Periodic Variations of Pitch in Sound Reproduction by Phonographs*

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Summary—Causes of recurrent variations of pitch frequently encountered in the reproduction of phonograph records, commonly called "wow," are discussed. Considerations for the design of an instrument to measure the magnitude of these variations are given, minimum requirements and limitations of the design of such equipment are presented, and an instrument for use with 78-revolutions-per-minute turntables built to meet these requirements is described.

Introduction

FOR MANY years phonograph records have been by far the most popular means of recording music and speech. Extensive efforts have been made by manufacturers of recording and reproducing equipment to render the reproduction as natural as possible.

In addition to the requirements of other reproducing or communication equipment as far as volume distortion, harmonic distortion, power-line hum, etc., are concerned, it is also necessary that all intelligence be reproduced at the original frequency. Although a slight shift of all frequencies can be tolerated as long as each frequency is changed by the same factor, it is important that the frequency ratios of all reproduced sounds are not disturbed and that the frequencies themselves do not vary during the time of reproduction. In other words, any frequency (or phase) modulation of the recorded signals due to the process of recording and reproducing should be avoided. In this paper, those causes of such modulation due to imperfections of the design of the turntable mechanism only will be discussed.

In addition to this frequency or phase modulation, amplitude modulation of the signal might be caused also by variations of the frequency response of the pickup over the range of these frequency variations, if the frequency characteristic of the reproducing pickup is not flat. This effect makes the occurrence of these disturbances even more objectionable to the listener, but it will be disregarded in this paper.

This described variation of frequency is commonly called "wow," and a number of papers have been published which discuss this subject.[1-6] A generally accepted definition of "wow" is given in an earlier issue of this publication.[7]

Causes of Wow

The causes of wow discussed in this paper can be classified into three groups: those originating in the turntable itself, those originating in the driving motor, and those originating in the driving mechanism necessary to transfer the motion of the motor to the turntable. In the first group, most variations of pitch can be traced to at least one of the following causes:

The turntable might be tilted from its correct position normal to the axis of rotation, or it might be warped, or its rim might be eccentric, oval, or irregular in shape. The latter cases are serious only when the turntable is rim-driven. It can be seen easily that an eccentricity of the rim-driven turntable produces variations of its angular speed, and these variations of speed produce phase or frequency modulation. The modulating frequency in this case equals the rotating speed of the turntable. Similarly, an oval rim-driven turntable will cause a modulating frequency of twice that of the turntable, and this can be generalized even further for other irregular shapes. These conditions can be treated by simple mathematics, and the results of such calculations are given in the appendix of this paper.

If the turntable is tilted from its normal position, the apparent angular speed of that point of the turntable on which the needle of the tone arm rests varies, because the normal distance of this point from the axis of rotation changes due to the inclination of the turntable. This variation in apparent speed also produces frequency modulation of the reproduced signal at twice the rotating speed of the turntable. In addition to this effect, it should be noted also that the needle point of the tone arm follows the up-and-down motion of the tilted turntable. The distance of the needle point from the tone-arm pivot is fixed, and, therefore, the needle point moves along a circle with its center in the pivot, rather than along a straight line parallel to the axis of the turntable. This causes a tangential motion of the needle point relative to the turntable. It can be seen easily that this relative motion produces phase modulation of the reproduced signal at a frequency of twice the speed of the turntable. This phase modulation adds to the variation of pitch described before, because these two effects are in quadrature. Obviously, the amount of this phase modulation depends also on the length of the tone arm and decreases with an increase of its length. A similar effect takes place when a relative motion between the center of the turntable and the pivot of the

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[1] E. G. Shower and R. Biddulph, "Differential pitch sensitivity of the ear," *Jour. Acous. Soc. Amer.*, vol. 3, part 1, pp. 275–287; October, 1931.

[2] T. E. Shea, W. A. MacNair, and V. Subrizi, "Flutter in sound records," *Jour. Soc. Mot. Pic. Eng.*, vol. 25, pp. 403–415; November, 1935.

[3] R. R. Scoville, "A portable flutter-measuring instrument," *Jour. Soc. Mot. Pic. Eng.*, vol. 25, pp. 416–422; November, 1935.

[4] E. W. Kellog and A. R. Morgan, *Jour. Acous. Soc. Amer.* vol. 7, p. 271; April, 1936.

[5] W. J. Albersheim and D. MacKenzie, "Analysis of sound-film devices," *Jour. Soc. Mot. Pic. Eng.*, vol. 37, pp. 452–479; November, 1941.

[6] H. E. Roys, "The measurement of transcription-turntable speed variation," Proc. I.R.E., vol. 31, pp. 52–56; February, 1943.

[7] H. A. Chinn, "Glossary of disk-recording terms," Proc. I.R.E., vol. 33, pp. 760–763; November, 1945.

tone arm exists. This occurs, for instance, on certain inexpensive phonographs when one of these elements, or both, are shock-mounted. In practice, the last mentioned causes produce considerably less wow than the irregularities of the turntable rim.

Another factor which might influence the wow output of a phonograph turntable is the mechanical unbalance of the turntable itself. Even if the turntable is centered on its shaft and its plane is exactly normal to the axis of rotation, irregularities can be produced by the unbalanced forces if the center of gravity of the turntable is not in the axis of rotation. The effect of this unbalance depends very much on the design of the turntable bearings and is especially pronounced if the bearings are loose. This might, for instance, happen after an extended period of operation when the bearing surfaces are worn. The rigidity of the piece supporting the bearing also has an influence on the irregularities caused by unbalance. It is not possible to calculate this influence under general assumptions because of the differences of the details in the design on different turntables. The amount of unbalance which can be tolerated, therefore, has to be determined separately in each case.

