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ENGINEERING METHODS IN THE DESIGN OF THE CATHODE RAY TUBE

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HILARY MOSS

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List of Principal Symbols Used

k - constant of scale (linear size or voltage)

K_1 and K_3 - initial velocities in axial and radial directions respectively

K_2 and K_4 - coordinates of point of emission

V - general symbol for potential on bounding electrode

φ - general symbol for potential at any point in space

$\bar{\varphi}$ - height of negative potential hump necessary to just suppress emission

V_c - negative grid bias necessary to just cut-off beam when anode voltage is V_a

V_c^* - negative grid bias necessary to just cut-off beam when anode voltage is zero

V_1 - potential at crossover point

V_2 - potential at image point

A, B, - 'constants of potential', functions only of the space coordinates

λ - scanning angle, i.e. angle between deflected ray and axis

θ - half angle of beam subtended by final anode hole at screen

n_1 - refractive index of crossover point

n_2 - refractive index of image point

z - distance measured along beam axis

r - distance measured from beam axis

t - general symbol for time

y_1 - size of crossover

y_2 - size of image

v - distance from plane of equivalent focussing lens to image

u - distance from plane of equivalent focussing lens to object, i.e. crossover

M - geometrical magnification, i.e. v/u

ρ, ρ' - space charge densities

x,y,z - space coordinates

X,Y,Z - space coordinates in transformed system

Definitions

Deflectional discrimination - the ratio $\frac{\text{Sensitivity of deflection}}{\text{Spot diameter}}$

Electron gun - That portion of the electrodes excluding the deflector system

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SUMMARY

This paper discusses the application of the general theories of scale, dimensional homogeneity and energy conservation to cathode ray tube designing. From these simple bases it is shown that many important deductions can be made about the general form which the tube geometry should assume. There is no appeal to advanced electron-optics, and the approach should therefore commend itself to the engineer.

GENERAL

In the design of almost any scientific instrument, there are two distinct lines of approach. The first consists of evolving a complete theory of the mode of operation of the device. This is the comprehensive way, which will yield the maximum of information, and will enable all aspects of designing to be done with rigour and exactitude. But the task of evolving such a theory is often difficult and, moreover, much of the information it will yield belongs more properly to the field of development and research than to routine design work. If one is content to deal with the latter only, much can be done by far simpler methods. These methods are not peculiar to any particular branch of scientific designing, but are based on quite general theories of scale, energy and dimensional homogeneity. One of their interesting features is that they require only a most rudimentary knowledge of the theory of the particular device to which they are being applied.

A very striking instance of this, familiar to most physics students, is the derivation of Poiseuille's equation relating to the flow of viscous liquids through pipes. Without any knowledge whatever of hydro-dynamics, and by purely dimensional methods, it is readily shown that the volume of liquid discharged/unit time through the pipe is given by $V = K.p.r^{4+\beta}/l^{1+\beta}.\eta$ where p is the pressure difference, r is the pipe radius, l its length and η the coefficient of viscosity. An intelligent guess or simple experiment gives $\beta = 0$, whereas the derivation of this law from hydro-dynamical principles is quite difficult. On the other hand, such an analysis yields the value of the constant K as $\pi/8$, about which the dimensional method gives no information at all. But

1.1. PRINCIPLE OF VOLTAGE SIMILITUDE

When space charge is absent, the potential at any point in space must satisfy Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \ldots\ldots\ldots\ldots (1)$$

or briefly

$$\nabla^2(V) = 0$$

Suppose that $V = f(x,y,z)$ is a solution of (1). Then it is clear that $V = k.f(x,y,z)$ is equally a solution, since

$$\frac{\partial^2 (kV)}{\partial x^2} = k.\frac{\partial^2 V}{\partial x^2} \quad \text{etc.}$$

Thus

$$\nabla^2(kV) = k.\nabla^2(V) = 0$$

since

$$\nabla^2(V) = 0 \text{ by hypothesis.}$$

Note that this does not apply when space charge is present, since in this case Poisson's equation holds and

$$\nabla^2(V) = 4.\pi.\rho$$

If this equation is satisfied by $V = \psi(x,y,z)$ it is not in general satisfied by $\phi = k.V$.

the value of the constant could be easily determined by a single experiment following the dimensional analysis, so the power of the latter method is evident.

A very similar position occurs in cathode ray tube design. The methods which we shall now treat do not alone give any information as to how a design should proceed, but taken in conjunction with experimental investigation and prototype tubes, they do provide an easy basis for further designing. This approach is a useful complement to, but not a substitute for, a complete theory of the tube.

PART 1.

FOUR BASIC RULES IN CATHODE RAY TUBE DESIGN

The following four rules form the basis of the 'relaxation' methods of cathode ray tube design. The first three 'rules' are really laws, and are quite rigorous within their limiting postulates. The fourth is merely a rule, which has some theoretical justification, but for which the main support is experimental.

1.1. Principle of Voltage Similitude

In any electron optical system, in which space charge is negligible, and in which the electrons start from rest, the electron trajectory is unaltered by multiplication of all electrode potentials by a constant factor (k). The transit time between any two fixed points in the system varies as $1/\sqrt{k}$.

1.2. Principle of Geometrical Similitude

In any electron optical system in which the total current flow is constant, the shape of the field and of the electron trajectory is unaltered by

1.1. PRINCIPLE OF VOLTAGE SIMILITUDE (CONT.)

When space charge is negligible, the two fundamental equations defining the motion of an electron in an axially symmetrical field are

$$\frac{d^2z}{dt^2} = \frac{e}{m}\cdot\frac{\partial V}{\partial z}, \quad \frac{d^2r}{dt^2} = \frac{e}{m}\cdot\frac{\partial V}{\partial r}$$

If <u>all</u> electrode potentials are multiplied by k, these equations become

$$\frac{d^2z}{dt^2} = \frac{e.k.}{m}\frac{\partial V}{\partial z}, \quad \frac{d^2r}{dt^2} = \frac{e.k.}{m}\frac{\partial V}{\partial r} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Integrating each equation twice, w.r.t., to obtain the displacements gives

$$z = \frac{e}{m}\iint k.\frac{\partial V}{\partial z}\,dt\,dt + K_1.t + K_2 \quad \ldots\ldots\ldots\ldots(2)$$

$$r = \frac{e}{m}\iint k.\frac{\partial V}{\partial r}\,dt\,dt + K_3.t + K_4 \quad \ldots\ldots\ldots\ldots(3)$$

K_1 and K_3 are the initial velocities in axial and radial directions respectively. Provided these are zero, $K_1 = K_3 = 0$. K_2 and K_4 are merely the coordinates of the starting point. They may be eliminated by shift of origin effected by writing $Z = z - K_2$, $R = r - K_4$. Making this substitution, and dividing (2) by (3) then gives

$$\frac{Z}{R} = \frac{f(t)}{\phi(t)} \quad \text{which is independent of k.}$$

Hence the shape of the trajectory and its scale are independent of k.

multiplication of the size of all the bounding electrodes by a constant factor (k). The transit time between corresponding points in the two systems is proportional to k.

