TRACING DISTORTION IN STEREOPHONIC DISC RECORDING

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Introduction

Although the theory of tracing distortion in vertically and laterally cut records has been known for many years, it will be reviewed briefly to show the method of computation and the modifications required for stereophonic recordings. In previous work, the tracing distortion was obtained by a harmonic analysis of the curve traced by the center of a spherical stylus. The amplitudes of the harmonic frequencies were found by (1) finding the coordinates of the curve and making a numerical harmonic analysis[1] or (2) by expanding an explicit expression for the curve in a power series.[2,3] In either event, the amount of work necessary was prodigious when more than one frequency was involved. When the record groove is modulated laterally and vertically simultaneously, the amount of labor required to compute the various distortion components, using a desk calculator, would be too great to attempt. In view of this, it was decided to make the computations by programming an electronic digital computer to use method (1).

No Crosstalk Between Channels

In the 45°-45° stereophonic record system with a groove angle of 90° as shown by Fig. 1,

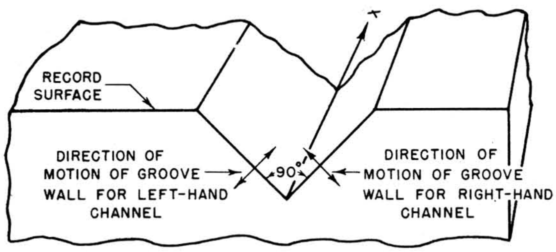

FIG. 1 GROOVE WALL MOTIONS FOR 45°-45° SYSTEM

the outer wall of the groove is modulated by moving it parallel to itself with amplitude proportional to the signal in the right-hand channel. The inner groove wall is moved in a similar manner in accord with the signal in the left-hand channel. If identical signals are fed into both channels, the phasing is chosen so the groove motion is lateral only.

The pickup is designed with two output circuits and the axes of the elements are arranged so each circuit responds to the motion of one groove wall only. If adequate care is taken in the pickup design there will be no crosstalk between channels since the groove walls move independently.

Equations for Stylus Motion

Assume that a cosine wave is recorded in one channel. A cross section of the wall of the groove, looking in a direction parallel to the elements of the cylindrical surface, will be as shown by the solid curve of Fig. 2. It is assumed that the curvature of the stylus is always greater than that of the groove wall, so there is only one point of tangency (x_1, y_1).

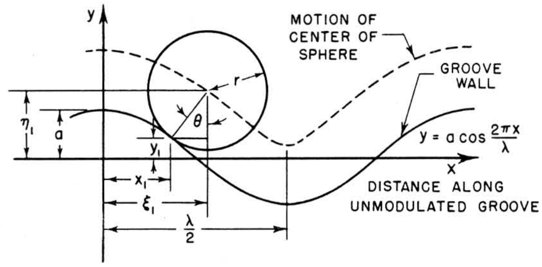

FIG. 2 SPHERICAL STYLUS TRACING A COSINE WAVE

The equation of the center of the sphere (ξ_1, η_1) using the coordinate system shown is

$$\xi_1 = x_1 + r \sin\theta \qquad (1)$$

$$\eta_1 = y_1 + r \cos\theta \qquad (2)$$

where $y_1 = a \cos k\, x_1$ and $k = 2\pi/\lambda$. The angle θ is defined by

$$\tan\theta = - \left.\frac{dy}{dx}\right|_{x=x_1} = - y_1' \qquad (3)$$

so that $\sin\theta = - y_1'/\left[1 + (y_1')^2\right]^{\frac{1}{2}}$ and $\cos\theta = 1/\left[1 + (y_1')^2\right]^{\frac{1}{2}}$.

When the curve traced by a spherical stylus of radius r is an arbitrary function $y = f(x)$, the coordinates of the center of the sphere can be written

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$$\xi_1 = x_1 - \frac{r\, y_1'}{\sqrt{1 + (y_1')^2}} \qquad (4)$$

$$\eta_1 = y_1 + \frac{r}{\sqrt{1 + (y_1')^2}} \qquad (5)$$

where $y_1' = - \left.\frac{df}{dx}\right|_{x=x_1}$.

The coordinates of the center of the sphere for the cosine wave y = a cos kx become

$$\xi_1 = x_1 + \frac{r\, a\, k \sin k\, x_1}{\sqrt{1 + a^2k^2 \sin^2 k\, x_1}} \qquad (6)$$

$$\eta_1 = a \cos k\, x_1 + \frac{r}{\sqrt{1 + a^2k^2 \sin^2 k x_1}} \qquad (7)$$

If equations (6) and (7) are each multiplied by k and the substitutions

$$A = ka \qquad H = \eta_1 k$$
$$R = kr \qquad \Xi = \xi_1 k$$
$$X = kx_1 \qquad (8)$$

are made, the normalized form of equations (6) and (7) are

$$\Xi = X + \frac{RA \sin X}{(1 + A^2 \sin^2 X)^{\frac{1}{2}}} \qquad (9)$$

$$H = A \cos X + \frac{R}{(1 + A^2 \sin^2 X)^{\frac{1}{2}}} \qquad (10)$$

Solution of Equations for Stylus Motion

To obtain a table of corresponding values of Ξ and H, equations (9) and (10) must be solved simultaneously by eliminating X, the normalized distance down the unmodulated groove. H should be tabulated at equidistant steps in Ξ to simplify the numerical harmonic analysis.

