LB-1032

DESIGN CONSIDERATIONS IN THE FIRST STAGE OF TRANSISTOR RECEIVERS

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Approved

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This bulletin includes a discussion of noise performance of transistor r-f stages utilizing capacitive antennas and of transistor mixer stages utilizing loop antennas. Examples of the noise performance to be expected with each type of antenna are included. Comparisons are drawn between transistor stages and corresponding tube stages. Consideration is given to design compromises between image rejection and insertion loss for an r-f stage employing both tuned input and interstage transformers. The procedure for transformer design for optimum insertion loss-image rejection performance is outlined.

Noise Performance

Capacitive Antenna

A typical transistor r-f stage is shown in Fig. 1a. A capacitive type antenna is assumed (as for example the rod antenna of an automobile receiver). Fig. 1b shows an equivalent circuit from the antenna to the r-f stage input where the antenna has been replaced by a voltage source, V_A, and an internal impedance consisting of a capacitance C_A. The transistor is represented by its input resistance R_i and the transformer tuned impedance by R_o. The signal power delivered to the r-f stage input by an unmodulated signal is given by

$$P_s = \left[n\, V_A \frac{C_A}{C_T}\, Q_{01} \frac{R_i}{n_2 R_o + R_i} \right]^2 \frac{1}{R_i}, \qquad (1)$$

where n is the transformer turns ratio, N_s/N_p, as indicated in Fig. 1 (essentially unity coupling is assumed), C_T is the sum of the antenna capacitance and the total shunt capacitance across the antenna transformer, and Q_{01} is the unloaded Q of the antenna transformer. This expression is derived in Appendix I.

The equivalent thermal noise generator referred to the secondary of the equivalent circuit of Fig. 1b is shown in Fig. 1c. The antenna radiation resistance is negligible compared to R_o and is ignored. The thermal noise power delivered to the transistor is

$$P_{TH} = 4KT\, n^2 R_o\, \Delta f \left[\frac{R_i}{n^2 R_o + R_i} \right]^2 \frac{1}{R_i} \qquad (2)$$

where K is Boltzman's constant, T is absolute tempera-

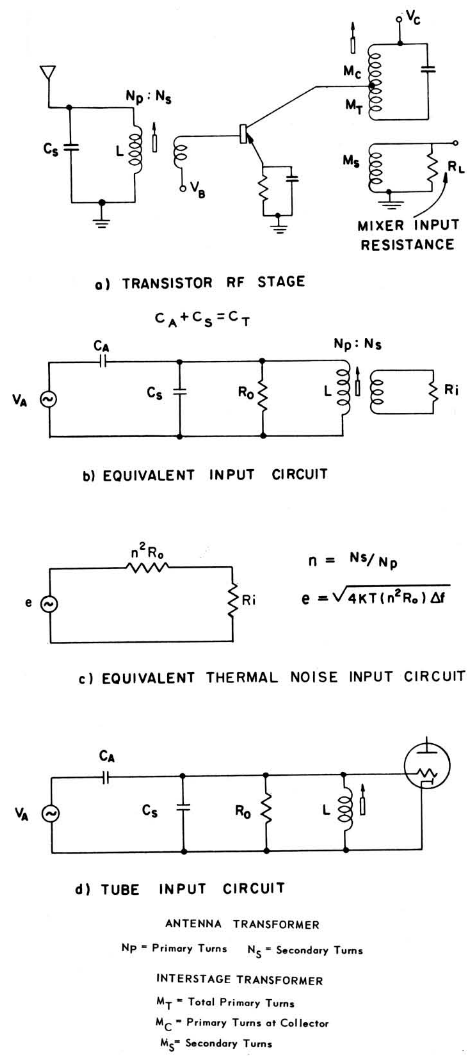


Fig. 1 – RF stage utilizing capacitive antenna.

ture, and Δf is the noise bandwidth. Since the r-f transistor noise factor, F, may be expressed as P_N/P_{TH}, where P_N is the total noise power referred to the transistor input,

$$P_N = F\, P_{TH} \qquad (3)$$

(assuming that noise from the second and succeeding stages is small and may be neglected). The signal-to-noise ratio is obtained from Eqs. (1), (2), and (3) and is given by

$$\frac{P_S}{P_N} = \frac{\left[V_A \frac{C_A}{C_T} Q_{01}\right]^2}{4KT\, R_o\, \Delta f\, F} \qquad (4)$$

For a receiver employing a linear detector the signal-to-noise ratio at the output of the audio amplifier, $(\frac{P_S}{P_N})$ A, is related to the r-f signal-to-noise ratio by

$$(\frac{P_S}{P_N})_A = m^2 \frac{P_S}{P_N} \frac{\Delta f}{\Delta f_A} \qquad (5)$$

where m is the percent modulation and Δf_A is the overall noise bandwidth of the receiver. (For the usual case where the receiver audio bandwidth is less than half of the bandwidth to the detector input, the overall noise bandwidth is twice the audio noise bandwidth[1]). For a 30 percent modulated signal and an output signal-to-noise ratio of 20 db, $(\frac{P_S}{P_N})_A = 100$, the antenna voltage required for this signal-to-noise ratio is obtained by combining Eqs. (4) and (5) and is given by

$$V_A = \frac{10\sqrt{4KT\, R_o\, \Delta f_A F}}{0.3 \frac{C_A}{C_T} Q_{01}} \qquad (6)$$

