Class-A Push-Pull Amplifier Theory*

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Summary—Two tubes operating in push-pull, class-A_1, will produce more than twice the power output of a single tube using the same operating voltages, and with optimum load values in each case. This is demonstrated analytically by means of an equivalent circuit. The change in load impedance seen by one tube due to the effect of coupling to the other tube is considered and used to explain the results obtained. Experimental data are presented to verify the theory.

Introduction

SINCE THE first paper on the operation of push-pull amplifiers by B. J. Thompson,[1] relatively little has been added to our knowledge of the circuit operation. In explaining the increased power output available (with a given percentage distortion) from push-pull amplifiers as compared to single-tube amplifiers, the statement is usually made that because of the elimination of even-order harmonics by the push-pull connection the tubes may be driven harder to obtain a greater output per tube without exceeding the prescribed maximum distortion. However, the case of single and push-pull tubes operating with the *same* plate voltage, grid bias, and grid signal has not been considered. Under this condition, the push-pull connection will deliver more than twice the output power obtainable from a single tube, using load resistances to give maximum power output in each case,

Symbols

The circuit diagram is shown in Fig. 1, and the symbols are defined in the accompanying list. Subscripts 1 and 2 are used to differentiate between corresponding quantities for the two tubes as indicated in Fig. 1.

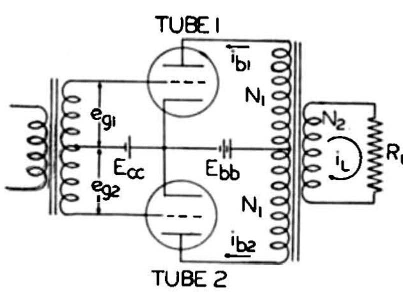

Fig. 1—Push-pull amplifier circuit.

N_2 = number of turns on output-transformer secondary winding

N_1 = number of turns on one-half the primary winding

E_{bb} = plate-supply voltage

E_{cc} = grid-bias voltage

e_b = total instantaneous plate voltage

e_c = total instantaneous grid voltage

e_g = instantaneous varying component of grid voltage

E_g = effective value of the varying component of grid voltage

I_{b0} = quiescent plate current of one tube

i_p = instantaneous varying component of plate current

$i_b = I_{b0} + i_p$ = total instantaneous plate current

$i_d = i_{b1} - i_{b2}$ = net instantaneous magnetizing component of current in the transformer primary

i_L = instantaneous load current

r_p = instantaneous dynamic plate resistance

r_{p0} = dynamic plate resistance of either tube at the quiescent operating point

r_d = dynamic plate resistance of the composite tube

R_L = load resistance on the transformer secondary

$R_{pp} = 4\left[\frac{N_1}{N_2}\right]^2 R_L$ = plate-to-plate reflected load resistance due to R_L

$R_L' = \left[\frac{N_1}{N_2}\right]^2 R_L$ = reflected load resistance due to R_L across one-half the transformer primary,

Theory of Push-Pull Operation

The usual assumptions are made that the tubes and related circuit elements are identical, and that the load is coupled to the tubes through an ideal transformer having no resistance or leakage reactance. Signal voltage applied to the grids is assumed to be sinusoidal, and to be limited to values giving class-A_1 operation ($\sqrt{2}E_g \leq E_{cc}$).

From an examination of Fig. 1 it is apparent that for the quiescent operating condition (no signal applied) the grid voltages, plate voltages, and plate currents of the two tubes will be identical. Thus,

$$e_{c1} = e_{c2} = E_{cc} \tag{1}$$

$$e_{b1} = e_{b2} = E_{bb} \tag{2}$$

$$i_{b1} = i_{b2} = I_{b0} \text{ of one tube.} \tag{3}$$

When a signal voltage is applied to the input, the center-tapped transformer connections cause the instantaneous changes in grid and plate voltages to be equal and opposite for the two tubes.

$$\Delta e_{c1} = -\Delta e_{c2} \tag{4}$$

$$\Delta e_{b1} = -\Delta e_{b2}. \tag{5}$$

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[1] B. J. Thompson, "Graphical determination of performance of push-pull audio amplifiers," Proc. I.R.E., vol. 21, pp. 591–601; April, 1933.

Since the currents i_{b1} and i_{b2} flow in opposite directions in the transformer primary, the net flux-producing current is

$$i_d = i_{b1} - i_{b2}, \tag{6}$$

which is related to the load current (i_L) by

$$i_L = \frac{N_1}{N_2}(i_d) \tag{7}$$

where i_d is assumed flowing in N_1 turns (one-half the primary winding).