The above-described causes can be remedied only by proper design and suitable manufacturing methods. There is no possibility of reducing the influence of the speed variations of the turntable itself on the reproduced signals. Fortunately, this is not the case with the irregularities produced by the turntable motor and by the drive connection between motor and turntable. It is customary to provide some resilient member as a portion of this drive. This resilient member, together with the mass and the inertia of the turntable, acts as a mechanical low-pass filter, and its cut-off frequency can be chosen so low that it is considerably lower than the lowest wow frequency, e.g., the frequency of the turntable rotation. The cut-off frequency of this filter is determined by the compliance of this resilient member and the moment of inertia of the turntable. The moment of inertia can be easily calculated from the dimensions of the turntable; the compliance of the resilient coupling link can be found best by experiment in most cases.

Due to this filter action, the influence of other wow-producing factors is much smaller than the irregularities of the turntable itself. In the driving motor an electrical or mechanical unbalance of the rotor, or eccentricity of the motor shaft, or of the driving pulley on this shaft, might produce a modulating frequency equal to the frequency of rotation of the motor. This frequency is approximately equal to the line frequency if a two-pole induction motor is used, and approximately equal to one half the line frequency with a four-pole induction motor. These causes of wow can be eliminated by careful design and manufacture of the motor.

The influence of the third group of irregularities, those originating in the driving mechanism which transfers the motion of the motor to the turntable, depends, of course, on its design. If gears are used to reduce the speed of the motor to the turntable speed, the frequency of rotation of intermediate gears might be one of the modulating frequencies; others are related to the number of teeth of each gear, their multiples, fractions, sums, and differences. Usually, only a few of these combinations are predominant over the rest, and in many cases their frequency is high enough to be damped out by the fly-wheel effect of the turntable.

Another possible source of disturbances is ball bearings, where the frequency of disturbance might be related to the frequency of the shaft through the number of balls.

If the turntable is rim-driven through an idler wheel which transmits the motion by friction between the motor pulley and the idler wheel on one side, and the idler wheel and the turntable rim on the other side, the frequency of rotation of the idler wheel might be one of the modulating frequencies, also. This is especially true if the idler wheel is eccentric, or if its circumference is irregular, or if the plane of the wheel is not parallel to the plane of the turntable. This design is used in most phonographs for home use, and it is customary to place a rubber tire around the idler wheel. Also, the aforementioned effects can occur if the elastic properties of the rubber tire vary along its surface, or if there are "bumps" on certain spots of the circumference.

It should be noted that all the disturbances under discussion are periodic but not strictly sinusoidal, but they can always be resolved into a Fourier series in which the fundamental frequency predominates.

The Wow Factor

The general considerations above raise the question as to how wow actually can be determined. But any quantity must first be defined before it can be measured, since the results of the measurement otherwise would be meaningless. From the equations given in the appendix, it seems to be most practical and simple to express the amount of wow present in a particular signal by its peak-frequency excursion given as a fraction or percentage of the average frequency. This definition would have the additional advantage that peak-reading instruments can usually be built more easily than those measuring average or root-mean-square values. On the other hand, for practical reasons it would be desirable to express wow in quantities of its "nuisance value," i.e., frequency fluctuations which sound equally objectionable to the average listener should be characterized by the same value of wow. Unfortunately, the physiological properties of the human ear do not seem to follow the desired simple relations, and additional research would be needed to produce a better definition of wow which also meets the physiological requirements. Shower and Biddulph[1] and Albersheim and MacKenzie[6] show that not only the amount of frequency deviation but also the frequency and rate of change of this frequency deviation, as well as the frequency and the level of the signal which is subjected to wow, have a bearing on this "nuisance value." Some authors propose to use a root-mean-square value of the frequency fluctuation rather than the peak value to characterize the amount of wow, but this opinion seems to be based on theoretical speculations rather than on experimental results. For these reasons, it is suggested that the peak value of the frequency variations be used, and a coefficient which may be called the "wow factor" shall be defined as the value of peak-frequency deviation expressed as a percentage of the average frequency of the signal.

Fig. 1—Front view of the wow meter.

Requirements for a Wow Meter

An instrument which is capable of measuring this wow factor without tedious computations should fulfill the following requirements:

1. It should produce a direct reading of the peak-frequency deviation, preferably directly in terms of per cent of the average frequency.
2. Its reading should not be affected by amplitude variations of a signal containing wow.
3. It should allow the use of an easily available signal source (for instance, a standard 1000-cycle test record for phonographs) and its range of operation should accommodate possible variations of the average rotating speed of the turntable.
4. It should have a "flat" response curve for all wow frequencies, i.e., modulating frequencies which are of interest. This means that, for domestic phonograph turntables which operate at 78 revolutions per minute, the frequency-response curve must be flat from less than 1.3 to at least 60 cycles per second. In this range the phase shift should be zero or, at least, proportional to the frequency, because peak readings are greatly affected by any phase shift of higher-frequency components. For commercial turntables with a speed of 33⅓ revolutions per minute, the low-frequency range ought to be extended below ½ cycle per second in a similar manner.
5. It is desirable that the relation between the value of the wow factor and the meter indication be linear by nature to facilitate calibration of the instrument. This is important, as it is rather cumbersome to produce signals containing exactly known amounts of wow for the calibration of a nonlinear instrument.

Design of a Wow Meter

An instrument designed to measure wow of phonograph turntables operating at 78 revolutions per minute and which meets these requirements is shown in Fig. 1. A block diagram is shown in Fig. 2.

Because the problem of measuring wow is essentially that of detecting the amount of frequency modulation of a low-frequency carrier, the obvious thought was to use the well-known limiter and frequency-discriminator circuits of frequency-modulation radio receivers. Actually, an instrument using one of these circuits has been described by Miner.[8] Unfortunately, all of these discriminators require tuned circuits, and the fact that a "carrier" of only 1 kilocycle is used makes it difficult to design them. To obtain the Q necessary for a satisfactory frequency response, iron-core inductors or transformers must be used whose inductance varies with the current. In addition, fixed capacitors are needed because no variable capacitors are available with a sufficiently high capacitance. Due to these two facts, it is not easy to tune these circuits to the desired frequency and to maintain this tuning adjustment. For this reason, a different system of frequency discrimination has been sought. It is described in detail in a later section of this paper.

