Design criteria are presented for L-C and R-C phase shifting networks that develop a constant phase difference over a wide frequency band. The technique is to introduce two phase shifts such that, although they vary with frequency, their difference does not

**Network Development**

One way of producing the two voltages is to derive both of them from a single source and arrange that the output of either channel is independent of frequency, but that both have phase angles with respect to the input voltages varying in such a way that over a wide band of frequencies

\[ \phi_1 - \phi_2 = K \]  

where \( \phi_1 \) : phase angle of output No. 1
\[ \phi_2 \] : phase angle of output No. 2
\( K \) : a constant
\( \phi_1 \) and \( \phi_2 \) are each their own function of \( f \), the frequency

One possible configuration for these functions is

\[ \phi_1 = C + \log f \]  
\[ \phi_2 = C + \log kf \]  

where \( C \) and \( k \) are constants

Substituting Eq. 2 and 3 in Eq. 1

\[ \phi_1 - \phi_2 = C + \log f - C - \log kf \]  
\[ = \log f - \log f - \log k \]  
\[ = -\log k = K \]  

Thus it is only necessary to find a network configuration which will yield a phase angle which varies as the logarithm of the frequency over a wide range of frequency. The network must also have no change in output amplitude with frequency. The latter limitation usually restricts the final network configuration to lattice types because finite ladder types with any phase shift must be accompanied by amplitude variations. In order to avoid the use of transformers in the lattice structure, a half-lattice will be chosen with the input terminals excited by two equal voltages 180 degrees out of phase. Such voltages are readily available from vacuum tube phase inverters consisting of a tube with equal cathode and anode loads.

**L-C Lattice**

A circuit having enough independent parameters to permit the designer to shape the phase angle curve to the required logarithmic form and the basic design equations of the circuit are shown in Fig. 1A. It will be noted from the equation for the phase angle (Fig. 1A) that an arbitrary factor \( s \) is included. The choice of \( s \) is up to the designer.

If \( s \) lies between 3 and 5, a fairly straight line for \( \phi \) is obtained when plotted against a logarithmic frequency scale. As will be shown later, if a second curve for a second similar network, but with a resonant frequency \( f_0 \), which is 4.53 times the resonant frequency of the first curve, is plotted, the phase

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**FIG. 1**—Three networks for producing phase shift. Networks are used in pairs to obtain two outputs which have a constant phase difference over a wide range of frequency. Phase shift equation given for network of Fig. 1A holds for other two networks.
angle difference between these two curves is maintained within $90 \pm 4$ degrees over a fairly wide range of frequency if $s = 4$, and is quite satisfactory for voice frequencies because the upper to lower frequency ratio is approximately 28 to 1, as for example would occur in a band from 130 to 3,600 cps.

The band can be widened indefinitely by adding more elements to the bridge arms with the result that the frequency range is expanded more until remarkably good performance is obtained over the full audio range from 30 to 15,000 cps.

The use of inductances in filters is to be avoided if possible because of the inability to obtain a pure inductance; there are always the inevitable series resistance and shunt capacitance. Other objections to the use of inductors are that they may pick up unwanted signals from stray magnetic fields and that they are not constant in inductance with applied voltage. As a consequence, a resistance-capacitance type of half-lattice network would be preferable providing the required type of phase curve were obtainable from them.

**R-C Lattice**

In order to provide a wide range of operation, two simple resistance-capacitance networks can be used in cascade with isolating tubes between them as shown in Fig. 1B. A single network has restricted bandwidth. No terminating resistor can be used in this type of network.

The phase angle of the final output voltage can be adjusted exactly as in the L-C circuit of Fig. 1A because the phase angle equation is identical. The only limitation with this network is that $s$ must be greater than 2, however as previously found, for best results $s$ should be approximately 4, and hence lies well above 2.

The number of stages included in cascade may, of course, be increased to any desired number. When six such stages are used, the frequency ratio for which a phase angle difference between two sets of networks can be maintained within 90 degrees $\pm 3$ degrees becomes approximately 200 to 1, or what would cover an audio frequency band of 50 to 10,000 cps, which is adequate for high quality broadcast service.

Another R-C network, which does not require an isolation tube, is shown in Fig. 1C. This network somewhat resembles the L-C network of Fig. 1A but differs in output voltage. The output voltage is a proper fraction of the input voltage. A factor $\alpha$ must be less than 0.25, or $s$ must be greater than 2, when using this network. As previously pointed out, for best results $s$ should be approximately 4, or $a = 0.167$, which will yield an output voltage of about 0.33 times the input voltage.

The phase angle equation (Fig. 1A) is seen to be identical to those for the previously described networks. In order to find the proper values for $f_n$ in the two networks which are to yield a phase angle difference of 90 degrees, assume a geometric mean frequency between the upper and lower frequency limits of usable phase angle difference. This mean frequency is not critical, but a value of 700 cps has been suggested as a practical frequency (O. B. Hansen, Down to Earth on 'High Fidelity', Radio Technical Planning Board, Report on Standards and Frequency Allocations for Post-War FM Broadcasting, Panel 5, June 1, 1944, Section VIII). Having decided on the mean frequency of the system, it is merely necessary to design the two phase shift networks so that one has a phase angle of $180 - 45$ degrees at 700 cps, and the other a phase angle of $180 + 45$ degrees at 700 cps.