This expression is retained in this form to facilitate comparison with the tube case, which will be discussed below, (the mutual dependence of R_o, C_T, and Q_{01} should be noted, i.e., $Q_{01} = \omega\, C_T\, R_o$). Note that the above expression does not appear to be directly dependent on the turns ratio or operating Q of the antenna transformer. The transistor noise factor, however, is dependent on the turns ratio to the extent that the turns ratio determines the driving source impedance. This noise factor dependence is quite small in the vicinity of the matched source impedance. For example, a four-to-one mismatch in either direction causes an increase in F of about 2 db,[2] corresponding to an increase of about 25 percent in the signal required for 20 db signal-to-noise ratio.

A comparison of the noise performance of a tube and transistor r-f stage may be made by comparing Eq. (6) with a like expression for a tube stage. Fig. 1d shows a typical antenna connection for a tube r-f stage where the primary of the antenna transformer of Fig. 1b is connected directly to the grid. The tube noise factor, F_T, may be expressed as

$$F_T = \frac{R_o + R_{eq}}{R_o} \qquad (7)$$

where R_{eq} is the equivalent noise resistance of the tube. For typical remote-cutoff pentodes, the equivalent noise resistance is of the order of 3000 to 5000 ohms. For this type of connection at broadcast frequencies R_o is usually large compared to 5000 ohms and F_T is very nearly unity. The signal required for 20 db signal-to-noise ratio is then

$$V_A = \frac{10\sqrt{4KT\, R_o\, \Delta f_A}}{0.3 \frac{C_A}{C_T} Q_{01}} \qquad (8)$$

A comparison of Eqs. (6) and (8) shows that the tube r-f stage will be $\sqrt{F}$ times better than a transistor stage.

Example of Noise Performance Calculation

As an example, consider the r-f stage of a receiver for which the input circuit parameters are $C_A = 30\ \mu\mu f$, $C_T = 100\ \mu\mu f$, and $Q_{01} = 70$. The noise factor of the r-f transistor is 6 db and the overall noise bandwidth is 3 kc. Substituting the pertinent values in Eq. (6),

$$V_A = \frac{10\sqrt{1.6 \times 10^{-20} \times 112 \times 10^3 \times 3 \times 10^3 \times 4}}{0.3 \times \frac{30}{100} \times 70} = 7.37\ \mu v$$

for a 20 db signal-to-noise ratio at 1 mc.

Loop Antenna

A mixer or converter first stage is frequently used in receivers employing a loop antenna. A transistor mixer first stage is shown in Fig. 2a. The antenna circuit typically consists of a coil wound on a ferrite rod, L, a tuning capacitor, C; and a secondary winding for impedance transformation from antenna to transistor input.

1. *LB-775, Noise Factor and Its Measurement.*

2. Determined experimentally.

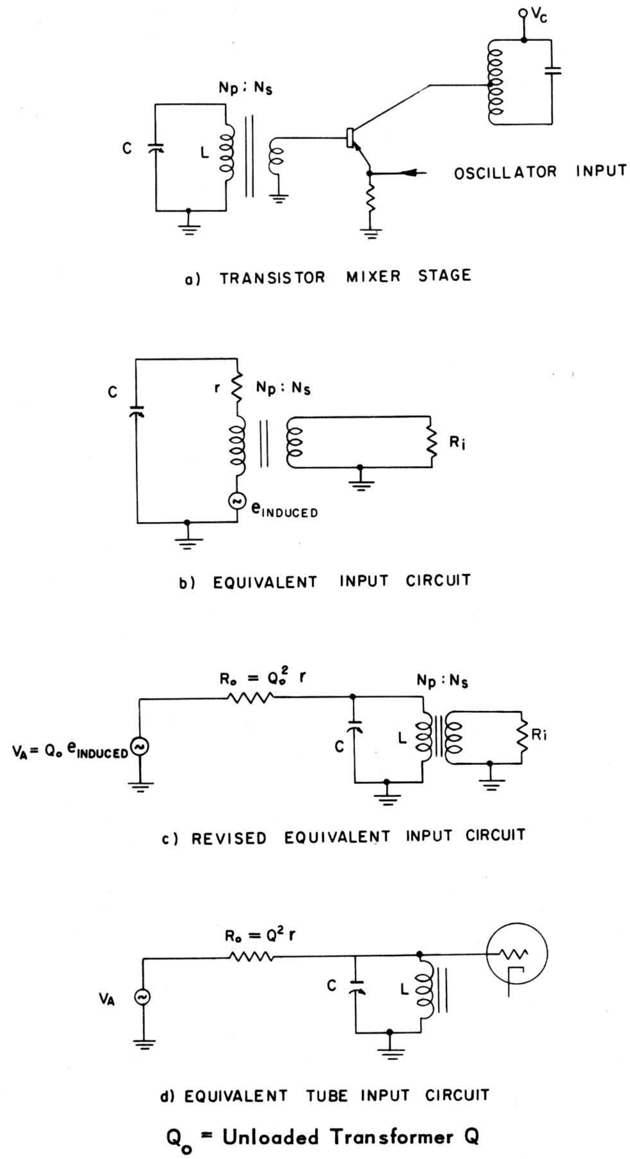


Fig. 2 – Mixer stage utilizing loop antenna.