1.3. Spot Size/Crossover Size Relationship

If the crossover and spot are formed in regions of the same potential, then

Spot size = crossover size × geometrical magnification (M)

More generally, if V_1 is the crossover potential, and V_2 the spot potential, then

$$\text{Spot size} = \text{crossover size} \times M \times \sqrt{V_1}/\sqrt{V_2}$$

1.4. Dependence of Crossover Size on Voltage on Crossover forming Electrode

To a close approximation, in any system where the space charge is negligible, the crossover diameter is inversely proportional to the square root of the potential on the crossover-forming electrode.

The proofs of these four principles are discussed below.

1.1 Proof of Voltage Similitude Principle

The proof of this principle is in two parts. Firstly, we prove on sheet 1 that the shape of the field bounded by any electrode system is independent of the absolute magnitude of the potentials on the bounding electrodes, and depends only on their ratios. Note that this is true only when space charge is negligible. Since the size of the electrode system is postulated as constant, it immediately follows that the electric field strength, at any point, is proportional to the voltages on the electrodes.

Next, on sheet 2 by integrating the equations of motion of an electron in an axially symmetrical field, it is shown that the shape of the trajectory is independent of the potentials on

1.2. PRINCIPLE OF GEOMETRICAL SIMILITUDE

In general the potential at any point in space must satisfy Poisson's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 4.\pi.\rho \quad \ldots\ldots\ldots\ldots (1)$$

Now suppose that the dimensions of all the bounding electrodes are multiplied by a factor k. Then any point (x,y,z) becomes transformed to a corresponding point (X,Y,Z) where $X = k.x$, $Y = k.y$, $Z = k.z$. Transforming (1) then gives

$$k^2\left\{\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2}\right\} = 4.\pi.\rho \quad \ldots\ldots\ldots\ldots (2)$$

so that provided the new density $\rho' = \rho/k^2$ it follows that (1) and (2) are identical. Now suppose that the electronic device being considered comprises some ray or beam, and that we twist the reference axes so that the beam axis lies along the z dimension. Then the condition $\rho' = \rho/k^2$ merely means that the density of the beam at right angles to its length has been reduced to $1/k^2$ of its original value. But the cross sectional area of the beam has increased by k^2 times. Hence the condition $\rho' = \rho/k^2$ implies that the total current in the ray has been kept constant. This proves the invariance of the field shape with change of electrode scale.

Next consider the fundamental equations of motion of an electron in an axially symmetrical field. Taking only the z-directed term gives

$$\frac{d^2 z}{dt^2} = \frac{e}{m}.\frac{\partial V}{\partial z} \quad \ldots\ldots\ldots\ldots (3)$$

using the transformation $Z = k.z$ as above, converts (3) into

$$\frac{1}{k}.\frac{d^2 Z}{dt^2} = \frac{e}{m}.k.\frac{\partial V}{\partial Z}$$

which may be re-expressed

$$\frac{d^2 Z}{dT^2} = \frac{e}{m}.\frac{\partial V}{\partial Z} \quad \ldots\ldots\ldots\ldots (4)$$

where $T = kt$. (3) and (4) are identical in form so that the trajectories are geometrically similar, but with a transit time k times as large in the transformed system.

the electrodes. Note particularly that the theorem is true generally only when the electrons start from rest, since only then are the constants K_1 and K_3 equal to zero. A special case corollary, of importance in connection with deflector-plate theory, occurs when both K_1 and K_3 are proportional to k, for the theorem still holds in that case. (Physically speaking, this is the case where the 'injection' volt velocity of an electron is raised in proportion to the rise of potential on the subsequent electrodes).

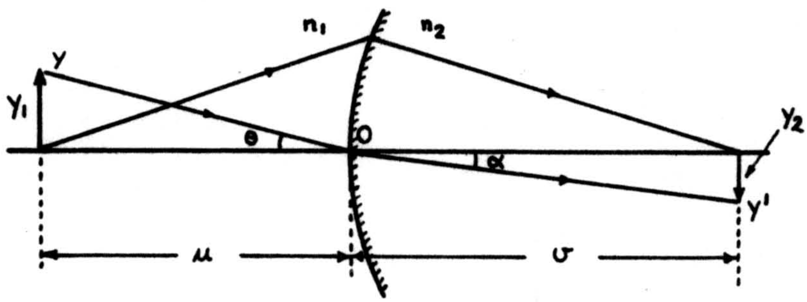
Finally, from sheet 2, we note that the transit time of an electron between any two fixed points is inversely proportional to $\sqrt{k}$, where k is the 'scale' factor. Thus, for instance, a multiplication of all electrode potentials by four results in a reduction of the transit time to one half its previous value. This result is of interest in connection with design problems at ultra high frequencies.

1.2. Proof of the Principle of Geometrical Similitude

Again the proof of this principle is in two parts. Firstly, we establish on sheet 3 the invariance of the geometrical form of the field shapes with change in the scale of the bounding electrodes. This principle holds even when the space charge is present, and is thus of extreme generality. The proof is based simply on the invariance of Poisson's equation with change of scale, provided $\rho' = \rho/k^2$, where, as usual, k is the scale factor. The condition $\rho' = \rho/k^2$ is merely an assertion that the current in the rays is constant.

Secondly, on sheet 3, a transformation of the equations of motion into the enlarged coordinate system shows the identity of the transformed system, provided the new time scale is

1.3. LAGRANGE'S LAW



Applying Snell's law to the ray YOY' gives

$$\frac{\sin\theta}{\sin\alpha} = \frac{n_2}{n_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again, since θ , α are small we may write

$$\sin\theta = y_1/u$$
$$\sin\alpha = y_2/v \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting from (2) into (1) gives

$$y_2 = y_1 . \frac{v}{u} . \frac{n_1}{n_2} = y_1 . M. \sqrt{\frac{V_1}{V_2}}$$

multiplied by k. Hence the transit time between corresponding points on the two systems is proportional to the scale constant k.

1.3. Proof of the Lagrange Law

This law is also sometimes attributed to Abbe. A proof is given on sheet 4. The only important point to note is that it holds only for paraxial rays. In point of fact this restriction is of far less consequence in electron-optics than in the light optical case for which the law was originally derived, since electron beams are in general far thinner, and make smaller angles to the axis.

1.4. Dependence of Crossover Size on Voltage

This relationship is in no sense a law, but is a rule which appears to have useful accuracy over a wide range of voltages, and which appears to be largely independent of the form of the electrode system used to produce the crossover.