But if Δe_{c1} is positive,

$$i_{b1} = I_{b0} + \Delta i_{b1}, \quad \text{and} \quad i_{b2} = I_{b0} - \Delta i_{b2}. \tag{8}$$

Substitution in (6) gives

$$i_d = \Delta i_{b1} + \Delta i_{b2}, \tag{9}$$

in which the varying components of i_{b1} and i_{b2} add as far as i_d (or i_L) is concerned. This suggests an equivalent circuit for the varying quantities in which the two tubes may be considered as generators in parallel supplying the common load resistance. Let the instantaneous values of the varying components of current be i_{p1} and i_{p2}, and let

$$R_L' = R_L\left[\frac{N_1}{N_2}\right]^2$$

be the reflected load resistance seen across N_1 turns of the primary.

The equivalent circuit is shown in Fig. 2, with the current and voltage directions established by the foregoing discussion. The plate resistances of the two tubes are called r_{p1} and r_{p2}, respectively, since they are not necessarily equal except at the quiescent operating point. The values of μ for the two tubes are assumed equal and constant throughout the operating cycle.[2] This equivalent circuit holds for the nonlinear as well as the linear region of tube operation, as long as the nonlinearity can be expressed by variations in r_{p1} and r_{p2} only.

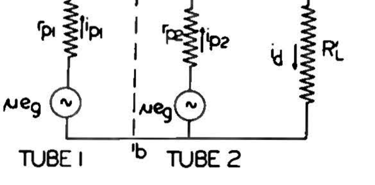

Fig. 2—Equivalent circuit for the push-pull amplifier.

Now consider the composite plate characteristics, load line, and individual-tube operating line (A-A')[3] shown for triode-connected 6L6's in Fig. 3, with the operating voltages chosen the same as those which would give good class-A$_1$ operation with a single tube, and with the load line (representing R_L') chosen to give maximum power output ($R_L' = r_d$). The individual-tube operating line (A-A') is not straight, so it represents a

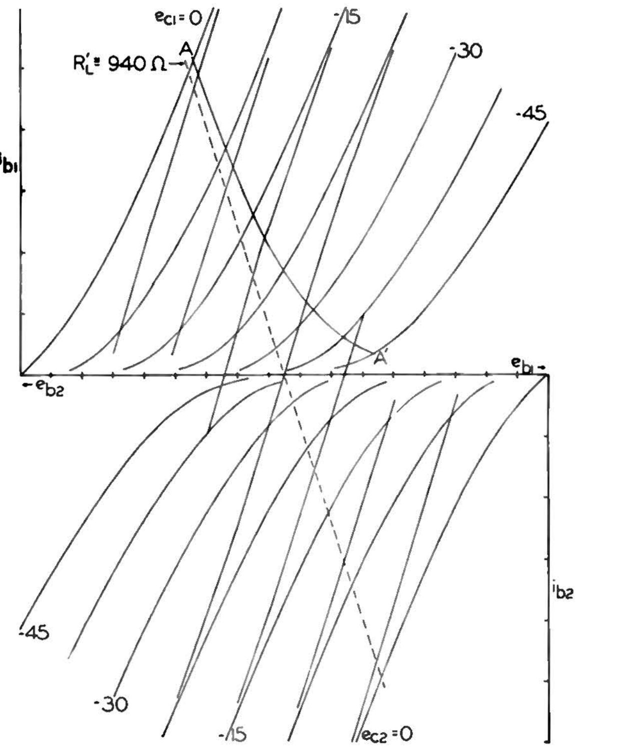

Fig. 3—Composite characteristics for a push-pull amplifier using two 6L6 tubes, triode-connected, with $E_{bb}=255$ volts, $E_{cc}=-22.5$ volts, $R_L'=r_d=940$ ohms.

varying load resistance presented to the tube. However, for the optimum load chosen, the slope of A-A' is approximately equal to the negative of the slope of the individual-tube plate characteristics at each of their intersections; so the dynamic plate resistance of the tube sees an equal load resistance throughout the cycle. Thus, the conditions for maximum power transfer[4] are satisfied at each instantaneous point in the operating cycle; whereas this is not true in large-signal operation of single-tube amplifiers where the load and tube impedances can be matched at only one point in the cycle. Thus, each tube in the push-pull amplifier should be able to deliver more power to the load than it could in single-tube operation, because of the continuous impedance-match.

The equivalent circuit of Fig. 2 will now be solved to verify the preceding discussion. All quantities except the reflected load resistance R_L' and μ are *instantaneous* values, including the varying plate resistances r_{p1} and r_{p2}. Let the load resistance seen by tube 1 be r_{ab} = impedance of the circuit to the right of line a-b in Fig. 2. This impedance cannot be expressed merely as the parallel combination of r_{p2} and R_L' ,because of the action of the two equivalent generators in the circuit; i.e., r_{ab} includes

[2] M.I.T. Staff, "Applied Electronics," John Wiley and Sons, New York, N. Y., 1943, p. 182.
[3] See pp. 440–446 of footnote reference 2.