An input equivalent circuit of Fig. 2a is shown in Fig. 2b where e is the voltage induced by the signal field, r is the series resistance of the tuned transformer, and R_i is the transistor input resistance. Another equivalent circuit is shown in Fig. 2c which is in the form of Fig. 1b. The voltage, V_A, corresponding to a 20 db signal-to-noise ratio is given by

$$V_A = \frac{10\sqrt{4\,KT\,R_o \Delta f_A\,F}}{0.3} \tag{9}$$

which is derived in Appendix II. In terms of the induced voltage, Eq. (9) may be written

$$e_{\text{induced}} = \frac{10\sqrt{4KT\,R_o\,\Delta f_A F}}{0.3\,Q_o} \tag{10}$$

The signal input is usually described in terms of field strength so that making use of Eq. (10):

$$\text{Field Strength} = \frac{e}{h} = \frac{10\sqrt{4KT\,R_o\,\Delta f_A\,F}}{0.3\,Q_o\,h} \tag{11}$$

for a 20 db signal-to-noise ratio, where h is the effective height of the antenna.

A comparison of the noise performance of a transistor mixer stage and a tube converter stage may be made by the use of Eq. (11). An equivalent input circuit for a tube stage is shown in Fig. 2d where the primary of the antenna transformer of Fig. 2c is connected directly to the signal grid. The value of the tube noise factor to be used in Eq. (11), depending on the input circuit parameters and the type of converter tube, may be determined from Eq. (7). For typical converter tubes, R_{eq} is of the order of 150,000 to 300,000 ohms and the corresponding F_T is of the order of 1.5 to 6 db.

Examples of Noise Performance Calculations for Loop Antennas

Noise performance calculations will be made for two battery-operated receivers employing ferrite cored loop antennas, one receiver having a transistor mixer first stage and the other a pentagrid converter tube first stage. The following information applies for both receivers. The antenna core consists of a ferrite rod 0.25 inches in diameter by 7 inches long. The variable tuning capacity is 8 to 172 $\mu\mu f$. The effective height of the antenna is about 0.004 meters at 1000 kc.[3] The noise bandwidth of the receivers is 3 kc.

The receiver employing the transistor mixer has an unloaded antenna circuit Q of 200. Substituting the pertinent values in Eq. (11):

$$\text{Field Strength} = \frac{10\sqrt{1.6 \times 10^{-20} \times 612 \times 10^{3} \times 3 \times 10^{3}\,F}}{0.3 \times 200 \times 4 \times 10^{-3}} = 226\sqrt{F} \quad \mu v/m$$

for 20 db signal-to-noise ratio at 1000 kc. For a transistor converter having a noise factor of 6 db, the field strength for a 20 db signal-to-noise ratio is 452 $\mu v/m$.

The second receiver employs a 1R5, a popular converter tube in battery portables, having an equivalent noise resistance of about 200,000 ohms. The antenna

3. This value of effective height applies for a particular core material and inductance. It should be noted that for a given core the effective height increases with increasing inductance, but for a constant Q the ratio $\sqrt{R_o}/Q_o h$ appearing in Eq. (11) is unchanged.

unloaded Q is usually limited to about 100 by other circuit considerations. (Signal feedback, loss of high frequency audio response, and loss of sensitivity due to tracking errors are all aggravated by increased Q.) Substituting the required terms in Eq. (11):

$$\text{Field Strength} = \frac{10\sqrt{1.6\times10^{-20}\times306\times10^{3}\times3\times10^{3}\,F}}{0.3\times100\times4\times10^{-3}} = 320\sqrt{F}\ \ \mu v/m$$

for a 20 db signal-to-noise ratio at 1000 kc. The converter noise factor, as obtained from Eq. (7) is 2.2 db at 1000 kc, resulting in a field strength of 412 $\mu v/m$ for a 20 db signal-to-noise ratio.

Insertion Loss and Image Rejection

Equivalent input and output circuits for the r-f stage of Fig. 1a are shown in Fig. 3. These circuits are convenient for the consideration of insertion loss and image rejection. In Fig. 3a, the voltage source of Fig. 1b is replaced by an equivalent current source, i, and the transistor input resistance is referred to the transformer primary and designated R_1. The transistor input power is

$$P_1 = i^2 R_o^2 \frac{R_1}{(R_o + R_1)^2} \qquad (12)$$

which maximizes at $i^2R_1/4$ when $R_1 = R_0$. A curve of input power relative to maximum transistor input power (hereafter referred to as insertion loss) for varying R_1/R_0 is shown in Fig. 4.