Equation (9)

$$X + \frac{RA \sin X}{(1 + A^2 \sin^2 X)^{\frac{1}{2}}} - \Xi = 0 \qquad (11)$$

is solved for X for each value of Ξ by Newton's method[4] of successive approximations. If X_1 is an approximate value of the desired root, Newton's formula for a more accurate value, X_2, for the root of the equation $f(X) = 0$ is

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)} \qquad (12)$$

where X_1 is the first approximation and $f'(X_1)$ is the value of the first derivative at X_1. This formula can be used successively until the desired degree of accuracy is obtained. When X has been found, equation (10) is used to find the value of H corresponding to the assumed value of Ξ. The result is a table of values of H for uniformly spaced values of Ξ as shown by Table I. The increment in Ξ is determined by how rapidly the modulation varies and by the accuracy required.

TABLE I

Ξ	H
0°	—
5°	—
10°	—
15°	—
—	—
—	—

Harmonic Analysis of Stylus Motion

From the table of values of Ξ and H just obtained, the Fourier series representation

$$H = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos n\Xi \qquad (13)$$

where

$$A_n = \frac{2}{\pi} \int_0^{\pi} H \cos n\Xi \; d\Xi \qquad n=0, 1, 2, \ldots \qquad (14)$$

can be obtained by the Newton-Cotes formulas[5] for numerical integration. The evaluation of integral (14) for the harmonic amplitudes was programmed so that the entire calculation was done by the computer. Although equations (9) and (10) were derived on the basis of a single tone, it is obvious that the analysis can be extended to multitone operation by using equations (4) and (5).

Analysis of Vertical-Lateral System

Cross Modulation Obtained

In the vertical-lateral system of stereo-recording, the cutting stylus is modulated vertically and laterally simultaneously. The lateral channel produces a second-harmonic term in the vertical channel because of the pinch-effect. The tones in the vertical and lateral channels beat together to produce sum and difference frequencies in the lateral channel.

Equations for Stylus Motion

The vertical-lateral system can be analyzed in the following manner: Let the vertical and lateral modulation of the cutting stylus be given by $F_1(x)$ and $F_2(x)$ respectively, where x is the distance down the unmodulated groove. The vertical and lateral motions may be resolved into components $f_1(x)$ and $f_2(x)$ respectively perpendicular to the groove walls OB and AO as

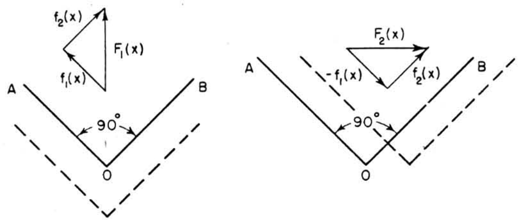

FIG. 3 VERTICAL AND LATERAL MOTION OF CUTTING STYLUS

shown in Fig. 3. In terms of $f_1(x)$ and $f_2(x)$, the vertical and lateral displacements become

$$F_1(x) = \frac{1}{\sqrt{2}}\left[f_1(x) + f_2(x)\right] \qquad (15)$$

and

$$F_2(x) = \frac{1}{\sqrt{2}}\left[- f_1(x) + f_2(x)\right]. \qquad (16)$$

Solving equations (15) and (16) for $f_1(x)$ and $f_2(x)$ gives

$$f_1(x) = \frac{1}{\sqrt{2}}\left[F_1(x) - F_2(x)\right] \qquad (17)$$

$$f_2(x) = \frac{1}{\sqrt{2}}\left[F_1(x) + F_2(x)\right] \qquad (18)$$

By use of equations (4) and (5), the distance from the center of the sphere to the unmodulated sidewalls OA and OB may be found as a function of the distance, x, along the unmodulated groove.