On sheet 6 is given a justification for this form of relationship. Another justification, based on quite different reasoning, has been given by Langmuir[1] in his fundamental paper, and has been discussed by the present author[2].

The primary evidence for the truth of this relationship is, however, experimental. Careful measurements have been made in these laboratories, over the range 900 to 4,000 volts, which show that the spot diameter varies as $1/\sqrt{V}$ to a close approximation. The spot diameter is here defined as that diameter corresponding to a current density of 1/5 that on the beam centre. The current distribution in the spot was measured by the method of slit scanning, first described by Jacob[3].

1.4. DEPENDENCE OF CROSSOVER SIZE ON VOLTAGE

When space charge is negligible, the two equations of parametric form defining the electron displacement have been shown to be

$$z = \frac{e}{m}\iint \frac{\partial V}{\partial z}\, dt\, dt + K_1 . t + K_2$$

and .... (1)

$$r = \frac{e}{m}\iint \frac{\partial V}{\partial r}\, dt\, dt + K_3 . t + K_4$$

Here, K_1 and K_2 are the initial emission velocities along, and at right angles to, the beam axis respectively. K_2 and K_4 are zero if the origin of coordinates is taken to coincide with the point of emission. For the very high voltages used in cathode ray tubes, $\iint \frac{\partial V}{\partial z}\, dt\, dt \gg K_1 . t$. Again, for an electron starting on the beam (z) axis, $\partial V/\partial r = 0$ initially, and since its distance (r) from the beam axis is always small, it is perhaps justifiable to write $\iint \frac{\partial V}{\partial r}.\, dt\, dt \ll K_3 . t$. Hence, equations (1) degenerate into

$$z = \frac{e}{m}\iint \frac{\partial V}{\partial z}\, dt\, dt \quad \text{.......................... (2)}$$

$$r = K_3 . t \quad \text{.................................... (3)}$$

If we now assume that $\partial V/\partial z$ is constant over the very short axial distance involved, we may put $\partial V/\partial z = k.V$, where V is the voltage on the crossover-forming electrode. Integrating (2) then yields

$$z = \frac{e}{m} . k . V . t^2/2$$

whence for constant z (fixed cathode-crossover distance), $t = K/\sqrt{V}$ and thus substituting in (3), $r = K/\sqrt{V}$ at the crossover.

1.5. Spot Size and Deflection Defocussing

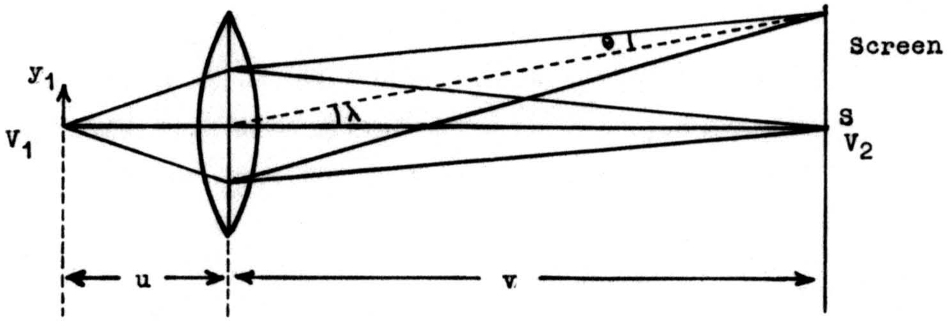
Before passing on to discuss the application of these principles to specific problems, we shall treat briefly some aspects of spot size and deflection defocussing.

Firstly, as regards spot size only, it is necessary to point out that any apparent inconsistency between principles 1.1 and 1.4 is resolved by the fact that the electrons from the cathode do not start from rest. If the initial emission velocity were zero, then principle 1.1 relating to voltage similitude would hold; the crossover and spot sizes would be quite independent of anode potential, and this would be at variance with 1.4. In fact, however, the electrons have a Maxwellian velocity spread on emission, which means that both crossover (and therefore spot) sizes are finite, and are dependent on the accelerating potential in the way indicated in the analysis on sheet 5 and elsewhere[1,2].

When the spot is deflected, it suffers distortion, the form of which varies widely. In all cases, however, the spot increases in area. This increase in area is due to the distortions of the deflecting field, i.e. to the geometry of the deflecting region. Hence, to this part of the deflected spot size, the principles of geometrical and voltage similitude hold. Thus, for instance, the increase in spot size on deflection (through constant angle) is not affected by anode potential. Hence, a tube with severe deflection defocussing is not improved by operation at increased anode voltages. Only the central focus is improved.

Sheet 6 expresses these facts in symbolic form.

1.5. SPOT SIZE AND DEFLECTION DEFOCUSSING RELATIONS



Effective Magnification between crossover and spot $= M.\sqrt{\frac{V_1}{V_2}}$

where M is the 'geometrical magnification' and equals v/u. Thus if y_1 is the crossover size, the size of the undeflected spot is

$$S_{undeflected} = y_1 \cdot M. \sqrt{\frac{V_1}{V_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But we have shown that $y_1 = K/\sqrt{V_1}$. Thus substituting in (1)

$$S_{undeflected} = k_1/\sqrt{V_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By experiment we find that when the spot is deflected, it increases in size, and also that this increase in 'size' is a function of the deflecting geometry (G) and the angle of deflection (λ). Thus we may write for the deflected spot size

$$S_{deflected} = k_1/\sqrt{V_2} + k.f(G,\lambda) \quad \ldots\ldots\ldots (3)$$

where k is a constant of scale. Note that $f(G,\lambda) = 0$ when $\lambda = 0$. Note that only the first term of (3) is a function of anode voltage. Hence if $k.f(G,\lambda) > k_1/\sqrt{V_2}$ (severe deflection defocussing) little improvement is made by raising V_2.

PART 2.

APPLICATION TO SPECIFIC PROBLEMS

2.1. Elementary Applications of Principle of Voltage Similitude 1.1.

The following two very well known properties of the cathode ray tube are direct results of the principle of voltage similitude.

(1) If the final anode potential is multiplied by k, then the deflector plate voltages for equal spot displacements are also multiplied by k.

(2) If, in any electrostatically focussed cathode ray tube, the potentials of all the accelerating electrodes except the focussing anode are multiplied by k, then the focussing anode potential must also be multiplied by k to maintain focus. In practice some slight deviation from this may be detected, and this is due either to space charge effects or possibly to shift of crossover position with variation of grid bias.

2.2. Applications of Principle of Voltage Similitude 1.1. - Prediction of Relative Triode Performance - Conditions at Cut-off

Some prediction of, and justification for, the behaviour of the triode portion of the electron gun is given by application of voltage similitude. Caution is necessary, however, since the two basic postulates, - namely, absence of space charge, and zero starting velocity for the electrons - , are not wholly satisfied.