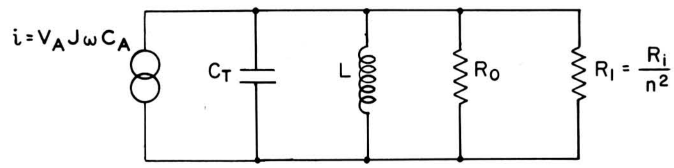
The operating Q of the antenna transformer, designated Q_1, is related to the unloaded Q, by

$$Q_1 = \frac{1}{1 + \frac{R_o}{R_1}} Q_{01} \qquad (13)$$

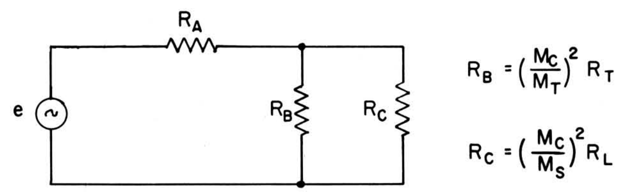
and the image rejection ratio, I, is very nearly

$$I = Q_1 \left(\frac{f}{f_o} - \frac{f_o}{f}\right) \qquad (14)$$

where f_o is the frequency of the desired signal and f is the image frequency. Eq. (12) is valid[4] for values of Q_1 greater than 10. Maximum image rejection is obtained when the transformer is unloaded, ($Q_1 = Q_{01}$). The loss of image rejection relative to this maximum is also plotted in Fig. 4.



(a) EQUIVALENT INPUT CIRCUIT



(b) EQUIVALENT INTERSTAGE CIRCUIT

Fig. 3 – Equivalent circuits of Fig. 1 rf stage.

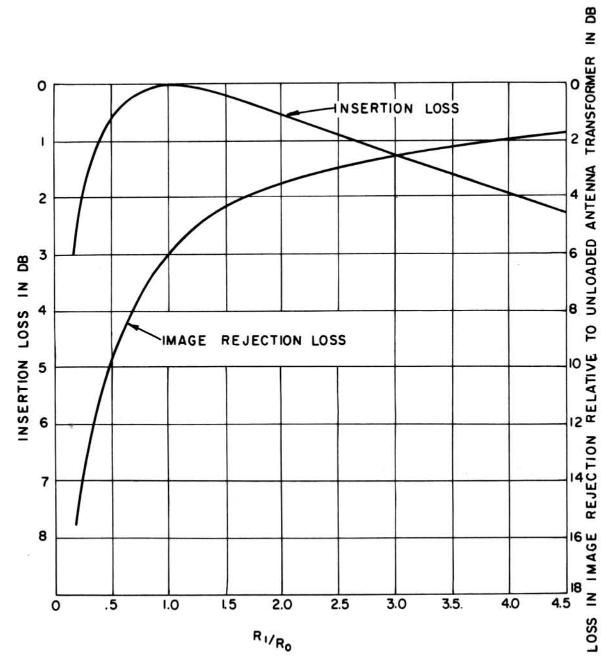


Fig. 4 – Variation of insertion loss and image rejection with the ratio R_i/R_o for the antenna circuit of Fig. 1.

4. F. E. Terman, RADIO ENGINEERS HANDBOOK, p. 144, McGraw Hill, New York, 1943.

A significant improvement in image rejection at a small sacrifice in insertion loss results from operation at values of R_1/R_O greater than unity. For instance, a change from $R_1/R_O = 1$ to $R_1/R_O = 2.5$ results in an improvement of 3 db in image rejection at the expense of less than 1 db in insertion loss.

An output equivalent circuit which consists of a generator coupled to a load by means of a tuned transformer is shown in Fig. 3b. The elements R_A, R_B and R_C represent the output resistance of the r-f stage, the interstage transformer tuned resistance, and the load resistance, respectively. Maximum operating Q (maximum image rejection) for a given insertion loss obtains for the condition that $R_C = R_A$. For this condition the power into the load resistance is

$$P_2 = e^2 \frac{\left(\frac{R_B}{R_A}\right)^2}{R_A\left(2\frac{R_B}{R_A} + 1\right)^2} \qquad (15)$$

and the operating Q of this transformer is

$$Q_2 = \frac{1}{1 + 2\frac{R_B}{R_A}} Q_{02} \qquad (16)$$

where Q_{02} is the unloaded Q.