Single Tone in Each Channel: Let the cutting stylus be modulated by the tones

$$F_1(x) = a_1 \cos k_1 x \qquad (19)$$

and

$$F_2(x) = a_2 \cos k_2 x \qquad (20)$$

in the vertical and lateral directions respectively, where a_1 and a_2 are the amplitudes of motion and k_1 and k_2 are the angular frequencies of the tones. Using equations (17) and (18), the equations for the sidewall motion are

$$y_1 = f_1(x) = \frac{1}{\sqrt{2}}\left[a_1 \cos k_1 x - a_2 \cos k_2 x\right] \qquad (21)$$

and

$$y_2 = f_2(x) = \frac{1}{\sqrt{2}}\left[a_1 \cos k_1 x + a_2 \cos k_2 x\right] \qquad (22)$$

The coordinates for the center of the sphere are obtained by substituting equations (21) and (22) into equations (4) and (5). After normalization, the equations of the stylus motion are given by

$$\Xi_1 = X_1 - \frac{RW_1}{\sqrt{1 + W_1^2}} \qquad (23)$$

$$H_1 = A \cos X_1 - \beta A \cos \alpha X_1 + \frac{R}{\sqrt{1 + W_1^2}} \qquad (24)$$

$$\Xi_2 = X_2 + \frac{RW_2}{\sqrt{1 + W_2^2}} \qquad (25)$$

$$H_2 = A \cos X_2 + \beta A \cos \alpha X_2 + \frac{R}{\sqrt{1 + W_2^2}} \qquad (26)$$

where

$$W_1 = - A \sin X_1 + \alpha\beta A \sin \alpha X_1,$$

$$W_2 = A \sin X_2 + \alpha\beta A \sin \alpha X_2,$$

$\alpha = k_2/k_1$, $A = a_1k_1/\sqrt{2}$, $X_1 = k_1x_1$, $\Xi_1 = k_1 \xi_1$, $H_1 = k_1 \eta_1$, $\beta = a_2/a_1$, $R = k_1r$, $X_2 = k_1x_2$, $\Xi_2 = k_1 \xi_2$, $H_2 = k_1 \eta_2$. x_1 is the distance to the point of contact on groove wall OA.

Since the normalized coordinate of the center of the sphere measured along the unmodulated groove is the same for both sidewalls, $\Xi_1 = \Xi_2 = \Xi$. H_1 and H_2 are, therefore each expressible as a function of Ξ. $H_1(\Xi)$ is found by solving equation (23) for X_1 for each successive assumed value of Ξ by Newton's method and substituting this value of X_1 into equation (24). $H_2(\Xi)$ is found in the same manner using equations (25) and (26).

Stylus Displacement: Fig. 4 shows the position

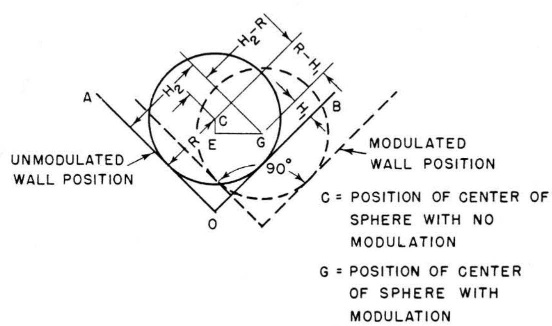

FIG. 4 COORDINATES OF MOVEMENT OF SPHERE

of the groove in the modulated and unmodulated states. In this figure, H_1 represents the distance of the center of the sphere from the unmodulated right-hand groove wall OB at any displaced position. The distance of the center of the sphere from the other unmodulated sidewall AO is given by H_2. Let the quantities D_1 and D_2 denote the normalized vertical and horizontal displacements of the center of the sphere. In Fig. 4 these distances are given by D_1 = CE and D_2 = EG respectively. When expressed in terms of H_1 and H_2 with the aid of Fig. 5, the vertical and horizontal displacements are

$$D_1 = \frac{H_1 + H_2 - 2R}{\sqrt{2}} \tag{27}$$

$$D_2 = \frac{H_2 - H_1}{\sqrt{2}} \tag{28}$$

A harmonic analysis of equations (27) and (28) by the method previously described can be made to find the harmonic and crosstalk distortion in the two channels. To solve for the

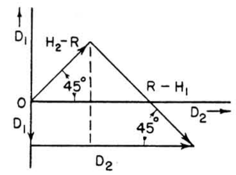

FIG. 5 RELATION BETWEEN H_1, H_2, D_1 AND D_2

harmonic coefficients, the integration indicated by equation (14) must extend over π radians of the difference frequency $k_1 - k_2$, which may be several cycles of each signal frequency.

The analysis described above can be extended to cases where there is more than one tone in each channel. The computation time will increase rapidly, since the integration interval must be one half cycle of the lowest possible beat frequency of the tones involved.