Consider, for example, the question of variation of cut-off voltage with variation of first anode potential. We know that the electrons are emitted with a Maxwellian velocity distribution, so that on an average they have some initial velocity.

Hence it is reasonable to suppose that the emission will be suppressed by the creation of a small <u>negative</u> potential barrier in front of the cathode. Suppose that a potential $-V_c$ on the grid cuts off the triode when the anode potential is V_a, this cut-off being due to a small negative potential hump of height $-\bar{\Phi}$. What can be predicted about the value of the grid bias necessary to cut-off the tube when the first anode potential is $k.V_a$? By voltage similitude a grid voltage of $-k.V_c$ will now create a negative barrier of height $-k.\bar{\Phi}$. Now k is inherently positive, and if, furthermore, $k > 1$, then it follows that $-k.\bar{\Phi} < -\bar{\Phi}$, so that the triode <u>must</u> be cut-off under the new conditions. Thus it is <u>certain</u> that if a tube is just cut-off with first anode voltage V_a and grid voltage $-V_c$, then it will be cut-off for all <u>higher</u> anode voltages, for which the modulus of the grid voltage is raised in proportion.

We can, however, carry our predictions considerably further, as indicated by the following analysis. It is fundamental in potential theory that however complex a field may be, the potential at any fixed point in it is linearly related to each of the potentials existing on the bounding electrodes. Thus it follows that the potential at some fixed point on the beam axis in front of the cathode surface can be expressed as

$$\Phi = A.V_g + B.V_a \quad \text{.....................(1)}$$

where A and B are constants depending only on the electrode geometry and the position of the point at which the potential is Φ. Now suppose that the critical potential $\bar{\Phi}$ necessary to just cut-off the triode is created by a grid potential of $-\bar{V}_c$ and an anode potential of $\bar{V}_a$. Then from (1)

$$\bar{\Phi} = A.\bar{V}_c + B.\bar{V}_a \quad \text{.....................(2)}$$

We next multiply the anode potential by k, and we wish to determine the new grid potential which will maintain the same critical retarding potential $\bar{\Phi}$, (and hence presumably will just cut-off the triode). Clearly, if V_c is this grid potential,

$$\bar{\Phi} = A.V_c + B.k.\bar{V}_a \quad \ldots\ldots\ldots\ldots(3)$$

and by equating (2) and (3) we readily obtain

$$V_c = \bar{V}_c - \frac{B}{A}.\bar{V}_a(k - 1)$$

whence, dividing both sides by $\bar{V}_c$

$$\frac{V_c}{\bar{V}_c} = 1 - \frac{B.\bar{V}_a}{A.\bar{V}_c}(k - 1) \quad \ldots\ldots\ldots\ldots(4)$$

It will be noted that this last equation (4) incorporates the principle of voltage similitude. For if the required critical retarding potential $\bar{\Phi}$ to just cut-off the triode were zero, then from equ. (2) it follows that $A.\bar{V}_c = -B.\bar{V}_a$, whence substituting this latter relation in (4) yields $V_c/\bar{V}_c = k$. This of course is merely a direct application of voltage similitude, and could be predicted immediately without the analysis given.

The value of the full analysis, incorporating both the principle of voltage similitude and the linearity concept of potential, as exemplified by (4), is that it permits a deduction to be made about the extent of the departures of V_c and V_a from proportionality, in terms of the grid voltage necessary to cut-off the triode when $V_a = 0$. For, dividing (2) throughout by $A.\bar{V}_c$ gives

$$\frac{B.\bar{V}_a}{A.\bar{V}_c} = \frac{\bar{\Phi}}{A.\bar{V}_c} - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

and using this relation (5) in (4) yields

$$\frac{V_c}{\bar{V}_c} = 1 - \left\{\frac{\bar{\Phi}}{A.\bar{V}_c} - 1\right\}(k - 1) \quad \ldots\ldots\ldots\ldots(6)$$

Now consider the position when the first anode voltage V_a is zero. Let V_c^* be the negative grid voltage then required to suppress emission. Thus, from (2), putting $V_a = 0$ gives

$$\bar{\Phi} = A.V_c^* \quad \text{.......................................(7)}$$

since in all cases regardless of the actual potentials on the grid and anode, the only criterion of cut-off is the creation of the same negative potential barrier of height $\bar{\Phi}$. Substituting from (7) into (6) yields

$$\frac{V_c}{\bar{V}_c} = 1 - \left\{\frac{V_c^*}{\bar{V}_c} - 1\right\}(k - 1) \quad \text{...............(8)}$$

Equ. (8) is a remarkable deduction from such simple postulates. It shows that the V_c/V_a relation is uniquely defined, once the value of grid bias for cut-off without applied anode voltage is known. The only assumption made in its deduction is neglect of space charge. In view of the cut-off conditions this would seem quite reasonable.

Fig. 1 shows a sketch of the form of the V_c/V_a curve, deduced from equation (8). Direct experiment[†] has shown that this type of relation is followed in practice so closely that no deviations are detectable, within the measurement accuracy as limited by the difficulty of deciding when the triode is actually cut-off.

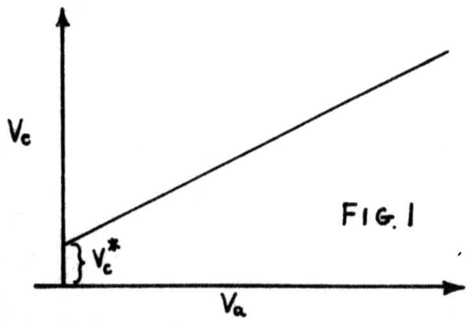


FIG. 1

[†]Experimental evidence presented in author's paper, 'The Electron Gun of the Cathode Ray Tube', Part 2. See Fig. 14.

2.3. Extended Applications of Voltage Similitude - Prediction of Relative Triode Performance - Conditions near Zero Grid

It is important to have some knowledge of the way in which the cathode current of any C.R. tube depends on the first anode voltage, for a specified grid potential. For the moment we shall restrict the investigation to the case when the grid potential is zero.

The usual method of computing the form of the required relation is to solve the reduced Poisson's equation, as was first done by Childs for the case of the planar diode. In this case, and also in one other of practical importance - namely the cylindrical diode - the problem is relatively easy, since the reduced form of Poisson's equation presents no difficulty. But in our problem the position is much more serious, for we are confronted with the difficulty of solving a three dimensional form of the equation in which the only simplification lies in the fact that the field distribution possesses rotational symmetry. In fact the equation is

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}.\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 4.\pi.i\sqrt{\frac{m}{2.e.V}} \quad \ldots\ldots(9)$$

which is not very hopeful.