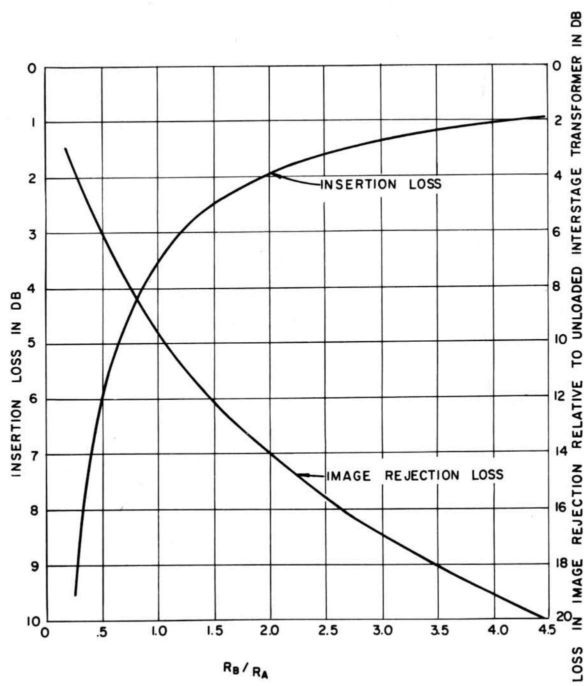


Fig. 5 – Variation of insertion loss and image rejection with the ratio R_B/R_A for the interstage circuit of Fig. 1.

The insertion loss, determined from Eq. (15), is shown in Fig. 5 as a function of R_B/R_A. The image rejection relative to an unloaded tuned transformer, also shown in Fig. 5, is obtained from Eqs. (16) and (14) for varying R_B/R_A.

Example of RF Stage Design

The design of an r-f stage which provides near optimum noise performance and 66 db image rejection, in a receiver employing a 455 kc i-f, will be considered. The available antenna and interstage transformers each have an unloaded Q of 70.

Near optimum noise performance (determined experimentally) is obtained in the range of R_1/R_o from 0.4 to 2.5; for this example let $R_1/R_o = 1.5$. From Fig. 4, the image rejection is 4.3 db below maximum. The maximum image rejection as calculated from Eq. (14) would be 39.7 db at 1000 kc for $Q_o = 70$. Therefore, the antenna transformer provides 35.4 db of image rejection and an insertion loss of 0.2 db over that obtaining for the maximum input power condition. The remainder of the required image rejection, 30.6 db, is to be obtained from the interstage transformer. This allows for a tolerable loss of image rejection of 9.1 db, relative to an unloaded transformer. As determined from Fig. 5, the ratio R_B/R_A is 0.92 and the consequent insertion loss is 3.8 db.

The r-f transistor has matched terminating resistances of 250 ohms input and 10,000 ohms output and 30 db matched gain; the mixer input resistance is 500 ohms. The capacitance required to tune the antenna transformer is 100 $\mu\mu f$. From the above information the turns ratios of both transformers may be determined. The tuned resistance of the antenna transformer at 1000 kc is

$$R_o = Q_o X_{C_T} = 70 \frac{1}{2\pi\, 10^6 \times 10^{-10}} = 112{,}000 \text{ ohms.}$$

For the turns ratio, n, of the antenna transformer

$$1.5 = \frac{R_1}{R_o} = \frac{R_i}{n^2 R_o} \quad or \quad n = \sqrt{\frac{R_i}{1.5\, R_o}}$$

Inserting the values of R_i and R_o in this expression,

$$n = \sqrt{\frac{250}{1.5 \times 112{,}000}} = \frac{1}{26} \quad ,$$

assuming unity coefficient of coupling.

The turns ratios of the interstage transformer may be calculated in a similar manner. The tuned resistance at 1000 kc is given by

$$R_B = Q_{o2}\, X_c = 70 \; \frac{1}{2\pi\, 10^6 \times 5 \times 10^{-10}} = 22{,}400 \quad ohms.$$

The turns ratio from the collector to the top of the tuned circuit, $\frac{M_C}{M_T}$, is given by

$$\frac{M_C}{M_T} = \sqrt{\frac{0.92 \times 10{,}000}{22{,}400}} = \frac{1}{1.56}$$

and the collector-to-secondary turns ratio, $\frac{M_C}{M_S}$, is given

$$\frac{M_C}{M_S} = \sqrt{\frac{10{,}000}{500}} = \frac{1}{0.224}$$

The r-f stage gain from the base of the r-f transistor to the base of the mixer will be (30 - 3.8) or 26.2 db. The circuit with normalized turns ratios is shown in Fig. 6.

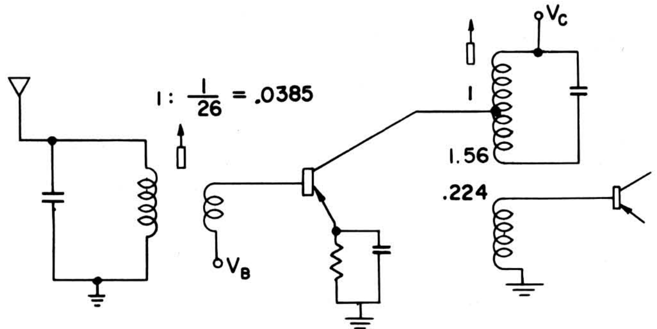


Fig. 6 – Example of rf stage input and interstage design.

