, TRANSISTOR ELECTRONICS, Prentice Hall.

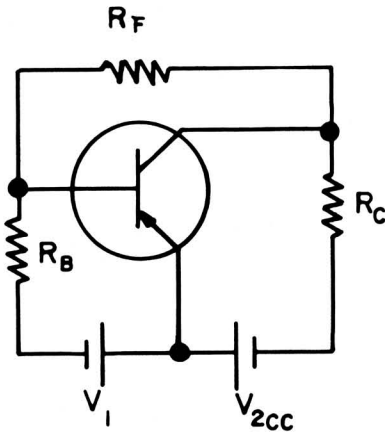


Fig. 5 - D-C negative feedback circuit to increase the maximum power dissipation.

degeneratively fed back to the base thus reducing the change in collector current.

Solving for the voltage between the emitter and the base, it can be found that

$$V_{B'E} = V_{CC} - \frac{R_C + R_F}{R_C + R_F + R_B} (V_{CC} - V_1) - \left[\frac{R_B R_C + (R_B R_C + R_B R_F) a_{CB}}{R_C + R_F + R_B} + \frac{r_{BB'}}{a_{CB}} \right] I_C + \left[\frac{R_B R_C + R_B R_F}{R_B + R_F + R_C} \right] (I_S) \delta T \quad (11)$$

Comparing Eq. (11) with Eq. (6), the effective base lead resistance is $r_{BB'} + \frac{R_B}{1 + R_B/(R_F + R_C)}$, the effective current amplification factor,

$$a_{CB} / \left[1 + \frac{R_B R_C a_{CB}}{(R_B + r_{BB'}) (R_F + R_C) + R_B r_{BB'}} \right]$$

latter small compared with a_{CB} , substantial increase in P_m can be realized. To do this

$\frac{R_B R_C a_{CB}}{(R_B + r_{BB'}) (R_F + R_C) + R_B r_{BB'}}$ should be chosen to be much greater than unity.

As in Case (c), P_m can be determined graphically when effective values of base resistance and current amplification factors are used in Eq. (9).

Case (f) Circuits with Thermally-coupled Temperature Compensation.

The forward base-to-emitter biasing voltage required to give a constant collector current decreases with increasing temperature. Methods for deriving a bias using diodes, thermistors, etc. to compensate for ambient temperature variation were described in LB-979, *Effect of Temperature in Junction Transistor Circuits*. These methods are used to maintain the quiescent operating point constant over wide temperature range for certain circuits such as detectors or Class B amplifiers, so that the transistor would not cut off at low temperature or run away at high temperature even at no-signal condition. When the compensating element is thermally coupled to the transistor, the tendency for the collector current to vary with temperature is reduced and the maximum power dissipation can be increased. For example, it was mentioned in LB-979 that a compensating diode may be built inside a transistor, sharing a common electrode to provide tight thermal coupling. The arrangement is shown in Fig. 6.

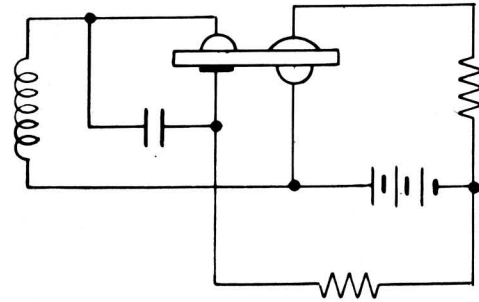


Fig. 6 - Compensation diode built inside a transistor to provide tight thermal coupling.