But consider the following chain of reasoning. From the fundamental definition of potential we may write

$$\phi = \int_{\text{volume}} \frac{\rho\, d\sigma}{r} + \int_{\text{anode}} \frac{\sigma\, dS}{r} + \int_{\text{grid}} \frac{\sigma\, dS}{r} + \int_{\text{cathode}} \frac{\sigma\, dS}{r} \quad \ldots\ldots\ldots(10)$$

(10) merely expresses the fact that the potential ϕ at any fixed point in the field of the triode is equal to the sum of the potentials due to the volume distribution of charge, and to the surface distributions on the three bounding electrodes. Now

consider the position when no space charge is present, when the anode potential is V_a and the grid potential zero. From equation (1) we see that the potential is then expressible in the form $\phi = B.V_a$. But under these conditions, a certain charge distribution exists on the bounding electrodes, and the resulting potential is expressed by the sum of the last three terms of equ. (10). Hence we may rewrite (10) in the form

$$\phi = \int_{\text{volume}} \frac{\rho d\sigma}{r} + B.V_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)^*$$

At first sight this does not seem hopeful, but we now recollect that in all these derivations of the saturated emission/ voltage law, we make the fundamental postulate that the mechanism of the emissive process is such as always to maintain zero potential gradient at the cathode surface. Differentiating (11) therefore with regard to z, and equating to zero gives

$$0 = \frac{\partial}{\partial z}\int \frac{\rho d\sigma}{r} + B_1.V_a \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Note that voltage similitude is the justification for the form of the second term in (12). This latter equation indicates proportionality between ρ and V_a, for zero cathode gradient. Hence the important conclusion - <u>in any space charge limited device in which the cathode potential gradient is zero, the total space charge is proportional to the anode voltage</u>.

By application of the energy equation, $mv^2/2 = e\phi$, it immediately follows that the electron velocity at any fixed

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* This equation is interesting because it is homogeneous in V_a and ρ. Thus if both V_a and ρ are together multiplied by k the potential at any fixed point is also multiplied by k. This can be regarded as an extension of the principle of voltage similitude to the case where space charge is present. cf. Sheet 1.

APPLICATION OF VOLTAGE SIMILITUDE

ALL ANODE VOLTAGES ON TUBE MULTIPLIED BY k

Total cathode current and beam current (zero grid or at corresponding point on grid base)	$\times k^{3/2}$
Cathode loading, ditto	$\times k^{3/2}$
Beam power	$\times k^{5/2}$
Screen loading (watts/cm.^{2})	$\times k^{5/2}$
Crossover and undeflected spot size	$\times 1/\sqrt{k}$
Increase in spot 'size' on deflection	$\times 1$
Ratio of new to original deflected spot size	$\dfrac{f(G,\lambda) + a/\sqrt{k}}{f(G,\lambda) + a}$
Current density in undeflected spot	$\times k^{5/2}$
Sensitivity of deflection (mm. per volt)	$\times 1/k$
Deflectional discrimination	$\times 1/\sqrt{k}$
Electron transit time	$\times 1/\sqrt{k}$
Screen brightness assuming screen powder has square law response to voltage, and linear response to current density	$\times k^{7/2}$

Note that although the magnification ($M = v/u$) is constant, the ratio, current density in undeflected spot / cathode loading, has increased k times. This corresponds exactly to the optical analogue, - image brightness = object brightness multiplied by the ratio of the squares of the refractive indices of image and object spaces.

Note also the rapid increase in raster brightness with anode voltage.

TABLE 1.

point in the field is proportional to the square root of the voltage, provided the electrons start from rest, or at most with relatively negligible velocities. Thus the current, which is the product of the space charge density and velocity (under steady state conditions), must vary as the 3/2 power of the anode potential.

Thus finally

$$I_c = K.V_a^{3/2} \quad (V_g = 0) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where I_c is the total cathode current. This reasoning has not involved any special assumption as to the shape of the bounding electrodes.

2.4. Extended Applications of Voltage Similitude - Prediction of Relative Modulation Characteristic

The conclusions that the cathode current varies as the three halves power of the anode voltage, and that the cut-off voltage is fairly closely proportional to to the anode voltage have been adequately confirmed by experiment. Thus multiplication of the anode voltage by k also multiplies the cut-off by k and the cathode current at zero grid voltage by $k^{3/2}$. From this it immediately follows that for geometrically similar points on the grid base, the cathode current varies as $k^{3/2}$. By geometrically similar points is meant points which divide the grid base between zero and cut-off in the same ratio. Thus if the grid voltage were maintained constant, while the anode voltage were multiplied by k, the cathode current would certainly not increase by $k^{3/2}$, except in the special case where $V_g = 0$.

Table 1 summarises the main conclusions of the principle of voltage similitude as applied to the C.R. tube. The last row in the table follows immediately from energy conservation.

APPLICATION OF GEOMETRICAL SIMILITUDE

ALL DIMENSIONS MULTIPLIED BY k

Beam and cathode currents	×1
Beam power	×1
Cathode loading	$\times 1/k^2$
Crossover size	×k
Magnification (M)	×1
Undeflected spot size	×k
Increase in spot size on deflection	×k
Ratio of new to original deflected spot size	$\frac{k.f(G,\lambda) + k.a}{f(G,\lambda) + a} = k$
Current density in spot	$\times 1/k^2$
Screen size (linear)	×k
Sensitivity of deflection	×k
Deflectional discrimination	×1
Electron transit time	×k

Note there is no fundamental gain in performance. The spot density and cathode loading are equally affected, and there is no gain in deflectional discrimination.

TABLE 2.

2.5. Application of the Principle of Geometrical Similitude

Let all the dimensions of a C.R. tube be multiplied by k, while the operating voltages remain constant. Then by principle of geometrical similitude 1.2. the current remains constant, and the whole scale of the trajectories is multiplied by k.* The crossover diameter and spot diameter are multiplied by k; their density falls to $1/k^2$, as does also the cathode emission density, since the same current is being extracted from an area k^2 times as large. This last result can also be regarded as a consequence of the principle of dimensional homogeneity when applied to the system.

Table 2 summarises these questions.

2.6. Relaxed Geometrical Similitude - 1

More important applications of these ideas on scale theory involve what may be termed 'relaxed' similitude, in which only certain portions of the tube are changed. This type of computation is exceedingly rapid, and frequently leads to conclusions of a general sort about the form a tube should take to meet a specified demand.

Table 3 shows a specific example, which shows in general the superiority of the large tube, at least so far as the performance towards the screen centre is concerned. We might

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*The scale of the potential field is also multiplied by k, as can be seen from equation (11). For at corresponding points in the two systems the second term $B.V_a$ is constant. Let primed symbols refer to the transformed system. Since the current is constant, $\rho' = \rho/k^2$. Also $r' = k.r$, $d\sigma' = k^3.d\sigma$. Hence the first term in (11) is also constant, and thus ϕ is constant.