In the above design procedure, consideration was limited to operation at 1000 kc. This procedure is satisfactory only if performance at the extremes of the band are not severely compromised. The relative change in image rejection obtained with two tuned circuits which are either unloaded or loaded by a constant amount (R_o/R_1 and R_B/R_A constant) is shown in Fig. 7a, Curve A. If the transistor input and output resistances are constant over the band, the ratios R_o/R_1 and R_B/R_A will not be constant. Both the insertion loss and image rejection will differ, at the extremes of the band, from the mid-band value. The insertion loss will increase and the change in image rejection decrease, with respect to the constant load conditions, as the frequency increases. The amount of change will depend on the design values of R_o/R_1 and R_B/R_A, and may be obtained from Fig. 4 and 5 with the aid of Eqs. (11) to (15).

The calculated variation in image rejection and insertion loss over the broadcast band for the circuit of Fig. 6 is shown in Fig. 7, Curve B, assuming constant transistor input and output resistances. Note in Fig. 7b that while about ±2 db change in sensitivity results, the ratio of R_o/R_1 varies from 0.54 to 1.6 of the mid-band value and is thus maintained within the previously defined limits for good noise performance. For a practical case with many present day transistors, the input and output resistances may decrease almost as rapidly as R_o and R_B. Curves A of Fig. 7 will then be a closer approximation to the actual performance.

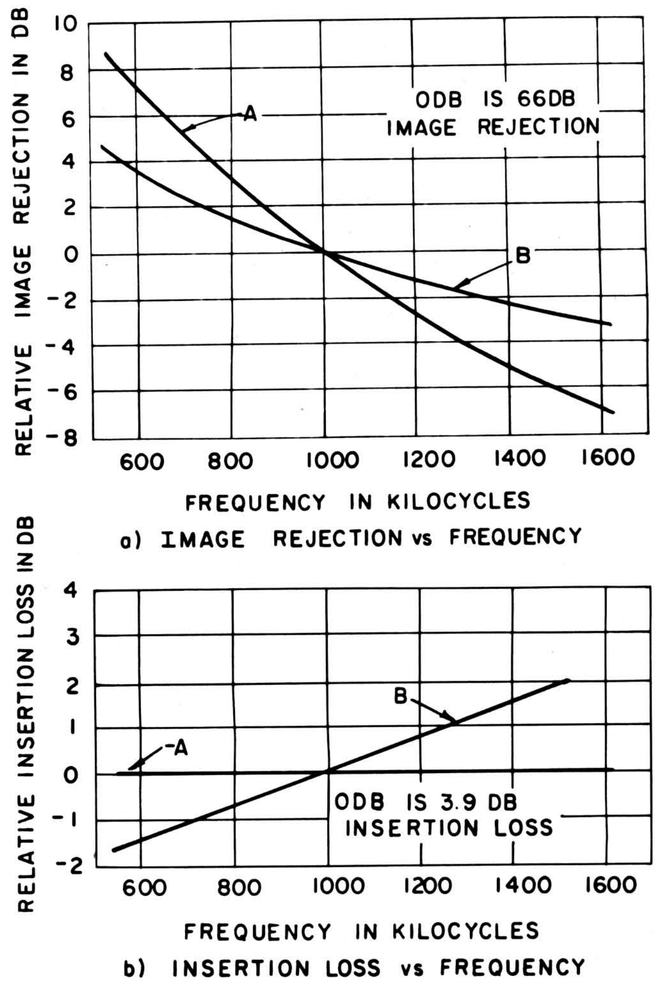


Fig. 7 – Image rejection and insertion loss of circuit of Fig. 6. A – constant loading, B – constant transistor input and output resistances.

Minimum Insertion Loss for Prescribed Image Rejection

While the above design procedure usually yields satisfactory results, it is interesting to consider another approach. The total insertion loss, ℓ, in the antenna and interstage transformers as obtained from Eqs. (11) and (14) is

$$\ell = \frac{\left(\frac{R_o}{R_1}+1\right)^2\left(2\frac{R_B}{R_A}+1\right)^2}{4\frac{R_o}{R_1}\left(2\frac{R_B}{R_A}\right)^2} \qquad (17)$$

and the total image rejection, obtained by combining Eqs. (13), (14), and (16) is

$$I = \frac{1}{\frac{R_o}{R_1}+1} \times \frac{1}{\frac{2R_B}{R_A}+1}\, Q_{01}\, Q_{02} \left(\frac{f}{f_o}-\frac{f_o}{f}\right)^2 . \qquad (18)$$

The values of R_o/R_1 and R_A/R_B which give minimum loss for a given image rejection may be obtained from Eqs. (17)′ and (18) and are derived in Appendix III. A plot of minimum insertion loss versus image rejection, as obtained from Eq. (17), for various values of $Q_{01}\ Q_{02}$ and an intermediate frequency of 455 kc is shown in Fig. 8. A change of 2-to-1 in the $Q_{01}\ Q_{02}$ product causes a 6 db change in image rejection for constant insertion loss. If, however, the image rejection is to be held constant, which is the usual case, the change in insertion loss for a change in $Q_{01}\ Q_{02}$ depends on both the value of image rejection and the intermediate frequency. A curve for an i-f of 260 kc is plotted for comparison. Note that the 260 kc curve indicates a loss of image rejection of 8.3 db (at 1000 kc) as compared to the 455 kc curve for the same $Q_{01}\ Q_{02}$. Corresponding curves of R_o/R_1 and R_B/R_A, from which the transformer turns ratios may be determined, are shown In Fig. 9.