Curves of Calculated Distortion

To obtain curves of harmonic distortion in terms of the tangential groove velocity v and the recording velocity u, the normalized amplitude A and radius R in equations (9) and (10) must be expressed in terms of these quantities. Since the tangential groove velocity for a frequency f is given by $v = \lambda f$ and the amplitude a for a recording velocity u is $a = u/2\pi f$, the normalized amplitude is

$$A = ka = \frac{2\pi a}{\lambda} = \frac{u}{v}. \tag{29}$$

The normalized radius is then

$$R = kr = \frac{2\pi r}{\lambda} = \frac{2\pi f r}{v} \tag{30}$$

Harmonic Distortion in Vertical and Lateral Systems

Using the proper values of A and R in equations (9) and (10), the coordinates of the stylus motion and its harmonic amplitudes have been found for a recording frequency of 400 cycles per second. The percent second harmonic amplitude is shown in Figs. 6(a), 6(b), and 7(a) for recording velocities of 7, 14, and 22 cm/sec for three different stylus radii. These curves correspond to the tracing distortion obtained in each channel of the 45°-45° system using an amplitude sensitive pickup. For a velocity sensitive pickup, the second harmonic amplitude would be multiplied by 2, the third by 3, etc.
The same curves also apply to the vertical channel of the vertical-lateral system when the crosstalk from the lateral channel is less than 0.1 percent. Higher order harmonics which are not shown in these figures have amplitudes which are in the order of or less than 0.1 percent. With a recording velocity u = 22 cm/sec the magnitude of the third component is more than 20 db down from the amplitude of the second harmonic component on an amplitude basis.

The harmonic distortion in a laterally-cut record, recorded at a velocity of 22 cm/sec, is shown in Fig. 7(b) for comparison purposes. In this case the only significant term is the third harmonic component which is more than 20-to-1 down in amplitude from the second harmonic component in the corresponding vertical recording.

Intermodulation Distortion in Vertical and Lateral Channels: The intermodulation distortion with two tones of frequencies 400 and 4,000 cycles in one channel has been plotted in Figs. 8(a) and 8(b) for the vertical and lateral channels.

For a vertically-cut record the intermodulation, I, has been defined as

$$I = \frac{\text{Amplitude of 3600 cycle tone + amplitude of 4400 cycle tone}}{\text{amplitude of 4000 cycle tone}} \tag{31}$$

For a laterally-cut record the intermodulation is

$$I = \frac{\text{Amplitude of 3200 cycle tone + amplitude of 4800 cycle tone}}{\text{amplitude of 4000 cycle tone}} \tag{32}$$

A recording velocity ratio of $u_1/u_2 = 4$ has been assumed for the 400 and 4000 cycle tones so that the amplitude of the 4000 cycle tone for a constant velocity recording is 1/40 of that for the 400 cycle tone.

It is noted that the intermodulation in the vertical channel is more than 10 times that in the lateral channel for the same recording conditions. The intermodulation limitations are the same in the vertical-lateral system as in the 45°-45° system since each uses a vertical channel. To decrease the intermodulation distortion it is necessary to reduce the recording level or the stylus radius.

A close approximation to the intermodulation is given by the formulas

$$I = \frac{800\,\pi\,u_1 r}{v^2}, \text{ (vertical recording)} \quad (33)$$

$$I = \frac{800\,\pi^2 u_1^2 r^2}{v^4}, \text{ (lateral recording)} \quad (34)$$

where

I = percent intermodulation,

u_1 = recording velocity in inches/sec (400 ∼),

r = stylus radius in mils,

v = groove velocity in inches/sec.

Equations (33) and (34) are accurate to within 5 percent at 75 percent intermodulation and within 1 percent at 25 percent intermodulation. These formulas are based on the relation between the harmonic amplitudes and the intermodulation distortion, and on the formulas derived by Corrington.[3]

<u>Combination Tone Amplitudes for Vertical-Lateral System</u>: Combination tones include all possible frequencies, harmonic, sum, and difference, etc., that are produced in a given channel. The curves shown in Figs. 9(a) and (b) were calculated with equal amplitude sine waves of frequencies 400 cycles and 300 cycles recorded in the vertical and lateral channels respectively. Combination tone amplitudes in the vertical channel include the 600-cycle pinch-effect crosstalk from the lateral channel and the 800-cycle second-harmonic distortion generated within the channel. Two recording amplitudes were chosen; these are u = 7 cm/sec and 14 cm/sec.

In the lateral channel the significant distortion frequencies are the sum and difference frequencies of 100 and 700 cycles per second. All other distortion terms are less than 0.1 percent. These distortion components are seen to be larger than the harmonic distortion or the pinch-effect component in the vertical channel.

The curves of Figs. 10 and 11 show the variation of the combination-tone amplitudes when the radius of the stylus is made 0.50 mils and 0.25 mils respectively.

Relation Between Groove Velocity and Record Diameter

The groove velocity was given as the abscissa of the preceding curves so that they would be independent of the particular choice of revolutions per minute. Fig. 12 can be used to find the record diameter corresponding to a given groove velocity.

Conclusions

Each wall of the groove of the 45°-45° system is equivalent to a channel with vertical recording. Since this is a single-sided system there will be second and third harmonic distortion. If the pickup is properly designed, there will be no cross modulation between channels. Curves are given for the harmonic and intermodulation distortion in either channel for various levels and stylus radii. The second harmonic component is the dominant distortion term and is approximately directly proportional to the stylus radius, the recording velocity and inversely proportional to the square of the groove velocity.