RELAXED GEOMETRICAL SIMILITUDE - 1

Suppose that in any cathode ray tube, the triode portion is maintained of constant size, but that all remaining dimensions are multiplied by k. Determine the main features of the new tube.

Beam and cathode currents at same grid voltage	×1
Beam power	×1
Cathode loading	×1
Magnification (M)	×1
Crossover size	×1
Undeflected spot size (space charge at screen assumed small)	×1
Increase in spot 'size' on deflection	×k
Ratio of new to original deflected spot size	$\frac{k.f(G,\lambda) + K}{f(G,\lambda) + K}$
Screen size (linear)	×k
Sensitivity of deflection (mm. per volt)	×k
Deflectional discrimination (near screen centre)	×k
Screen brightness for same relative scan	$\times 1/k^2$

Provided that the deflection defocussing on the original tube was small, we conclude that the derived tube will have a resolution k times as large.

TABLE 3.

further generalise by now imagining that the screen diameter is kept constant. Then the new (longer) tube has a much smaller scanning angle, so the increase in deflection defocussing is avoided, and the longer tube has a clear increase in deflectional discrimination for the same cathode loading and operating voltages. But there is a limit to this process, since table 3 assumes small space charge at the screen. If this does not obtain, so that the spot size is dominated by space charge swelling, then the spot size is a linear function of the scale factor k, and the conclusions of table 3 are invalid. This question was discussed very fully in another paper by the present author[2] to which close reference should be made.

2.7. Relaxed Geometrical Similitude - 2

Table 4 illustrates a similar type of problem, from which we may draw the important conclusion that the performance of any tube in which deflection defocussing is small is continuously improved by reduction in the scale of the triode portion. The price paid is an increase of cathode loading. This process has been applied in practice during the war to obtain a very high performance tube, without increase in length or operating voltage.

2.8. Relaxed Geometrical Similitude - 3

In this section we investigate the general effect of keeping the form and size of all the tube between anode and screen constant, but multiplying the linear scale of the remainder by k. The anode hole diameter is also kept constant. Table 5 illustrates the results and the following is the reasoning involved.

All dimensions of the triode are multiplied by k, - hence the crossover size is also multiplied by k. But unlike

RELAXED GEOMETRICAL SIMILITUDE - 2

Suppose that in any cathode ray tube, the dimensions of the triode portion are multiplied by k, all other dimensions remaining constant. Determine the main features of the new tube.

Beam and cathode currents at same grid voltage	×1
Beam power	×1
Cathode loading	$\times 1/k^2$
Crossover size	×k
Magnification (M)	×1
Undeflected spot size	×k
Increase in spot size on deflection	×1
Ratio of new to original deflected spot size	$\frac{f(G,\lambda) + a.k}{f(G,\lambda) + a}$
Screen size	×1
Sensitivity of deflection	×1
Deflectional discrimination (near screen centre)	×1/k
Screen brightness	×1
Current density in undeflected spot	$\times 1/k^2$

Provided deflection defocussing in original prototype tube is small, the overall performance of the new tube improves continuously as the triode is reduced in size. Limit set by permissible cathode loading and mechanical tolerances.

TABLE 4.

case 2.7, the crossover / anode distance is also multiplied by k, so that the magnification between crossover and. spot is multiplied by 1/k. Hence the undeflected spot size is unaltered.

Application of the principle of geometrical similitude also shows that for constant electrode voltages the total current is unaltered. But the anode hole has been postulated as being of constant size. Hence the beam current (i.e. current emerging from anode hole) is down to $1/k^2$ (approx.), assuming that the gun size is always sufficiently large to make the beam more than fill the anode hole. But again the cathode area is up k^2 times for the same cathode current. Hence the cathode loading is down $1/k^2$, i.e. in the same ratio as the beam current. Thus the net result is no gain in overall performance.† The focus conditions at the edge of the screen are clearly unchanged since the whole geometry of the deflecting region has been postulated constant.

At this stage it is desirable to issue a warning about drawing conclusions which are more far reaching than the postulates justify. Taken at its face value the foregoing reasoning would appear to imply that if the deflecting geometry of a cathode ray tube is maintained constant, then the actual size of the remainder of the gun has no effect, provided the gun size is adequate to fill the anode hole. But this conclusion has been based on certain postulates, one of which is that aberrations in the gun are independent of the physical size of the electrodes of which it is made. This is certainly not true without qualification. Experimental evidence shows that in

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†For, by driving the tube more heavily the spot density would be restored.

RELAXED GEOMETRICAL SIMILITUDE - 3

All dimensions of the gun only in a cathode ray tube are multiplied by k, except for the anode hole which remains of constant size. The deflector plates, anode to screen distance, screen diameter, etc., remain unchanged. Determine the main features of the new tube.

Cathode current at same grid voltage	$\times 1$
Beam current ditto (see text, section 2.8)	$\times 1/k^2$ (approx.)
Beam power	$\times 1/k^2$ (approx.)
Cathode loading	$\times 1/k^2$
Crossover size	$\times k$
Magnification (M)	$\times 1/k$
Undeflected spot size	$\times 1$
Increase in spot size on deflection	$\times 1$
Ratio of new to original deflected spot size	$\times 1$
Screen size	$\times 1$
Sensitivity of deflection	$\times 1$
Deflectional discrimination	$\times 1$
Screen brightness	$\times 1/k^2$

Conclusion -

Within the postulates set out in the text (section 2.8), no fundamental improvement gained by increasing gun size beyond the point where the anode hole is filled.

TABLE 5.

general for a fixed final anode hole diameter, the aberrations of the final focussing lens decrease as the size of the lens increases, up to a certain point at which further increases in size have little effect. Having regard to this fact, the practical conclusions of this section may be stated thus:

'In any cathode ray tube of fixed deflecting geometry in form and size, an advantage is gained as the size of the gun section alone increases (final anode hole diameter constant), up to the point when the anode hole is fully filled and reduction of lens aberrations is negligible. Further increases in size have then no useful effect!

2.9. Relaxed Geometrical Similitude - 4

This illustrates the important effect of keeping the size and form of the whole electron gun (including focussing portion and deflector plates) constant, and multiplying the geometry of the screen end of the bulb by k. More accurately, the tube neck diameter is kept constant, and the derived tube is formed by merely slicing off the bulb, and thus multiplying the anode to screen distance by k. Refer to table 6. Thus the new spot density at the screen centre is multiplied by $1/k^2$ for the same cathode loading, since the gun conditions are unchanged. If, as a usually justifiable first approximation, the relatively small distance between the anode and the centre of deflection is ignored, then for the same scanning angle the derived tube has a screen diameter multiplied by k.