An amplitude-limiting circuit is necessary to maintain the indication of the instrument independent of the amplitude of the input signal. A synchronized oscillator with fixed amplitude was found to offer greater advantages when a multivibrator is used than clipper circuits or a sufficient amount of automatic volume control. It can be easily synchronized by a single low frequency and produces a constant output voltage. The fact that its output is a square wave and not sinusoidal is an advantage for the discriminator circuit used.

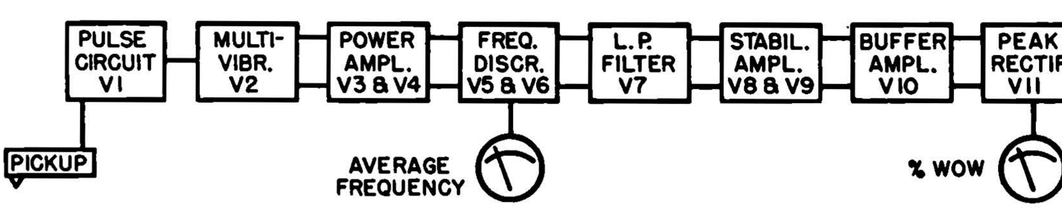

Fig. 2—Block diagram of the wow meter.

The output of this discriminator circuit contains the modulated and modulating frequencies, and a low-pass filter has to be used to suppress the former. The voltage produced by the modulating frequency is measured by a peak-reading vacuum-tube voltmeter consisting of a

[8] C. R. Miner, "Wow meter," *Gen. Elec. Rev.*, vol. 47, pp. 31–34; April, 1944.

stabilized amplifier, a conventional rectifier, and a meter. Fig. 3 shows the circuit diagram of the instrument. As may be seen from this diagram, the signal generated in the pickup of the phonograph is applied to the input terminals of an amplifier stage (V_1) which is overloaded under normal conditions to produce a flat-top output.

A differentiating circuit consisting of capacitor C_3 and resistor R_5 produces a fairly sharp pulse from this flat-top signal, and this pulse in turn is used to synchronize a conventional multivibrator (V_2) at the same frequency. Amplification of the output of this multivibrator by a stage of power amplification (V_3 and V_4) was

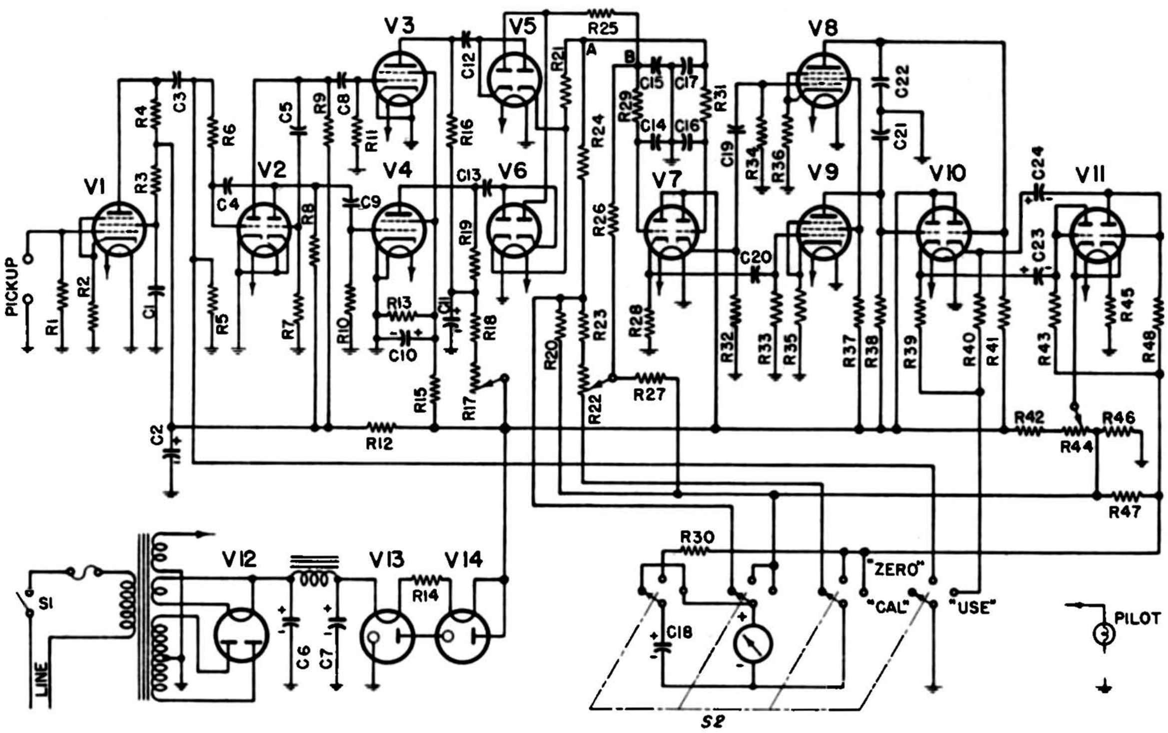

Fig. 3—Circuit diagram of the wow meter.

necessary to produce a signal of sufficient current in the discriminator circuit. For reasons described later, an advantage was seen in using push-pull amplifiers throughout. The square-wave output of this power amplifier is applied to two capacitors (C_{12} and C_{13}) which charge and discharge each through a pair of diodes (V_5 and V_6). These four diodes are connected so that each capacitor charges through one and discharges through the other one of its associated diodes. Due to the push-pull excitation, one of these capacitors charges while the other one discharges, and vice versa. The two charging and also the two discharging diodes are connected in parallel, and each capacitor causes one current pulse per cycle of the input signal to flow through each of its associated diodes. Due to this parallel combination, and the push-pull signal, two pulses per cycle flow through the charging and also through the discharging diodes. This doubling of the signal frequency aids in filtering it from the other lower frequencies. Actually, this is a regular full-wave bridge-rectifier circuit, and the discharging current of one capacitor is the charging current of the other one at the same time.