In either the 45°-45° or the vertical channel of the vertical-lateral system, the percentage distortion is the same, therefore the distortion limitations are the same for either system when the channels are considered individually.

To keep the distortion as low as in lateral recordings, the level should be lower than that presently used for single-channel lateral recordings, and the stylus radius should be less than the 1 mil commonly used. There is a limit to the allowable reduction in the recording level since the signal-to-noise ratio is also reduced as the level is decreased.

When the vertical-lateral system is used, the distortion will be different in the two channels. The second harmonic of the tone in the lateral channel will appear in the vertical channel because of pinch-effect. The vertical channel will also contain second and third harmonics of the modulation in the vertical channel.

The tones in the vertical and lateral channels will beat together and produce sum and difference tones in the lateral channel. These sum and difference amplitudes in the lateral channel are larger than the pinch-effect term produced in the vertical channel. There will also be third harmonic distortion in the lateral channel, however, this harmonic will be quite small in amplitude compared to the second harmonic in the vertical channel.

No attempt has been made to set maximum allowable limits for the distortion in a stereophonic system. This should be done after extensive listening tests where the levels and stylus radii are varied systematically. On the basis of a limited number of listening tests made to date, it may be that the present limits for single-channel systems are unnecessarily low for a pleasing stereophonic system.

Acknowledgment

The program for the computer was prepared by R. F. Kolar. Without his skill, persistence and efficient use of the computer, it would have been difficult to study so many cases.

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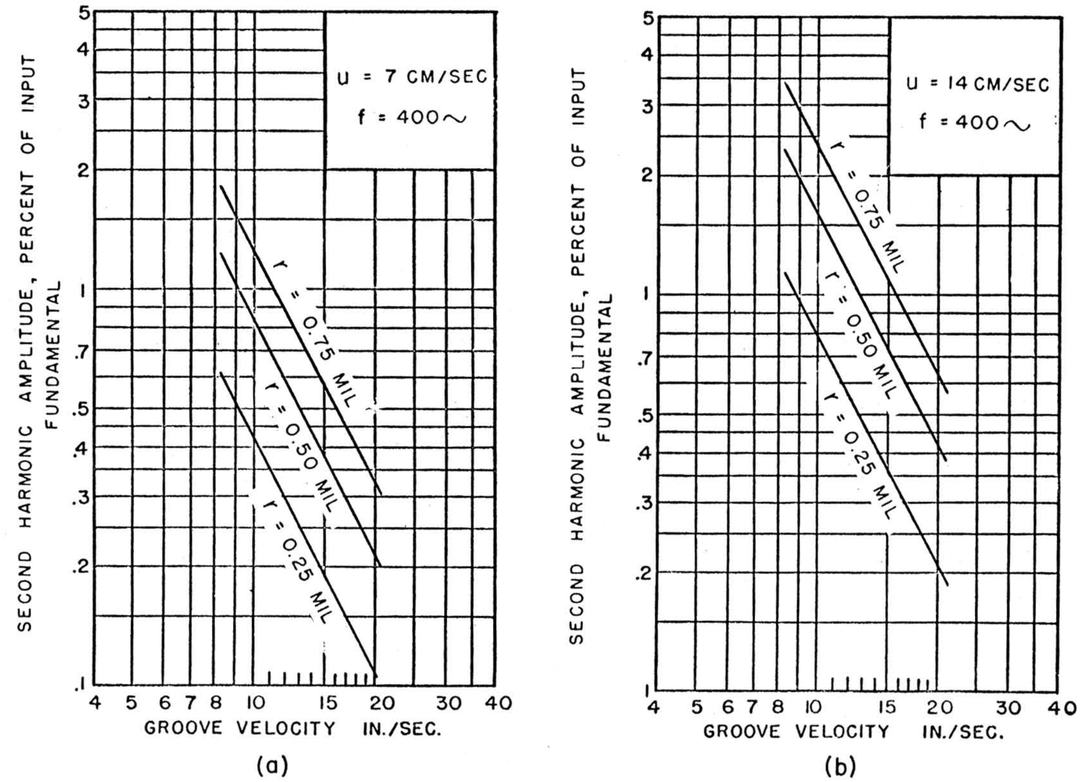