Approximations

Some approximations can be made for the expressions for the maximum power dissipations derived in the foregoing cases if the base resistance is high and the emitter or collector resistance is low. Base resistance is considered high when $r_{BB'} + R_B \gg \frac{\phi \theta V_{CE} a_{eff}}{\Lambda}$ where

a_{eff} is the effective collector-to-base amplification factor. Collector or emitter resistance is considered low when R_C or $R_E \ll V_{CC}^2 / 4P_m$. For resistance-coupled transistor circuits, these conditions are often satisfied. When such is the case, all the foregoing expressions for P_m can be generalized as follows:

$$P_m = \frac{1}{\phi \theta} \ln \frac{1}{\phi \theta (I_S)_{25^\circ C}} - \frac{1}{\phi \theta} \ln V_{CE} - \frac{T_a - 25}{\theta} - \frac{1}{\phi \theta} \ln a_{eff} \quad (12)$$

TABLE I

 α_{eff} For Use In Equation (12)

Case	Conditions	α_{eff}
a	$R_E = 0$	α_{CB}
d	$R_E \neq 0$	$(r_{BB'} + R_E) \alpha_{CB} / (r_{BB'} + \alpha_{CB} R_E)$
e	Collector-to-base feedback through R_F	$\frac{\alpha_{CB}}{1 + R_B R_C \alpha_{CB} / (R_B + r_{BB'}) (R_F + R_C) + R_B r_{BB'}}$

where $(I_{BS})_{25^\circ C}$ = saturation current at 25 degrees C, (see Example (4) Appendix 1) and T_A is the ambient temperature in centigrade.

The first term in the right-hand side is independent of external elements. The second term is a function of collector voltage. The third term is due to ambient temperature. $\frac{1}{\phi\theta} \ln \alpha_{eff}$ is a function of effective collector-

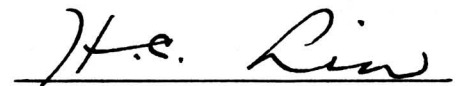
to-base current amplification factor and can be found from Table I.

Any external resistance in series with the base should be added to $r_{BB'}$.

Conclusions

The results obtained from the maximum power dissipation equations tend to be pessimistic, because the temperature coefficient of saturation current and d-c conductance is assumed to be constant, whereas actually the coefficient decreases somewhat at high temperatures. Also, the current amplification factor is assumed to be constant, whereas actually α_{CB} reduces at high current densities³. Furthermore, in neglecting $V_{B'E} \phi/V_g$ during the derivation of P_m , an additional factor of safety is incorporated.

³LB-917 On the Variation of Junction Transistor Current Amplification Factor With Emitter Current.



H. C. Lin

Examples

Example (1) Transformer-Coupled Class A Amplifier

A germanium power transistor is connected in a class A transformer coupled stage with $V_{CE} = 25 \text{ v}$ and a highest operating temperature of 40 degrees C. The transistor parameters are as follows: $\theta = 6 \text{ degrees C/W}$; $\alpha_{CB} = 10$; $I_S = 100 \text{ } \mu\text{amp}$ at 40°C; $r_{BB'} = 10 \text{ } \Omega$. Find the maximum permissible dissipation.

Solution:

Then $1/\phi\theta \alpha_{CB} V_{CE} I_{BS} = 6.67 \times 10^{-5}$

On Fig. 3, draw a vertical line from the abscissa where $1/\phi\theta \alpha_{CB} I_S = 6.67 \times 10^{-5}$ and intercept the curve corresponding to $I_S r_{BB'} = 1000 \text{ } \mu\text{v}$. Read from the ordinate $\phi\theta P_m = 3.9$. Thus $P_m = 5.8 \text{ watts}$.

Example (2) Transformer-Coupled Class B Stage

Two germanium transistors are connected in a transformer-coupled class B amplifier as shown in Fig. 7. Each transistor has the following characteristics: $\theta = 200^\circ\text{C/W}$; $\alpha_{CB} = 70$; $I_S = 20 \text{ } \mu\text{a}$ at 50°C. The amplifier is driven from a current generator, and operated from a 9-volt battery. If the instantaneous dissipation is not to exceed P_m at any time, find; at an ambient temperature of 50 degrees C, (i) the maximum sinewave output power; (ii) the maximum possible dissipation in each transistor; (iii) the least stable signal. If the lowest operating frequency has a period much shorter than the thermal time constant and if the resistance in series with the base including the base lead resistance is 250 ohms, find (IV) the maximum stable dissipation.