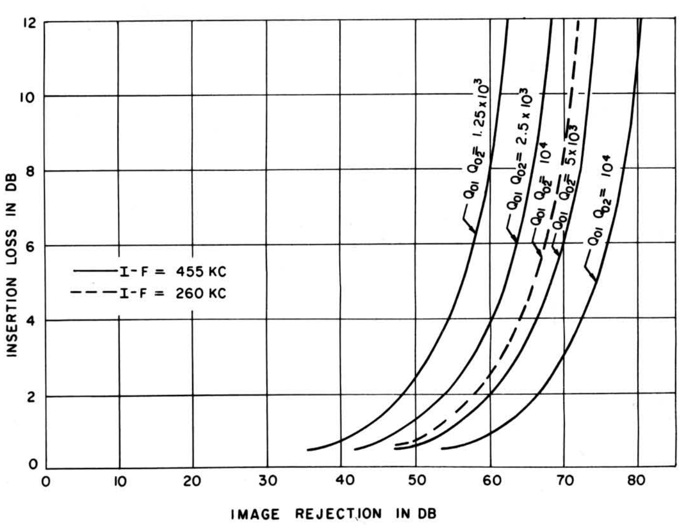


Fig. 8 – Minimum insertion loss vs image rejection at 1000 kc for rf stage of Fig. 1.

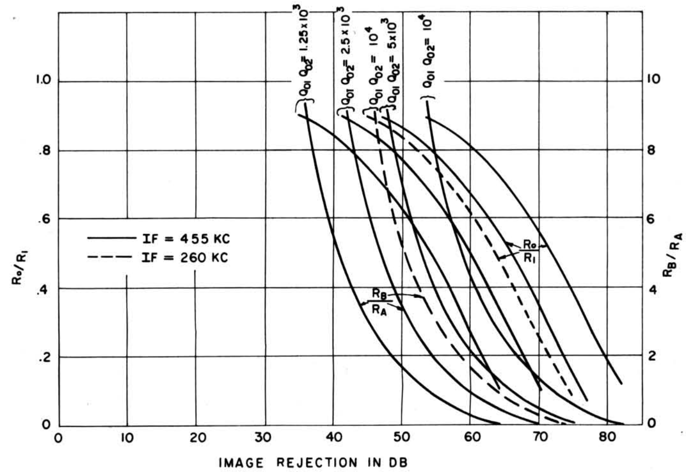


Fig. 9 – R_o/R_1 and R_B/R_A vs image rejection (these values provide minimum insertion loss at 1000 kc for the rf stage of Fig. 1).

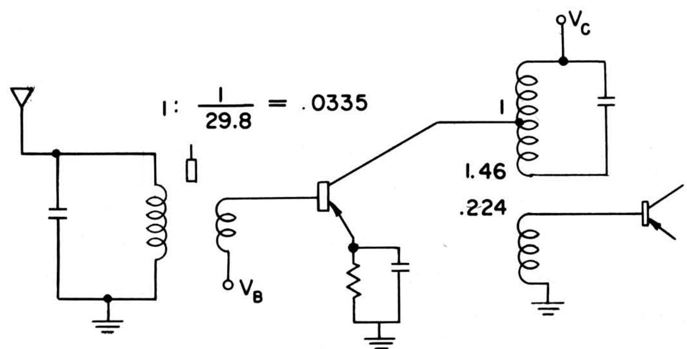
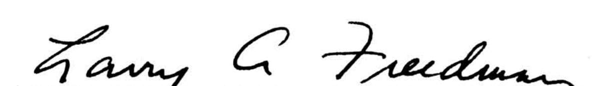


Fig. 10 – Example of rf stage input and interstage design for minimum insertion loss.

Referring to the sample design in the previous section for a 455 kc i-f and $Q_{01} = Q_{02} = 70$, Fig. 8 indicates an insertion loss of 3.7 db for a 66 db image rejection. The values of R_o/R_1 and R_B/R_A, from Fig. 9, are 0.505 and 1.05, respectively. Note that this value of R_o/R_1 is within the defined limits for good noise performance. The corresponding turns ratios, determined as in the above example, are shown on the circuit diagram of Fig. 10. For this design a negligible decrease in insertion loss (0.2 db) is obtained; other values of $Q_{01}Q_{02}$ and image rejection may provide a more striking difference. This method produces near optimum noise performance only by chance. Therefore, its chief value lies in evaluating the results of the design compromises of the previous section.