FIG. 6 SECOND HARMONIC AMPLITUDE FOR VERTICAL CUT RECORDING

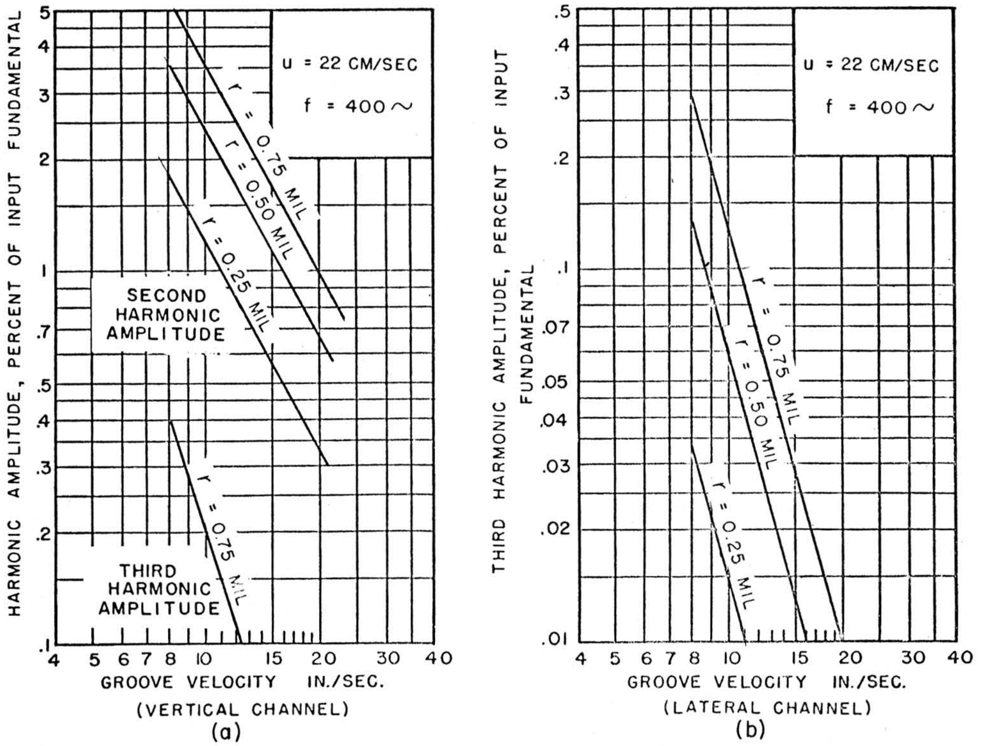

FIG. 7 HARMONIC DISTORTION IN VERTICAL AND LATERAL CHANNELS

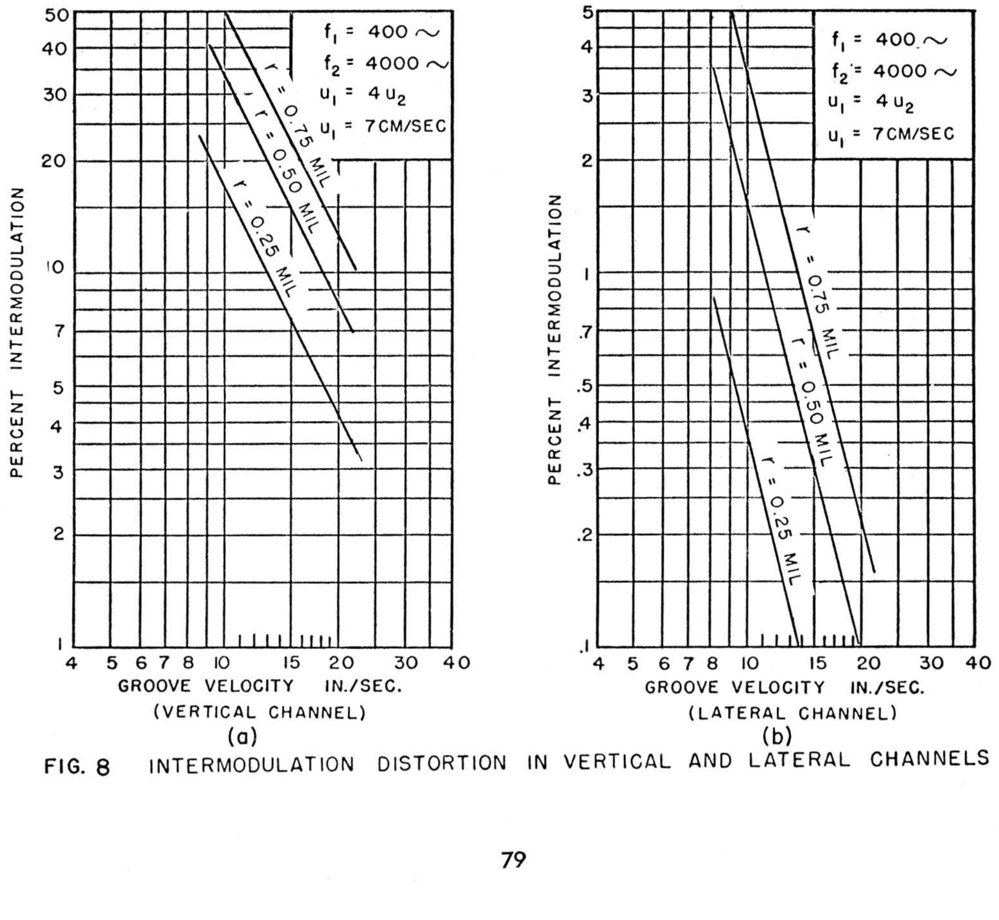