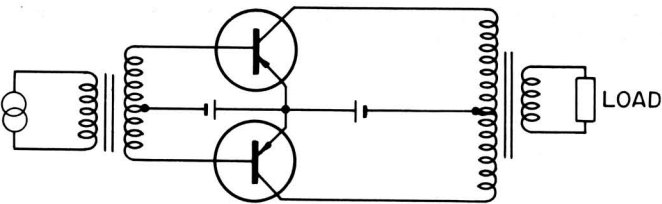


Fig. 7 – Transformer-coupled class B amplifier.

Solution:

(i) When the amplifier is driven from a current generator (such as the collector of a transistor or the plate of a pentode tube under ordinary operating conditions), the base input current to the class B amplifier is held by the current generator as though the base were fed from a voltage generator through a high resistance. Hence in using Fig. 4, the curve, $I_S r_{BB'} = \infty$, should be chosen. When the values of θ , α_{CB} , I_S and V_{CE} are substituted into the coordinates of Fig. 4, the maximum

permissible dissipation curve can be transformed into linear voltage-current scales as shown in curve "a", Fig. 8. From $V_o = 9 \text{ v}$ on the abscissa, draw a load line "b" tangent to curve "a". Since every point on line "b" lies below the maximum permissible dissipation curve "a", the amplifier is in stable operation regardless of the wave shape of the signal. The intercept of line "b" with the ordinate reads $I_o = 43 \text{ ma}$. For maximum sine-wave signal, I_o corresponds to the peak current, and V_o , the peak voltage. The maximum sine-wave output is therefore equal to $9 \text{ (v)} \times 43 \text{ (ma)} / 2 = 193 \text{ mw}$.

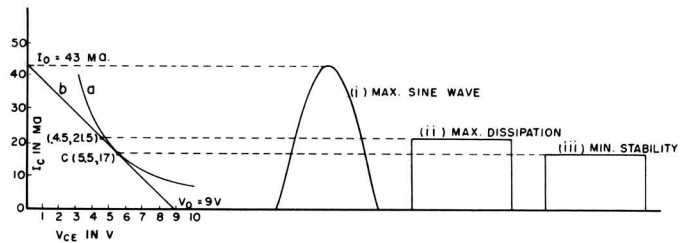


Fig. 8 – Voltage-current curves for the solution of the transformer-coupled-stage example.

(ii) The product of $V_{CE} I_C$ product occurs when $V_{CE} = V_o/2 = 4.5 \text{ v}$ and $I_C = I_o/2 = 21.5 \text{ ma}$ or $V_{CE} I_C = 97 \text{ mw}$. A square wave signal operating at this point during conduction therefore causes maximum dissipation in the transistor. Since each transistor conducts half of the time, the maximum possible dissipation is equal to $97/2 = 49 \text{ mw}$. Note that this dissipation is equal to one-quarter of maximum sine-wave output.

(iii) The most critical condition from the standpoint of thermal stability occurs when the line "a" touches line "b", or at point "c", ($V_{CE} = 5.5 \text{ v}$, $I_C = 17 \text{ ma}$). A square wave signal operating at this point during conduction is therefore the least stable signal.

(iv) The maximum stable dissipation for the case in which the lowest operating frequency is much shorter than the thermal time constant can be found from Fig. 3 as in the last example, using $V_{CE} = 9 \text{ V}$. and $I_S r_{BB'} = 20 \times 250 = 5000 \text{ } \mu\text{v}$. Finite d-c resistance of 250 ohms is substituted in $I_S r_{BB'}$ here since the change in saturation current is slow relative to the audio frequencies. The graphical solution gives $P_m = 55 \text{ mw}$. Since the maximum sine-wave output is four times the maximum possible average dissipation per transistor, $P_{out} = 4 P_m = 220 \text{ mw}$, which is greater than the 193 mw obtained in (i).