The current due to each pulse equals the product of the voltage of the square wave produced by the power amplifier and of the capacitance of its capacitor, and the average current through the diodes is, therefore, proportional to the number of pulses and, thus, to the instantaneous frequency of the input signal. This current flows through a voltage divider consisting of resistors R_{21}, R_{24}, R_{20}, R_{27}, R_{26}, and R_{25}, and the voltage across this voltage divider is, of course, also proportional to the instantaneous frequency. The average current is proportional to the average frequency of the input signal, and a highly damped meter measuring this current could be calibrated in terms of cycles per second. An electrolytic capacitor of 1000 microfarads (C_{18}) produces sufficient damping.

Obviously, if the average value of this voltage is proportional to the average frequency of the input signal, and the variations of this voltage are proportional in the same manner to the variations in frequency caused by wow, we can express these voltage variations as a percentage of the average voltage. Thus, we are able to measure the wow present in the input signal. This can be easily done by reducing the voltage of the square wave, and, therefore, the current through the diodes and the voltage divider, until a suitably shunted meter measuring its average value reads, for instance, full scale. To obtain this, a variable resistor (R_{17}) is connected in series with the power-amplifier tubes V_3 and V_4. An increase of this resistance reduces the voltage applied to these tubes and, therefore, also reduces the peak voltage of the square-wave signal produced in this stage. With the switch S_2 in the "CAL" (calibrate) position, the meter is connected across a portion of the above described voltage divider, and the shunt consisting of resistors R_{22} and R_{23} can be adjusted to the desired value to compensate for commercial tolerances of components in other parts of the instrument. This will be described later. This discriminator circuit is essentially a frequency meter similar to the one described by Hunt.[9]

The following signal components will be present between the points A and B of the voltage divider (see Fig. 3): a constant voltage proportional to the average current flowing through the diodes; low-frequency components caused by the variations of frequency of the original signal due to wow; and a group of frequencies of approximately twice the signal frequency, consisting of the frequency-doubled modulation products originally present in the signal containing wow. (These frequencies appear as pulses, and, therefore, also contain a large percentage of their harmonics.) The constant voltage is proportional to the average signal frequency, and the low-frequency components are proportional to the variations of the signal frequency. Therefore, their ratio equals the desired wow factor. The third group of frequencies is not wanted for the measurement of wow and must be suppressed by a low-pass filter before the amplitude of the low-frequency components can be determined. For this purpose, a balanced two-stage resistance-capacitance filter was used, consisting of the voltage divider, the resistors R_{29} and R_{31}, and the capacitors C_{14}, C_{15}, C_{16}, and C_{17}.

It was found necessary to connect a buffer amplifier to the output of this filter, and a twin triode (V_7) was used for this purpose connected as a double cathode follower. The output of this buffer stage is amplified by

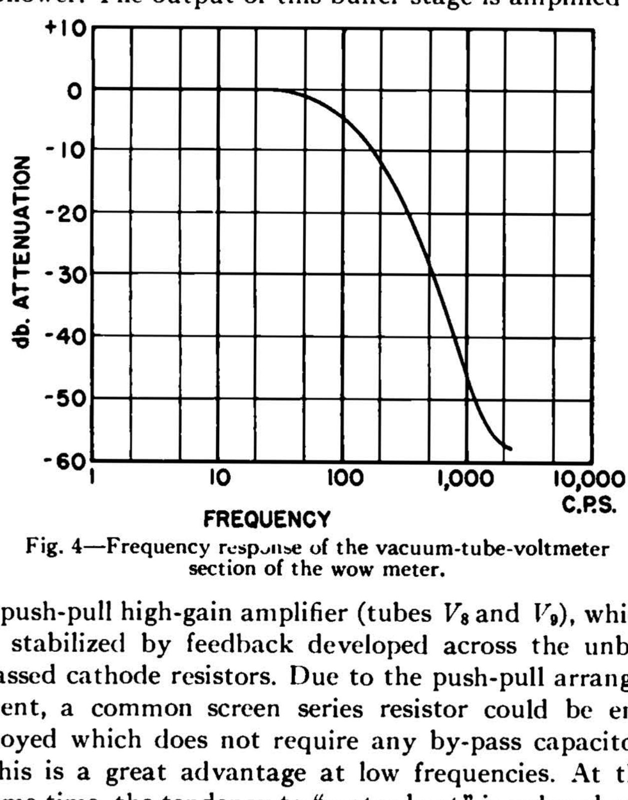

Fig. 4—Frequency response of the vacuum-tube-voltmeter section of the wow meter.

a push-pull high-gain amplifier (tubes V_8 and V_9), which is stabilized by feedback developed across the unbypassed cathode resistors. Due to the push-pull arrangement, a common screen series resistor could be employed which does not require any by-pass capacitor. This is a great advantage at low frequencies. At the same time, the tendency to "motor boat" is reduced and some in-phase feed-back is produced which aids in balancing the output of the amplifier.[10] The output of this amplifier was applied to another buffer amplifier (V_{10}), which is also connected as a double cathode follower. This was necessary to produce a signal at the low-impedance level needed for the peak-voltmeter rectifier tube (V_{11}). (A low-impedance level is required to obtain sufficient current in the indicating meter.)

Some difficulties were encountered in the design of this amplifier because of the low frequency at which it has to operate. High-grade oil capacitors of 12-microfarad capacitance were selected for the coupling capacitors C_{19} and C_{20}. The high-frequency response of the amplifier was further limited by the shunt capacitors C_{21} and C_{22}. The over-all frequency-response curve between the points A and B on the voltage divider and the push-pull output of the amplifier measured at the cathodes of V_{10} is shown in Fig. 4; this diagram shows that the unwanted frequencies near 2000 cycles are sufficiently attenuated, and that the response is "flat" up to approximately 60 cycles. This meets the requirements.