FIG. 8 INTERMODULATION DISTORTION IN VERTICAL AND LATERAL CHANNELS

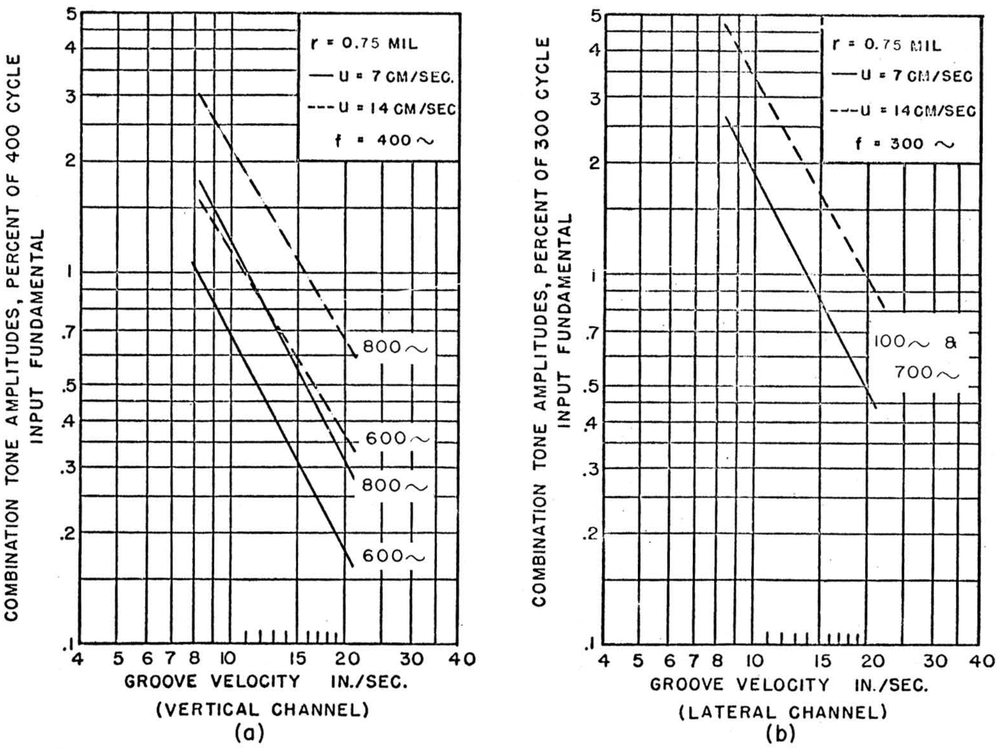

FIG. 9 COMBINATION TONE AMPLITUDES FOR VERTICAL-LATERAL SYSTEM

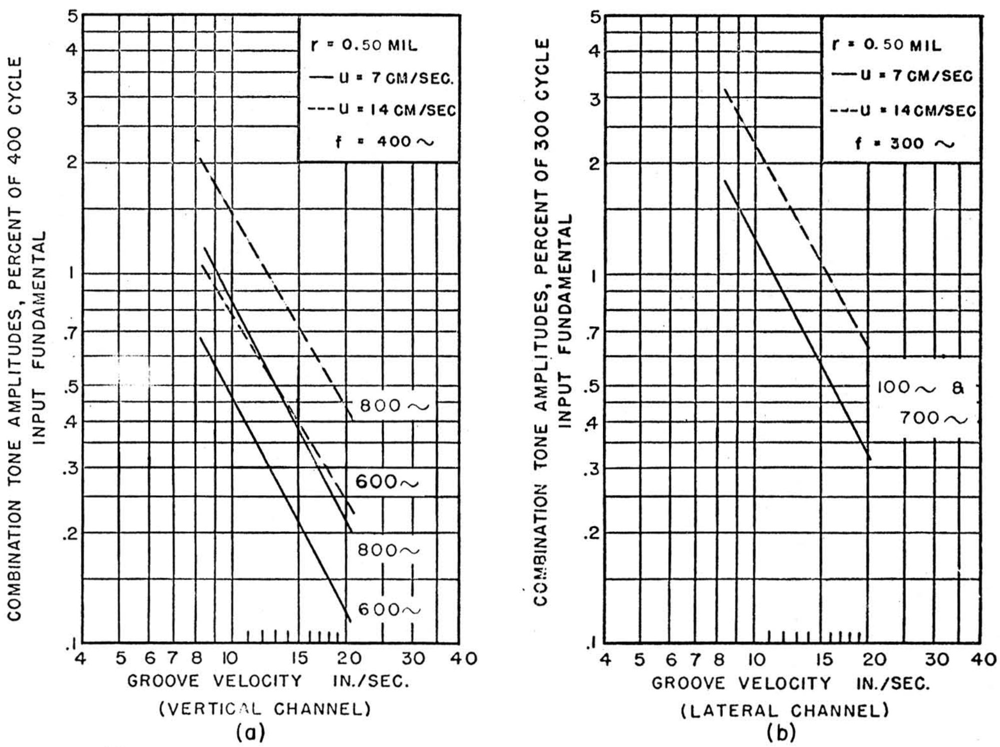

FIG. 10 COMBINATION TONE AMPLITUDES FOR VERTICAL-LATERAL SYSTEM