Example (3) Resistance-Coupled Class A Stage

A low-power germanium transistor is connected in a circuit with $R_B + r_{BB'} = 2000 \text{ } \Omega$, $R_C = 250 \text{ } \Omega$, $V_{CC} = 20 \text{ V}$. The transistor has the following characteristics:

$\theta = 250 \text{ C/W}$, $\alpha_{CB} = 40$, $I_{BS} = 2.5 \mu a$ at $25^\circ C$. Find P_m at $25^\circ C$.

Solution:

Eq. (9) can be solved by a graphical method as shown in Fig. 9. In this figure, the left-hand set of curves is a plot of the function $\frac{1}{\sqrt{1-4P_m R_C/V_{CC}^2}}$. The right hand set of curves is a plot of the function $(1/\sqrt{1-4P_m R_C/V_{CC}^2} + 1)/2\phi\theta P_m - 1$. The straight line in the middle is a natural logarithm plot. Use upper scale for the right and left hand sets of curves, and lower scale for the logarithm plot. From a point $X_1 = 1/\phi\theta I_S V_{CE} \alpha_{CB}$ in the lower scale, draw a curve (i) parallel to a curve in the left hand family corresponding to $n = 4R_C/\phi\theta V_{CC}^2$. Multiply the abscissa of either the right hand curve or the curve on Fig. 9 corresponding to the chosen n by $1/\Lambda r_{BB}$, and plot curve (ii). Subtract the product from curve (i) in the horizontal direction and a new curve (iii) is obtained. Then the intersection of curve (iii) and the logarithm curve (iv) gives the solution of P_m .

For this example,

$$1/\phi\theta = 0.04 \text{ watt}$$

$$n = \frac{4R_C}{\phi\theta V_{CC}^2} = 0.1$$

From a point $X_1 = 1/\phi\theta \alpha_{CB} V_{CC} I_S = 2 \times 10^{-5}$ in Fig. 9, draw curve (i) parallel to left hand curve corresponding

to $n=0.1$. Multiply $\frac{1}{I_{BS} \Lambda r_{BB}} = 5 \times 10^{-6}$ by the abscissa of the right hand curve corresponding to $n=0.1$. The product is represented by curve (ii). Subtract horizontally the magnitude of curve (ii), resulting in curve (iii). Then from the intersection of curve (iii) and line (iv), read $\phi\theta P_m = 3.0$. Thus $P_m = \frac{3}{\phi\theta} = 120 \text{ mw}$.

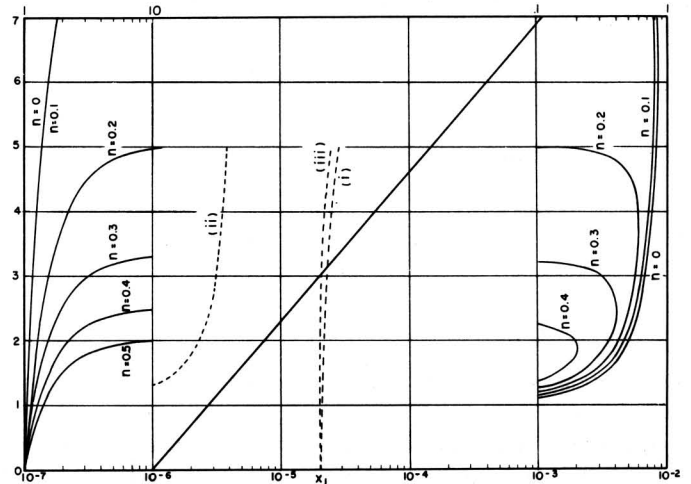


Fig. 9 – Graphical solution of Equation 9.