It was necessary to use electrolytic capacitors of 40 microfarads to couple the final rectifier to the stabilized amplifier, and considerable trouble was encountered due to leakage of these capacitors. This leakage produced excessive drift of the zero reading of the final meter. Fortunately, this drift was very slow and could be compensated during short periods by a bucking voltage developed across a portion of R_{44}; this resistor was adjusted so that the voltage drop across it was equal to the voltage drop developed across the rectifier load resistors R_{43} and R_{48} by the leakage currents. This adjustment also compensated for the contact potential developed in the diodes. To permit proper adjustment of R_{44}, a third position of the meter-selector switch S_2 was provided and marked "zero." In this position the condenser C_{18}, which is normally parallel with the meter, is discharged through R_{30} and disconnected from the meter circuit, so that the zero adjustment can be accomplished in a relatively short time.

Attempts were made to eliminate these large coupling capacitors by direct coupling, but without success. It was found that the circuit complications necessary to balance out the drift of the operating point of the tubes usually encountered in direct-coupled amplifiers were not worth the advantage of eliminating the 12-microfarad capacitors (C_{19} and C_{20}), as they did not cause much trouble. Unfortunately, no direct coupling was possible to eliminate the electrolytic capacitors C_{23} and C_{24} because they not only serve as coupling capacitors, but are also an integral part of the peak voltmeter. Their capacitance is determined by the time constant fixed by the low-frequency response of this voltmeter and by the resistance parallel to the rectifier (R_{43} and R_{48}) whose current is fixed in turn by the range of the indicating meter. It was because a direct indication of the wow factor was desired that capacitors of so high a capacitance were needed.

[9] F. V. Hunt, "A direct-reading frequency meter suitable for high-speed recording," *Rev. Sci. Instr.*, vol. 6, pp. 43–46; February, 1935.

[10] F. F. Offner, "Push-pull resistance coupled amplifiers," *Rev. Sci. Instr.*, vol. 8, p. 20; January, 1937.

Because the final amplifier has a constant gain, there exists a definite relation between the current through the voltage divider and the current developed by the output rectifier. The amplitude of the low-frequency components present between points A and B, which produce, say, full-scale deflection of the meter measuring the output current, expressed as a percentage of the current through the voltage divider, is also fixed. It is, therefore, only necessary to adjust the gain of this amplifier to obtain the desired full-scale reading. In practice it has been found simpler to add an adjustable meter shunt consisting of resistors R_{22} and R_{23} to produce full-scale reading of the meter in the "CAL" position, with a current through the voltage divider somewhat larger than calculated. This allows the use of components with commercial tolerances in the feedback amplifier. This meter shunt has to be readjusted only when tubes or other components of the amplifier are replaced. Because the wow factor is exactly proportional to the output current of the peak voltmeter, this adjustment may be made at one point of the scale only; a source of signals containing a known amount of wow is needed, and the meter shunt (R_{22}) is adjusted until the meter reads this known amount of wow after the instrument is calibrated for the average frequency of the signal. This average frequency need not be known.

The instrument is used in the following manner:

After a warming-up period, a phonograph pickup is connected to the input terminals. A standard 1000-cycle record is placed on the turntable and well centered, so that a minimum of wow is produced by the record itself. Test records with worn center holes should not be used.

The selector switch S_2 below the meter on the front panel (Fig. 1) is then set into the "CAL" position and the "calibrate" control adjusted until the meter reads full scale. Due to the long time constant of the meter and its parallel capacitor C_{18}, ten to fifteen seconds are required to make this adjustment. The selector switch is then set to the "ZERO" position and the "zero" control adjusted until the meter needle stays at zero. This, too, requires ten to twenty seconds. The selector switch can then be set to the "USE" position, and after the switching surge dies down, the steady deflection of the needle will indicate the wow factor. In the particular instrument described a full-scale value of 5 per cent wow was selected, and with some care readings can be duplicated within 0.1 per cent wow. It is unfortunate that the time constant of the instrument has to be that long, but this cannot easily be avoided, due to the low frequencies of wow. Despite this disadvantage, the instrument proved very valuable for checking the amount of wow present in phonographs and turntable assemblies.

Although this instrument was designed for phonographs with a speed of 78 revolutions per minute, there is no reason why a similar instrument for $33\frac{1}{3}$-revolutions-per-minute turntables could not be built, except that the difficulties encountered in the design of the amplifier and peak voltmeter would be increased due to the necessary extension of the low-frequency response.

Appendix

Symbols

In the following paragraphs a distinction is made between the quantities derived from the low-frequency motion of the turntable and those derived from the high-frequency recorded signal. The first group of values has been designated by lower-case letters, and the second one by capitals. The following symbols are used with this understanding:

a_n = eccentricity or other measure of geometrical irregularity of turntable, subscript indicating sections of appendix
c = constant peripheral velocity of rim-driven turntable
f = instantaneous frequency of turntable rotation
f_0 = average frequency of turntable rotation
$g_0, h_0, \cdots$ = average frequencies of wow-producing disturbances, other than turntable rotation
n = order of term in Fourier series, also number of irregularities per revolution of turntable
r = variable distance between irregular rim of turntable and its center of rotation
t = time co-ordinate
v = variable angular velocity of rotating turntable
α_n = phase angle in Fourier series for argument f_0 (see above), subscript indicating order of term
β = angle explained by Fig. 8
$\beta_n, \gamma_n, \cdots$ = phase angles in Fourier series for arguments $g_0, h_0, \cdots$ (see above)
θ = angular co-ordinate of turntable with irregular rim also tilt-angle of tilted turntable
ϕ = instantaneous phase angle of turntable rotation
A_n = Fourier coefficients expressing frequency f of rotation of turntable, subscript indicating order of term
$B_n, C_n, \cdots$ = like A_n, but for series with arguments $g_0, h_0, \cdots$, respectively (see above)
E = signal voltage at terminals of pickup in general case
E_n = like E, but subscripts indicating sections of appendix
F = instantaneous frequency of signal
F_0 = average frequency of signal
ΔF = peak value of difference between F and F_0
N = number of cycles of signal per revolution of turntable
R = average (nominal) radius of turntable
T = length of tone arm
W_n = wow factor, subscripts indicating harmonic causing it or sections of appendix
Φ = Instantaneous phase angle of signal.