Now consider the deflection defocussing. A direct application of geometrical similitude is inadmissible, since the deflecting region of the derived tube is not a scaled down replica

RELAXED GEOMETRICAL SIMILITUDE - 4

A cathode ray tube is derived from an established design by moving the plane of the screen only, so that the new anode to screen distance is k times that on the original. Determine the main features of the derived tube.

Beam and cathode currents at same grid voltage	×1
Beam power	×1
Cathode loading	×1
Crossover size	×1
Magnification (M)	×k
Undefected spot size	×k
Increase in spot size on deflection (see text)	$\times\sqrt{k}$
Ratio of new to original deflected spot size....	$\dfrac{\sqrt{k}.f(G,\lambda) + a.k}{f(G,\lambda) + a}$
Screen size (linear, for same scanning angle)...........	×k (approx.)
Sensitivity of deflection	×k (approx.)
Deflectional discrimination (near screen centre)	×1
Current density in undeflected spot	$\times 1/k^2$

Conclusions -

Large increase in spot density without corresponding increase in cathode loading, effected by reducing anode to screen distance and screen diameter. Absolute focus quality must improve over whole equivalent screen area. Relative focus quality unchanged at centre, and may not be appreciably worse at edges if deflection defocussing is small. Indicates the merit of correctly designed tubes of small screen diameter.

TABLE 6.

of the prototype. But consider the following chain of reasoning, based on dimensional homogeneity.

Let R be the radius of the anode hole, d the anode to screen distance, and λ the scanning angle. For constant deflector plate size and geometry, the linear increase in spot size on deflection (refer to section 1.5) must be a function of R, d and λ. Thus

$$\text{Linear increase in spot size} = K.R^a.d^b.\lambda^c \quad \text{..........(14)}$$

But λ is a pure numeric without dimensions, and since the left hand side must have the dimensions of a length, it follows that $a + b = 1$.

In the present problem, the only variable is the anode to screen distance d. Hence the linear increase in spot size is proportional to d^b.

It is obvious that $a > 0$, and $b > 0$. Thus $0 < b < 1$.

Thus if the anode to screen distance is multiplied by k, the linear increase in spot size on deflection through a constant angle is multiplied by k^b.

But here $0 < k < 1$. Therefore $k < k^b < 1$.

It is therefore certain that there will be some improvement in deflection defocussing. This improvement will tend to zero as $b \rightarrow 0$, and will tend to k times its original value as $b \rightarrow 1$.

A sensible first approximation would put $a = b = 0.5$, so that the increase in spot size on deflection is multiplied by $\sqrt{k}$. This has been done in table 6 which summarises the position.

The tube with the smaller screen size has a very marked superiority in central focus performance. The <u>absolute</u> quality of the edge focus <u>must</u> also be better than on the original tube. The relative quality of the edge focus <u>must</u> at best be slightly worse than on the original tube.

2.10. 'Philosophy' of Design - the 'Optimum' Tube

While all the statements in the foregoing sections are correct within their postulates, they do not form a very coherent pattern, and it is difficult to see in what way they point so far as the 'optimum' tube is concerned. This is natural enough, since a cathode ray tube is a perfect example of 'coordinated compromises' in which improvements in one feature appear to involve loss of performance in some other. But we shall now show that there is in fact a general 'pattern' which leads to the best type of design for most purposes.

The following postulates will be made, and it will be noted that these are essentially reasonable and practical.

(a) The cathode loading and final anode voltage are both fixed.

(b) The overall tube length is fixed.

(c) The screen diameter is fixed.

It has been clearly established that the general performance of a given <u>form</u> of tube improves as (a) and (b) are increased[2], and as (c) is reduced. Our problem is to find the optimum form, in general terms.

Ultimately, the design compromise breaks down into a contest between the central focus and spot density, and the permissible degree of deflection defocussing. In another paper by the present author[2] discussing the fundamental work of Langmuir[1], it was emphasised that when space charge is negligible the undeflected electron spot density is proportional to $\sin^2\theta$ where θ is the semi-angle of the electron beam converging on the screen. Hence constant θ should ensure constant density.

Let R be the radius of the final anode hole, and let d be the anode to screen distance. Then since θ is very small, $\theta = R/d$. So far, therefore, as the central spot density is concerned, the designer has freedom to put the final anode in any position provided that R/d is constant. What deduction, if any, can be made about its optimum position?

The whole answer to this question turns on the extent to which deflection defocussing varies with the scanning angle. Formal reasoning gives no answer to this last point; - λ in equ. (14) is a pure numeric, so that nothing can be deduced from dimensional considerations about c. But common experience shows that deflection defocussing rises very rapidly with scanning angle, so that if deflection defocussing were expressible as a simple power function of the scanning angle, then $c > 1$. From this if immediately follows that multiplication of R and d by a constant (k), (so as to preserve constant Θ), with corresponding reduction of the scanning angle λ to λ/k, results in reduced deflection defocussing.

Hence the general design procedure should be to make the anode to screen distance as large a fraction as possible of the total (fixed) tube length, and make the final anode hole as large as possible consistent with deflection defocussing.

Applying now the principles treated in section 2.8, the electron gun proper should be made sufficiently large to avoid aberration in the final lens.

Finally, applying the principles treated in section 2.7, the triode section of the gun should be reduced in scale to achieve the desired spot size.

These are the general principles behind the design of all-electrostatic cathode ray tubes of high performance and fixed length. It will be observed that this design technique not only ensures greatest focus uniformity, but also gives maximum deflector plate sensitivity and deflectional discrimination. The only exception to this principle would appear to be the case of high current, low voltage tubes which might become severely space charge limited at the screen[2,3].

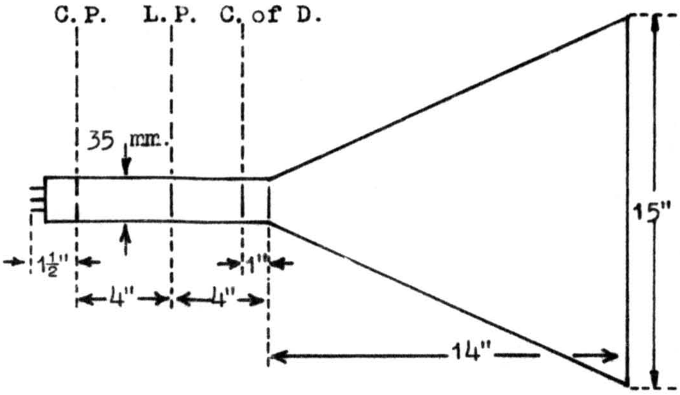
2.11. Application to the Problem of Projection Tubes

Initial design investigations on the possibilities of projection tubes afford an interesting application of the various methods discussed. All the foregoing ideas are used, and the analysis is an excellent example of the scope and limitations of the methods.