Larry A. Freedman

Appendix I

Derivation of Signal Power Delivered by a Capacitive Antenna

Referring to the equivalent circuit of Fig. 1b and using the notation of the text, the signal voltage at the primary of the transformer is given by

$$V_S = V_A \frac{C_A}{C_T} Q_1$$

where Q_1 is the loaded (operating) Q of the transformer. The signal voltage at the secondary winding is than

$V_{S2} = n V_A \frac{C_A}{C_T} Q_1$ and the signal power delivered to the transistor is

$$P_S = \frac{(V_{S2})^2}{R_i} = \left(n V_A \frac{C_A}{C_T} Q_1\right)^2 \frac{1}{R_i} .$$

Since the operating Q is related to the unloaded Q

$$Q_1 = \frac{R_o R_i/n^2}{R_o + R_i/n^2} \times \frac{1}{\omega L} = Q_{01} \frac{R_i}{n^2 R_o + R_i} ,$$

the signal power may be expressed as

$$P_S = \left(n V_A \frac{C_A}{C_T} Q_{01} \frac{R_i}{n^2 R_o + R_i}\right)^2 \frac{1}{R_i} . \qquad (1)$$

Appendix II

Derivation of Signal Required for 20 db Signal to Noise Ratio for a Loop Antenna

Referring to the equivalent circuit of Fig. 2c and using the notation of the text, the signal voltage at the primary of the transformer is

$$V_S = V_A \frac{R_i/n^2}{R_o + R_i/n^2} = V_A \frac{Q}{Q_o} .$$

The signal voltage at the secondary winding is then $V_{S2} = n V_A \frac{Q}{Q_o}$ and the signal power delivered to the transistor is

$$P_S = \frac{(V_{S2})^2}{R_i} = (n V_A Q/Q_o)^2 \times \frac{1}{R_i} .$$

The noise power referred to the transistor input is

$$P_N = 4KT\, n^2 R_o \Delta f \left[\frac{R_i}{n^2 R_o + R_i}\right]^2 \frac{1}{R_i} F,$$

where F is the transistor noise factor.

For an output signal to noise ratio of 20 db and 30 percent modulation

$$\left(\frac{P_S}{P_N}\right)_A = 100 = \frac{(0.3n V_A Q/Q_o)^2 \frac{1}{R_i}}{4KT\, n^2 R_o \Delta f_A \left[\frac{R_i}{n^2 R_o + R_i}\right]^2 \frac{1}{R_i} F}$$

or solving for V_A,

$$V_A = \frac{10\sqrt{4KT R_o \Delta f_A F}}{0.3} \qquad (9)$$

Appendix III

Derivation of Minimum Insertion Loss for a Specified Image Rejection

Eq. (18) of the text is

$$I = \frac{1}{\frac{R_o}{R_1} + 1} \times \frac{1}{\frac{2R_B}{R_A} + 1} Q_{01}\, Q_{02} \left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2 \qquad (18)$$

which may be rewritten as

$$\frac{R_o}{R_1} + 1 = \frac{\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2}{I}\, Q_{01}\, Q_{02}\, \frac{1}{\frac{R_B}{2R_A} + 1}$$

Introducing a constant

$$C = \frac{\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2}{I}\, Q_{01}\, Q_{02}$$

which depends on the required image rejection, the intermediate frequency, and the unloaded Q's, Eq. (18) becomes:

$$\frac{R_o}{R_1} + 1 = \frac{C}{2\frac{R_B}{R_A} + 1}$$

Substituting in Eq. (17):

$$\ell = \frac{C^2}{4}\, \frac{1}{(R_o/R_1)}\, \frac{\left[\left(\frac{R_o}{R_1}\right) + 1\right]^2}{\left[C - \left(\frac{R_o}{R_1}\right) - 1\right]^2} \qquad (17a)$$

Differentiating (17a) with respect to R_o/R_1

$$\frac{\partial \ell}{\partial\left(\frac{R_o}{R_1}\right)} = \frac{C^2}{4}\left[\frac{\left\{\left(\frac{R_o}{R_1}\right)^3 + 2\left(\frac{R_o}{R_1}\right)^2(1-C) + \left(\frac{R_o}{R_1}\right)(1-C)^2\right\}\left\{2\left(\frac{R_o}{R_1} + 1\right)\right\} - \left(\frac{R_o}{R_1} + 1\right)^2\left\{3\left(\frac{R_o}{R_1}\right)^2 + 4\left(\frac{R_o}{R_1}\right)(1-C) + (1-C)^2\right\}}{\left\{\left(\frac{R_o}{R_1}\right)^3 + 2\left(\frac{R_o}{R_1}\right)^2(1-C) + \frac{R_o}{R_1}(1-C)^2\right\}^2}\right]$$

Equating this expression to 0 to determine the value of R_0/R_1 for minimum ℓ

$$\frac{R_0}{R_1} = \frac{-(C+2)+\sqrt{C^2+8C}}{2}$$

and the corresponding value of R_B/R_A is

$$\frac{R_B}{R_A} = \frac{C}{-(C+2)+\sqrt{C^2+8C}} - \frac{1}{2}$$

The minimum loss for a given C is then given by (17a),

$$\ell = \frac{C^2}{2} \frac{\left[-C+\sqrt{C^2+8C}\right]^2}{\left[-(C+2)+\sqrt{C^2+8C}\right]\left[3C-\sqrt{C^2+8C}\right]^2} .$$