Example (4) Transistor Stage in Different Configurations

A germanium transistor has the following characteristics: $\theta = 250^\circ C/W$, $\alpha_{CB} = 40$, $(I_{CO})_{25^\circ C} = 2.5 \mu a$, $r_{BB}' = 200 \Omega$. An external resistance $R_B = 1800 \Omega$ is used. Find approximate P_m at $25^\circ C$ and $50^\circ C$ for (i) $R_E = 0$; (ii) $R_E = 200 \Omega$, (iii) $R_F = 10 \text{ K}\Omega$, $R_C = 250 \Omega$. From Eq. (7) and Table I, the approximate solution is shown in Table II.

Those results thus obtained are within 25 per cent from these obtained by the exact method.

TABLE II

Solutions For Example (4)

Case	Conditions	P_m in mw	
		$V_C = 20V, T_a = 25^\circ C$	$V_C = 20V, T_a = 50^\circ C$
a	$R_E = 0$	120	20
d	$R_E = 200$	180	80
e	$R_F = 10K,$ $R_C = 200$	141	41

Stability Criteria

In case (a) ($R_B=0$, $R_E=0$, $R_C=0$), the intrinsic base-to-emitter bias is expressed in Eq. (6). Substituting Eq. (6) into Eq. (1), the transistor characteristic equation becomes

$$I_C - I_{CS} = I_{CE} e^{\Lambda [V_{BE} + I_S r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}]} \quad (i)$$

The heat conduction equation is expressed in Eq. (4).

$$\delta T = \theta V_{CE} I_C \quad (ii)$$

In order that the curve representing Eq. (i) is tangent to that representing Eq. (ii), (1) the slopes of the two equations should be equal at the point of tangency; (2) the temperature and the collector current should satisfy both equations at the same point.

The slope of Eq. (i) can be found by differentiating I_C with respect to T . Thus

$$\frac{dI_C}{dT} - \frac{dI_{CS}}{dT} = \frac{dI_{CE}}{dT} e^{\Lambda [V_{BE} + I_S r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}]} + I_{CE} e^{\Lambda [V_{BE} + I_S r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}]} \left\{ \left[\frac{d\Lambda}{dT} [V_{BE} + I_S r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}] \right] \right.$$

$$\left. + \Lambda \left[r_{BB}' \frac{dI_S}{dT} - \frac{r_{BB}'}{\alpha_{CB}} \left(\frac{dI_C}{dT} - \frac{dI_{CS}}{dT} \right) \right] \right\} \quad (iii)$$

From LB-979, it was shown that both I_S and I_{CE} have a temperature coefficient equal to $e^{\phi \delta T}$, where $\phi \cong qV_g/kT^2$, hence $\frac{dI_{CE}}{dT} = \phi I_{CE}$; $\frac{dI_S}{dT} = \phi I_S$ (iv)

$$\frac{d(I_C - I_{CS})}{dT} = (I_C - I_{CS}) \left\{ \phi + [V_{BE} + I_S r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}] \frac{d\Lambda}{dT} + \phi \Lambda r_{BB}' I_{BS} - \right.$$

$$\left. \frac{\Lambda r_{BB}'}{\alpha_{CB}} \frac{d(I_C - I_{CS})}{dT} \right\} \quad (v)$$

Since the intrinsic base-to-emitter voltage $V_{B'E} = V_{BE} + I_{BS} r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}$ is normally much less than the energy gap V_g , the product

$$[V_{BE} + I_{BS} r_{BB}' - (I_C - I_{CS}) r_{BB}' / \alpha_{CB}] \frac{d\Lambda}{dT} = \frac{V_{B'E}}{V_g} \phi \quad (vi)$$

can be neglected in comparison with ϕ . Also negligible is I_{CS} which is much less than I_C in useful operation. Thus Eq. (v) becomes

$$\frac{dI_C}{dT} = I_C \left[\phi + \phi \Lambda r_{BB}' I_S - \frac{\Lambda r_{BB}'}{\alpha_{CB}} \frac{dI_C}{dT} \right] \quad (vii)$$

$$\text{The slope } dI_C/dT \text{ is then } \frac{dI_C}{dT} = \frac{\phi(1 + \Lambda r_{BB}' I_S) I_C}{1 + \Lambda r_{BB}' I_C / \alpha_{CB}}.$$