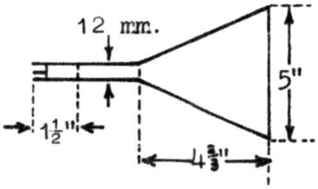
Insert 1 on table 7 shows the essential dimensions of a typical 15 inch direct viewing television cathode ray tube. This tube is regarded as the prototype. A justification of its form is irrelevant here, - the dimensions have been arrived at by long experience as representing a satisfactory compromise. The operating figures are similarly the result of experience. It has been stressed that all the ideas in this paper deal with relative, not absolute, performance. Therefore we must start from some existing design. What inferences can be drawn about the dimensions and operating conditions of derived projection tubes which will give the same final picture quality?

The essential point about the projection tube is that it is smaller, and thus avoids the fragile, rather expensive and even dangerous large glass envelope. Hence the first obvious derived tube to investigate is a proportionally scaled down replica. Insert 2 shows this with a three to one linear reduction, which seems reasonable. For practical reasons the base only is not scaled down.

The first step is to decide on the postulates. It will be assumed that the cathode loading remains constant, which is quite reasonable. From this it immediately follows that the beam

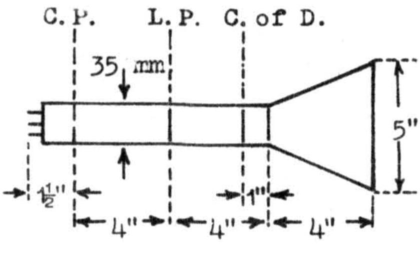


1. 15" DIRECT VIEWING TUBE



Triode portion only ⅔ scale. See text.

2. ⅓ SCALE PROJECTION TUBE



Gun section not reduced in size.

3. PRACTICAL PROJECTION TUBE

	Final picture illumination	Efficiency of optical system	Anode voltage	Beam current (Full white)	Cathode loading (Full white)	Beam power (Full white)	Screen loading (Full white)	Scanning power	Final picture size
1. 15" DIRECT VIEWING TUBE	10 e.f.c.	100%	8 k.V.	150 μA.	.35 A./cm^2	1.2 watts	1.6 mW/cm^2	18 watts	12" x 9.6"
2. ⅓ SCALE PROJECTION TUBE	10 e.f.c.	20%	28.4 k.V.	60 μA.	.35 Amp/cm.^{2}	1.7 watts	20.6 mW./cm.^{2}	21.3 watts	12" x 9.6"
3. PRACTICAL PROJECTION TUBE	10 e.f.c.	20%	18 k.V.	150 μA.	.35 Amp/cm.^{2}	2.7 watts	33 mW./cm.^{2}	40 watts	12" x 9.6"

C.P. = Crossover Plane, L.P. = Lens Plane, C. of D. = Centre of Deflection.

TABLE 7.

current in the scaled down tube can be only 1/9 of that of the prototype.

Next, in order to ensure equal final projected picture brightness, some assumption as to the efficiency of the projection lens system must be made. A figure of 20% is assumed, which can be achieved with a well designed Schmidt mirror system. Then if V_p, I_p are the beam voltage and current respectively in the projection tube, and V and I those of the direct viewing tube, the requirement of equal final picture brightnesses gives

$$.2\, V_p^2 . I_p = V^2 . I \qquad (15)$$

The screen efficiency is here assumed to be proportional to the square of the beam voltage, which is a good working approximation. In (15) only V_p is unknown, - its value works out at 54 k.V.

Next we investigate the picture definition. By geometrical similitude it immediately follows that if the anode voltage were unchanged, then the definition must also be unchanged. However, the anode voltage has been changed, and from section 1.4 it follows that the new spot size is multiplied by $\sqrt{\frac{8}{54}}$. Hence the scaled down replica will not give the constant picture quality required.

A solution involves application of the principles of section 2.7 and table 4. The spot size is restored by multiplying the scale of the triode by $\sqrt{\frac{54}{8}}$. This then means that cathode loading is only $\frac{8}{54}$ of that on the prototype. This suggests the current could be increased for the same cathode loading, so that the extreme voltage increase might be avoided.

To solve this problem, let k be the linear multiplying factor for the triode portion only. The remainder of the tube is, as before, multiplied by 1/3.

Inserting the values in table 7, insert 1, the following equations will be found to obtain:

(1) Energy balance

$$.2\ V_p^2 . I_p = 8^2 \times 150 \quad \ldots\ldots (15)$$

(2) Constant cathode loading (postulate)

$$I_p = 150 . k^2 \quad \ldots\ldots (16)$$

(3) Constant spot size

$$3 . k = \sqrt{\frac{V_p}{8}} \quad \ldots\ldots (17)$$

Solving this system gives V_p = 28.4 k.V., I_p = 60 μA., k = 0.63. Thus although the tube is reduced to one third size, the triode portion is reduced only to approximately 2/3 of its previous value.

The remaining columns in table 7, insert 2, are fairly obvious. The final column relating to scanning power depends on the fact that the latter varies directly as the anode voltage and neck diameter.

Another interesting solution for a derived tube begins with the method of section 2.9 and table 6 - by slicing off the bulb of the direct viewing tube. This spproach is suggested by the large resulting increase in spot density without loss in spot size / screen size ratio. Although the final tube must be larger than that derived from a complete scaling-down, since the neck dimensions are unchanged, this does not matter much, because the major bulb volume is in the conical portion.

The reasoning to obtain the values shown in table 7 is as follows.

Since the gun and deflector system is unchanged, a constant cathode loading requires the beam current to be unchanged. An energy equation as (15) then gives V_p = 18 k.V.

To investigate the definition, suppose for simplicity that the crossover size of the direct viewing tube were unity. Then the undeflected spot size of the latter would be 4.5. But the crossover size of the projection tube is down to $\sqrt{\frac{8}{18}}$ by section 1.4. The magnification of this tube is 2, - thus the spot size on the screen is $2.\sqrt{\frac{8}{18}}$, and magnifying this three times by the projection lens gives the final picture spot size as $3 \times 2 \times \sqrt{\frac{8}{18}} = 4$. This difference between 4 and 4.5 is hardly noticeable. Hence the definition at the screen centres is substantially the same.

The <u>relative</u> definition at the edges of the projection tube will be slightly worse by the reasoning given in section 2.9. It would be appreciably worse if the deflection defocussing on the prototype direct viewing tube were bad. In this case the calculations give only a very rough estimate of the position.

Literature

(1) 'Theoretical Limitations of Cathode Ray Tubes', David B. Langmuir, Proc. I.R.E., August 1937.

(2) 'The Electron Gun of the Cathode Ray Tube', Part 1, Hilary Moss, J. Brit. I.R.E., January 1945.

(3) 'A Space Charge Problem', Hilary Moss, Wireless Engineer, July 1945.