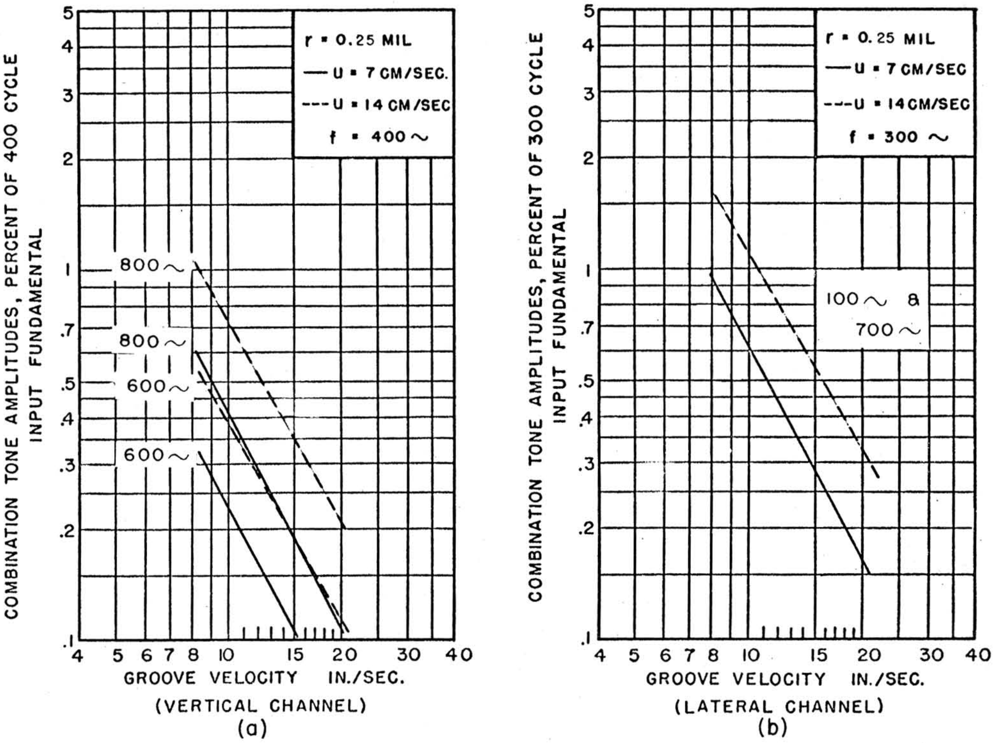

FIG. 11 COMBINATION TONE AMPLITUDES FOR VERTICAL-LATERAL SYSTEM

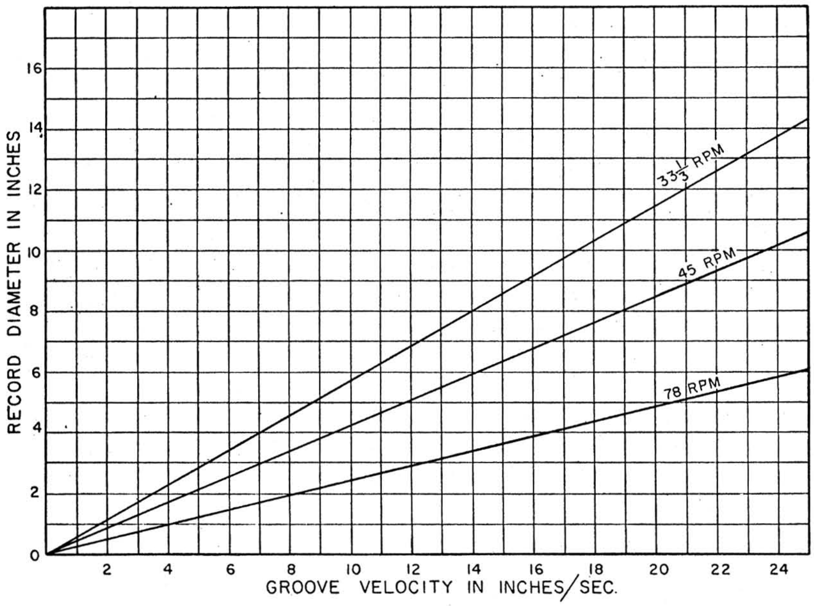

FIG. 12 RELATION BETWEEN RECORD DIAMETER AND GROOVE VELOCITY