First, let us consider the irregular motion of the turntable due to irregularities in the turntable itself, without investigating their actual causes for the time being. If we call the frequency with which the turntable rotates f, we can set up the following equation:

$$f(t) = f_0\left[1 + \sum_{n=1}^{\infty} A_n \cos(2\pi n f_0 t + \alpha_n)\right]. \qquad (1)$$

By multiplying with 2π and integrating equation (1), we obtain the total angle ϕ through which the turntable has rotated up to the time t:

$$\phi(t) = 2\pi f_0 t + \sum_{n=1}^{\infty} A_n/n \cdot \sin(2\pi n f_0 t + \alpha_n). \quad (2)$$

In the two equations, the first terms on the right side express the frequency of the turntable or the angle through which it would have moved, respectively, if no irregularities were present, and the second terms express these irregularities. Only periodical variations are considered whose period equals one average rotation of the turntable. This assumption enables us to express these variations in a Fourier series, where A_n means the amplitude and α_n the phase of each individual harmonic. (For convenience, we will start our considerations at such a time that α_0 equals zero.)

If we assume that N cycles of our signal are recorded on the record per revolution, we obtain the frequency F of our reproduced signal

$$F(t) = Nf = Nf_0\left[1 + \sum_n A_n \cos(2\pi f_0 n t + \alpha_n)\right]$$

$$= F_0\left[1 + \sum_n A_n \cos(2\pi n f_0 t + \alpha_n)\right]. \quad (3)$$

If we neglect all disturbances occurring more than once per revolution by breaking off our Fourier series after the first term, we obtain

$$F_1 = F_0(1 + A_1 \cos 2\pi f_0 t). \quad (3a)$$

This equation shows that the maximum frequency deviation equals

$$\Delta F_1 = F_0 A_1 \quad (4)$$

and we obtain our wow factor W_1 for this simplified condition

$$W_1 = \frac{\Delta F_1}{F_0} = A_1. \quad (5)$$

It is not practical to calculate a wow factor for the general case expressed by (3), due to effects of slight changes of the phase angles α_n on the peak value of the deviation, but additional information might be obtained by calculating a wow factor W_n due to the nth term of (3)

$$W_n = \frac{\Delta F_n}{F_0} = A_n. \quad (5a)$$

These wow factors are independent of turntable and signal frequencies. If $\Phi(t)$ is the total phase angle of the signal at the time t, we obtain from (2)

$$\Phi(t) = N\phi(t) = 2\pi F_0 t + \sum_n [A_n/n] N \sin[2\pi n f_0 t + \alpha_n]$$

$$= 2\pi F_0 t + \sum_n [A_n F_0/(nf_0)] \sin[2\pi(nf_0)t + \alpha_n]. \quad (6)$$

Then our signal E produced in the pickup will be proportional to

$$E \sim \sin \Phi t$$

$$= \sin\left[2\pi F_0 t + \sum_n [A_n F_0/(nf_0)] \sin[2\pi(nf_0)t + \alpha_n]\right]. \quad (6a)$$

The appearance of the modulating frequency (nf_0) in the denominator of the second term on the right side of (6) indicates that wow can be identified with frequency modulation of the reproduced signal similar to such modulation in communication applications. This is not the case, however, as conditions here are quite different from those in communication work; there we produce variable modulation of a constant carrier frequency, while here we obtain a constant modulation of a variable "carrier" frequency, our signal. It should be pointed out that we may obtain formally phase modulation or any other kind of angular modulation which has no particular name, if we use a different definition of the coefficients A_n in the Fourier series. Actually, this is of no real importance, as the modulation frequencies and their amplitudes are fixed by the mechanical and geometrical properties of the turntable, and their effects on the signal do not depend on the names or equations used to describe them.

In the same manner, other periodic disturbances can be treated, such as those originating at the motor or the driving mechanism. We would have to make one additional Fourier series to (1) and (2) for each source of

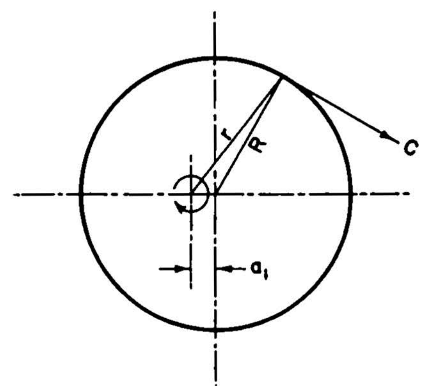

Fig. 5—Eccentric, round turntable.

disturbances of another basic frequency. We use another set of Fourier coefficients B_n, C_n, $\cdots$ and phase angles β_n, γ_n, $\cdots$ for each new series, and note that the new fundamental frequencies g_0, h_0, $\cdots$ are different from the frequency of rotation of the turntable f_0.

A number of simplified conditions will now be treated. Although they cannot be realized fully in practice, they are of interest because they show how the theoretical calculations can be applied to them.