The slope of the heat conduction equation (ii) is $1/\theta V_{CE}$. When the two slopes are equal

$$\frac{1}{\theta V_{CE}} = \frac{\phi [1 + \Lambda r_{BB}' (I_S) \delta T] I_C}{1 + \Lambda r_{BB}' I_C / \alpha_{CB}} \quad (viii)$$

Here the subscript δT indicates that the saturation current I_S is evaluated at equilibrium temperature δT above the initial temperature (of the heat sink).

$$\text{Since } \frac{(I_S) \delta T}{I_S} = e^{\phi \delta T} \quad (ix)$$

Substitute (ix) into (viii)

$$\frac{1}{\theta V_{CE}} = \frac{\phi (1 + \Lambda r_{BB}' I_S e^{\phi \delta T}) I_C}{1 + \Lambda r_{BB}' I_C / \alpha_{CB}} \quad (x)$$

To satisfy condition (2) for tangency of the two curves, substitute Eq. (ii) into Eq. (x) to eliminate δT

$$\frac{1}{\theta V_{CE}} = \frac{\phi(1 + \Lambda r_{BB'} I_S e^{\phi \theta V_{CE} I_C}) I_C}{1 + \Lambda r_{BB'} I_C / \alpha_{CB}} \quad (\text{xi})$$

If I_C is expressed in terms of $I_C = P_m / V_{CE}$, where P_m is maximum dissipation, Eq. (xi) becomes

$$\frac{1}{\theta P_m} = \frac{\phi(1 + \Lambda r_{BB'} I_S e^{\phi \theta P_m})}{1 + \Lambda r_{BB'} P_m / \alpha_{CB} V_{CE}} \quad (\text{xii})$$

Rearranging

$$e^{\phi \theta P_m} = \frac{1}{I_S} \left[\frac{1}{\phi \theta \alpha_{CB} V_{CE}} + \frac{1}{\Lambda r_{BB'}} \left(\frac{1}{\phi \theta P_m} - 1 \right) \right] \quad (\text{xiii})$$

Taking the natural logarithm of Eq. (xiii)

$$P_m = \frac{1}{\phi \theta} \ln \frac{1}{I_S} \left[\frac{1}{\phi \theta \alpha_{CB} V_{CE}} + \frac{1}{\Lambda r_{BB'}} \left(\frac{1}{\phi \theta P_m} - 1 \right) \right] \quad (\text{xiv}) + \frac{1}{\Lambda r_{BB'}} \left[\frac{1}{2 \phi \theta P_m} \left(\frac{1}{\sqrt{1 - 4 P_m R_C / V_{CC}^2}} + 1 \right) - 1 \right] \quad (\text{xvii})$$

For Case (c) due to the presence of external resistance in series with the collector, R_C , there is a voltage drop $I_C R_C$ between the power supply, V_{CC} , and the collector. The heat conduction equation should be modified as

$$\delta T = \theta (V_{CC} - I_C R_C) I_C \quad (\text{xv})$$

Differentiate Eq. (xv) with respect to temperature,

$$\frac{dI_C}{dT} = \frac{1}{\theta (V_{CC} - 2I_C R_C)} \quad (\text{xvi})$$

Combine Eqs. (xvi) and (vi), and substitute I_C in terms of P_m and R_C

$$P_m = \frac{1}{\phi \theta} \ln \frac{1}{I_S} \left\{ \frac{1}{\phi \theta V_{CC} \alpha_{CB} \sqrt{1 - 4 P_m R_C / V_{CC}^2}} \right.$$