[4] W. L. Everitt, "Communication Engineering," McGraw-Hill Book Co., New York, N. Y., 1937, pp. 49–52.

the effect of the generated voltage, μe_g, in series with r_{p2}, and this voltage must always be equal to the voltage μe_g in series with r_{p1} to satisfy the conditions for push-pull operation. Therefore, to find the load impedance seen by tube 1, solve for i_{p1} and use the relation

$$r_{ab} = \frac{\mu e_g}{i_{p1}} - r_{p1}. \tag{10}$$

The circuit equations for Fig. 2 are

$$i_d = i_{p1} + i_{p2} \tag{11}$$

$$\mu e_g - \mu e_g = i_{p1}r_{p1} - i_{p2}r_{p2} \tag{12}$$

$$\mu e_g = i_d R_L' + i_{p2}r_{p2}. \tag{13}$$

These combine to give

$$0 = i_{p1}r_{p1} - i_{p2}r_{p2} \tag{14}$$

$$\mu e_g = i_{p1}R_L' + i_{p2}(R_L' + r_{p2}). \tag{15}$$

Solving for i_{p1} gives

$$i_{p1} = \frac{\mu e_g r_{p1}}{r_{p1}(R_L' + r_{p2}) + R_L' r_{p2}} \cdot \tag{16}$$

Then

$$\frac{\mu e_g}{i_{p1}} = r_{p1} + R_L'\left[1 + \frac{r_{p1}}{r_{p2}}\right]. \tag{17}$$

From (10),

$$r_{ab} = R_L'\left[1 + \frac{r_{p1}}{r_{p2}}\right]. \tag{18}$$

To show that $r_{ab} = r_{p1}$ at all times, a relationship between r_{p1}, r_{p2}, and r_d (dynamic plate resistance of the composite tube) must be found. Let the dynamic plate resistance of the composite tube be expressed as

$$r_d = \frac{\Delta e_{b1}}{i_d} \cdot \tag{19}$$

But, as has been shown previously, at any point on the composite tube lines

$$i_d = \Delta i_{b1} + \Delta i_{b2} \tag{20}$$

$$\Delta i_{b1} = \frac{\Delta e_{b1}}{r_{p1}} \tag{21}$$

$$\Delta i_{b2} = \frac{-\Delta e_{b2}}{r_{p2}} = \frac{\Delta e_{b1}}{r_{p2}} \tag{22}$$ [5]

$$r_d = \frac{\Delta e_{b1}}{\dfrac{\Delta e_{b1}}{r_{p1}} + \dfrac{\Delta e_{b1}}{r_{p2}}} = \frac{r_{p1}r_{p2}}{r_{p1} + r_{p2}} \cdot \tag{23}$$

[5] The negative sign in the second expression is necessary in order to make it agree with the definition of Δi_{b2} in equation (8).

This equation states that, with $R_L' = r_d$, the plate resistance of the composite tube at any instantaneous operating point is equal to the parallel combination of the plate resistances of the individual tubes (thus further validating the use of the equivalent circuit of Fig. 2).

Since the composite-tube plate characteristics are very nearly straight, parallel lines for class-A_1 operation (see Fig. 3), r_d is almost constant throughout the cycle even though r_{p1} and r_{p2} are varying. This approximation to a constant value would become progressively poorer for class-AB or -B operation. However, for purposes of this analysis r_d will be considered constant, and the equivalent load resistance R_L' will be given the value

$$R_L' = r_d = \frac{r_{p0}}{2} \tag{24}$$

since the plate resistances of the two tubes are assumed equal at the quiescent operating point. Thus, the impedances of load and composite tube will be equal at all times, giving maximum-power-transfer conditions throughout the cycle. Considering the load impedance seen by the individual tube, (23) and (24) may be substituted in (18) to give

$$r_{ab} = r_{p1}, \tag{25}$$

indicating that each tube operates into a load resistance equal to its plate resistance at every point in the cycle. Thus, each tube is also delivering maximum power at all times.

Experimental Results

In order to check the theory, the fundamental-frequency power output of two triode-connected 6L6 tubes was determined both graphically and experimentally for both parallel and push-pull connections. The operating voltages were the same in all cases, and load values giving maximum power output were used. The calculated and measured values agreed perfectly within the limits of experimental error, giving 3.6 watts for parallel and 4.1 watts for push-pull operation. The parallel operation produced 14 per cent second-harmonic distortion, compared to less than 2 per cent third harmonic (negligible second harmonic) with the push-pull connection. The 11 per cent increase in power noted above would be even greater if the distortion in the output of the parallel connection had been held to a tolerable value. Thus, the theory is clearly verified.

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