1. Eccentric Turntable Rotating in Its Own Plane

We assume that a distance a_1 exists between the geometrical center of the circular turntable and its axis of rotation (see Fig. 5) and that it is driven at a constant peripheral speed; for instance, by a friction idler wheel making contact at the periphery of the turntable. Due to the varying distance of this point of contact from the axis of rotation, the instantaneous angular speed will also vary and will be inversely proportional to this distance. If we disregard the radial motion of the tone arm due to the spiral shape of the groove, the record will pass under the needle point of the tone arm with a velocity which is, in turn, proportional to the angular velocity of the turntable itself and, therefore, inversely proportional to the distance of the driven point of the periphery and the axis of rotation, provided the record is well centered in regard to the axis of rotation (and not in regard to the center of the turntable disc). If the peripheral constant velocity of the turntable is c, its radius

R, and its eccentricity a_1, then the angular velocity of the record will be proportional to c/r, the maximum velocity to $c/(R-a_1)$, and the minimum velocity to $c/(R+a_1)$. Then we can calculate our wow factor W_1 as follows:

$$W_1 = \Delta F/F_0 = \frac{\frac{1}{2}(F_{max} - F_{min})}{\frac{1}{2}(F_{max} + F_{min})} = \frac{v_{max} - v_{min}}{v_{max} + v_{min}}$$
$$= \frac{c/(R - a_1) - c/(R + a_1)}{c/(R - a_1) + c/(R + a_1)}$$
$$= a_1/R = W_1 \tag{7}$$

and the signal produced in the pickup

$$E_1 \sim \sin\left\{2\pi F_0 t + (\alpha_1/R)(F_0/f_0) \sin 2\pi f_0 t\right\}. \tag{7a}$$

The results of our calculations are approximate only, because we tacitly assumed that the distance r between the periphery of the turntable and its axis of rotation varies exactly sinusoidally, which is not really the case. Actually, this approximation will furnish us the same peak-to-peak deviation of the pitch of the signal. It is, therefore, permissible to use this simplified calculation and neglect the higher harmonics of this variation.

It should also be noted that, in practice, the turntable is always driven through a resilient member whose compliance, together with the inertia of the turntable itself, will tend to reduce the actual variations in angular speed or, in other words, impair the constancy of the peripheral speed. A wow factor smaller than the one calculated should, therefore, be expected in actual measurements.

Equation (7) also shows that an eccentricity of 0.025 inch will produce 0.5 per cent wow on a 10-inch turntable ($R=5$ inches). It is not impossible to maintain tolerances of considerably less than this amount of eccentricity in production.

2. Oval Turntable Rotating in Its Own Plane

To investigate the consequences of an oval turntable, we will assume that the turntable has the shape illustrated in Fig. 6 and is rotated around its center at a con-

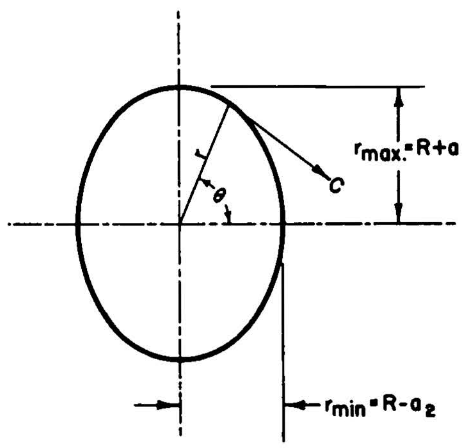

Fig. 6—Oval, rim-driven turntable.

stant peripheral speed c and that the radius of the turntable r varies between the values $R+a_2$ and $R-a_2$ sinusoidally as a function of the angle θ (or, rather, due to the double periodicity of this function, of the angle 2θ). Then we obtain equations equivalent to (7) and (7a) by the same reasoning:

$$W_2 = \Delta F/F_0 = \frac{c/(R - a_2) - c/(R + a_2)}{c/(R - a_2) + c/(R + a_2)} = a_2/R \tag{8}$$

and

$$E_2 \sim \sin\left[2\pi F_0 t + (a_2/R)(F_0/2f_0) \sin 2\pi(2f_0)t\right]. \tag{8a}$$

The same restrictions to the practical application of these equations exist which were found before for (7) and (7a).

3. Unround Turntable Rotating in Its Own Plane

The considerations developed in sections 1 and 2 of this appendix can also be applied to other types of irregularities, such as one occurring at three or four or more equally spaced points on the circumference of the turntable by substituting their linear dimensions in the current manner into (7) and the corresponding multiple of the frequency of rotation of the turntable into (7a), as was essentially done in (8) and (8a). In fact, we can

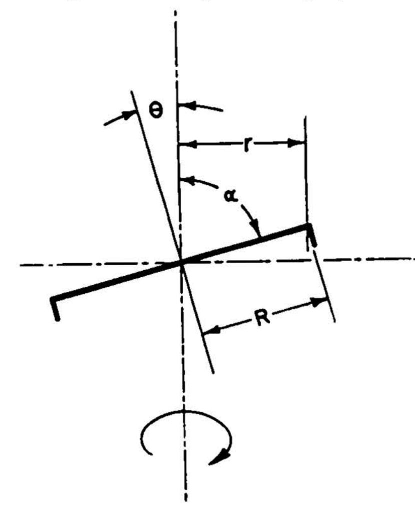

Fig. 7—Tilted turntable.

generalize even further by expressing the shape of the turntable in polar co-ordinates as a Fourier series of the angle with a_1/R, a_2/R, a_3/R, etc., as the Fourier coefficients and α_1, α_2, α_3, etc., as the corresponding phase angles of this series. The signal E_{gen} developed in the pickup will then be expressed by

$$E_{gen} \sim \sin\left\{2\pi F_0 t + \sum_{n=1}^{\infty} (a_n/R)(F_0/nf_0) \cdot \sin\left[2\pi(nf_0)t + \alpha_n\right]\right\}. \tag{6b}$$

4. Tilted Turntable

We assume now that a turntable is tilted so that its perpendicular and the axis of rotation include an angle θ as shown in Fig. 7, and that the turntable is rotated at a constant angular speed. The velocity of a particular point on the record surface passing under the point of the pickup needle varies, then, proportionally to the perpendicular distance of the particular point from the axis of rotation; it will reach a minimum value twice per revolution of the turntable when the needle is in the plane of symmetry of the turntable and its axis of rotation and maximum values after 90 degrees of rotation from each minimum position.

Using again the method which furnished (7) before, we first determine the extreme values of the groove velocity and find

$$v_{max} \sim R$$

and

$$v_{min} \sim R \cos \theta. \qquad (9)$$

These two equations then furnish the following expression for the wow factor W_4':

$$W_4' = (R - R\cos\theta)/(R + R\cos\theta)$$

$$= (1 - \cos\theta)/(1 + \cos\theta) = tg^2 \frac{\theta}{2} \qquad (10)$$

and for the signal voltage E_4':

$$E_4' \sim \sin \left\{ 2\pi F_0 t + tg^2 \frac{\theta}{2} (F_0/2f_0) \sin 2\pi(2f_0)t \right\}. \qquad (10a)$$

This shows that this effect produces wow at a frequency of twice the frequency of rotation of the turntable. This deduction assumes that the point of contact between the point of the needle and the record grooves lies in a vertical plane through the axis of rotation. Actually, the needle point is restricted to move on the surface of a sphere (or along a circle in a vertical plane if the effect of lateral motion of the tone arm is disregarded) as shown in Fig. 8, and, therefore, it moves

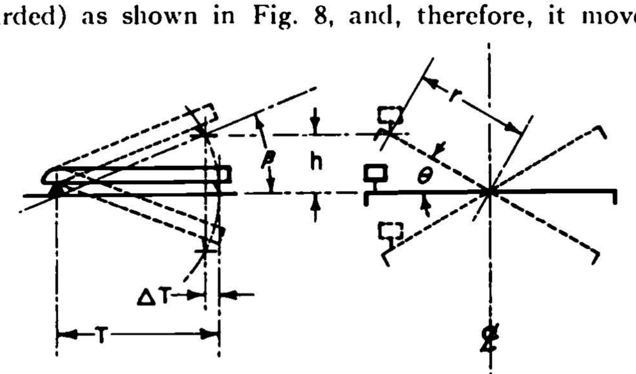

Fig. 8—Motion of tone arm caused by tilted turntable.

back and forth horizontally relative to the record. This relative motion causes an apparent lagging and advancing of the phase angle of the reproduced signal at a frequency of twice the frequency of rotation of the turntable and results in true phase modulation of the signal. Therefore, a different approach is required. This phase modulation is superimposed in quadrature over the frequency modulation calculated before, and can be of the same magnitude.

Fig. 8 illustrates the motion of the point of the tone-arm needle relative to the record when the plane of the record includes an angle θ with a plane normal to the axis of rotation. We see immediately that

$$h = r \cdot \sin\theta = T \sin\beta \qquad (11)$$

or

$$\sin\beta = (r/T)\sin\theta \qquad (11a)$$

and

$$\Delta T = T - T\cos\beta = T(1 - \sqrt{1 - (r/T)^2 \sin^2\theta}). \qquad (12)$$

The relative motion of the tone arm over the distance ΔT can be approximated by the peak-to-peak phase excursion $2\Delta\phi_4''$ on the turntable, measured in radians:

$$2\Delta\phi_4'' = \Delta T/r = T/r - \sqrt{(T/r)^2 - \sin^2\theta}. \qquad (13)$$

This, in turn, corresponds to a peak-to-peak phase shift of the signal of

$$2\Delta\Phi_4'' = 2N\Delta\phi_4''$$

$$= (F_0/f_0)\left[(T/r) - \sqrt{(T/r)^2 - \sin^2\theta}\right]$$

or

$$\Delta\Phi_4'' = (F_0/2f_0)\left[(T/r) - \sqrt{(T/r)^2 - \sin^2\theta}\right] \qquad (14)$$

to a phase angle Φ_4'':

$$\Phi_4'' = 2\pi F_0 t + (F_0/2f_0)\left[(T/r)\right.$$

$$\left. - \sqrt{(T/r)^2 - \sin^2\theta}\right] \cos 2\pi(2f_0)t$$

and to a signal E_4'' proportional to

$$E_4'' \sim \sin\left\{2\pi F_0 t + (F_0/2f_0)\left[(T/r)\right.\right. \qquad (15)$$

$$\left.\left. - \sqrt{(T/r)^2 - \sin^2\theta}\right] \cos 2\pi(2f_0)t\right\}. \qquad (15a)$$

By differentiating (15) and dividing by 2π, we obtain the instantaneous frequency F_4'':

$$F_4'' = F_0\left\{1 - \left[T/r - \sqrt{(T/r)^2 - \sin^2\theta}\right]\sin 2\pi(2f_0)t\right\} \qquad (16)$$

and from this equation the wow factor W_4'':

$$W_4'' = \Delta F_4''/F_0 = T/r - \sqrt{(T/r)^2 - \sin^2\theta}. \qquad (17)$$

The appearance of the turntable frequency f_0 in the denominator of (14) and in corresponding places in (15) and (15a) does not contradict our statement that we are dealing with phase modulation. Rather, it is an accidental coincidence that the relative motion of the tone arm on the record is related to the turntable speed. If we would produce a similar relative motion of the tone arm by other means at another frequency, this other frequency would only appear as the modulating frequency in the argument of the cosine function of (15) and (15a) but not in the denominator instead of f_0. Thus we obtain, also formally, phase modulation.

From (10) and (17), we could calculate the total wow factor W_4:

$$W_4 = \sqrt{W_4'^2 + W_4''^2} \qquad (18)$$

but the resulting formula is too unwieldy for a practical use.

As an example, (10) and (17) will be applied to a turntable which is tilted by an angle $\theta = 10$ degrees away from its normal position. For this case, we obtain from (10) 0.76 per cent wow (W_4'). If we play a 12-inch record on the turntable using a 7-inch tone arm, we obtain from (17) for the outside grooves ($r = 6$ inches) 1.3 per cent wow, and for the inside grooves ($r = 2$ inches) 0.7 per cent wow (W_4''). The over-all wow factor will then be 1.5 per cent and 1.0 per cent, respectively. In practice, manufacturing tolerances for the angle θ can be maintained at considerably smaller values than 10 degrees.

Similar methods can be worked out to calculate wow factors due to other irregularities, and the results of these calculations can be used to specify manufacturing tolerances so that the wow due to each possible cause can be kept below the desired levels.

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