**Basis for Design**

Suppose the value for $f_n$ of the first network is to be determined, then designating $f_n$ of the first network as $f_n$ and letting $\phi$ equal the mean frequency, designated as $F$, the phase angle equation (Fig. 1) becomes

$$
\tan \phi = \frac{2\omega V_{in} (F^2 - f_n^2)}{(F^2 - f_n^2)^2 - (\alpha f_n F)^2}
$$

Setting $\phi = 135$ degrees and $\alpha = 4$, clearing fractions and solving for $f_n$ gives $f_n = 2.126 F$, which, for $F = 700$ cps, gives $f_n = 1,488$ cps.

By reciprocal relationships, the value of $f_m$, $f_n$ of the second network, is $f_m = F/2.126$; at $F = 700$ cps, $f_m = 329$ cps.

The parameters of the two networks can now be calculated. For convenience and to establish values of impedance into which vacuum tubes can work satisfactorily, assume that $R_0 = 20,000$ ohms in each.
The calculations are summarized in Table I.

The phase shift curves for these two networks are shown in Fig. 2. Note the long frequency range over which these curves are substantially parallel. The difference angle is plotted to facilitate comparison. Note that this difference angle holds fairly close to 90 degrees over a range of frequencies from 130 to 3,600 cps. This range is quite adequate for a voice frequency channel.

Such a network can be used to build a simple single sideband transmitter without the necessity of using sharp cut-off filters and double or triple modulation. This simplification may be done by combining the outputs of two balanced modulators: one balanced modulator is fed with radio frequency \( \cos \omega t \) and audio frequency \( m \cos (\mu t - \phi) \), and the other balanced modulator is fed with radio frequency \( \cos (\omega t \pm \pi/2) \) and audio frequency \( m \cos (\mu t - \phi \pm \pi/2) \). If the plus signs are used and the balanced modulator outputs are added, the resultant output would be \( m \cos [(\omega - \mu)t + \theta] \), which is the lower sideband.

### Table I—Determination of Network Parameters

<table>
<thead>
<tr>
<th>First Network</th>
<th>Second Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 = 1488 \text{ cps} )</td>
<td>( f_0 = 329 \text{ cps} )</td>
</tr>
<tr>
<td>( s = 4.00 )</td>
<td>( s = 4.00 )</td>
</tr>
<tr>
<td>( a = 0.1666 )</td>
<td>( a = 0.1666 )</td>
</tr>
<tr>
<td>( R_1 = 20,000 \text{ ohms} )</td>
<td>( R_1 = 20,000 \text{ ohms} )</td>
</tr>
<tr>
<td>( C_1 = \frac{1}{2\pi f_0 R_1} = \frac{1}{2\pi 1488 \times 20000} )</td>
<td>( C_1 = \frac{1}{2\pi f_0 R_1} = \frac{1}{2\pi 329 \times 20000} )</td>
</tr>
<tr>
<td>( = 0.00535 \mu f )</td>
<td>( = 0.0242 \mu f )</td>
</tr>
<tr>
<td>( C_2 = a C_1 = 0.166 \times 0.00535 )</td>
<td>( C_2 = a C_1 = 0.166 \times 0.0242 )</td>
</tr>
<tr>
<td>( = 0.000892 \mu f )</td>
<td>( = 0.000806 \mu f )</td>
</tr>
</tbody>
</table>

### Single Sideband Telephony

An experimental system of this type has been constructed and operated using carrier frequencies in the broadcast band. No untoward difficulty was encountered and results were fully up to expectations. The transmitter was modulated with audio signals taken from the output of a broadcast receiver. The sideband transmission was received by a second broadcast receiver into whose input terminals was also fed an unmodulated continuous wave radio frequency from a signal generator to furnish the missing carrier. The signal generator frequency was adjusted until the reproduced program sounded most natural. No auxiliary filters were used in the radio section of the transmitter to aid in reducing the unwanted sideband.

The question may arise as to how much of the unwanted sideband is permitted to get through the system if not exactly 90 degrees phase difference is obtained between the two audio channels. The ratio of the weaker sideband to the stronger sideband is given by the equation

\[
\text{RATIO} = \sqrt{\frac{1 - \cos \delta}{1 + \cos \delta}} \cong \frac{\delta}{2}
\]

where \( \delta \) is the deviation of the difference phase angle from \( \pi/2 \), with \( \delta \) expressed in radians for the approximate formula. A deviation from 90 degrees of 6 degrees is required before the weaker sideband becomes 5 percent as strong as the stronger sideband, so that the phase angle curve of Fig. 2 is probably commercially useful over a range in frequencies included between 84 degrees on the low side of the center to 84 degrees on the high side of the center.

A second application for the system is that of providing a closer-to-zero frequency single sideband telephone or broadcast transmission when filters are used. For example, a single sideband transmitter may be constructed along conventional lines with a quartz crystal type of
band-pass filter to operate on 100 kc. Suppose the pass band extends from 100,600 to 108,000 cps. It is seen that frequencies below 600 cps are attenuated in such a system. Now if the single sideband system described in the present paper were to be used in conjunction with a filter, the filter could transmit from 100,000 to 108,000 cps, and the region between 27 and 600 cps could be taken care of by the audio phase shift system, letting the filter remove the undesired sideband from 99,400 down to 92,000 cps. Thus the resultant radiation could contain single sideband components corresponding to an audio frequency range of 27 to 8,000 cps instead of being limited to a range of 600 to 8,000 cps. The addition of the 27 to 600-cps range will add considerably to the naturalness of male speech and organ music. Frequencies between 0 and 27 cps should be removed from the original audio being fed into the system.

**High Efficiency Transmitter**

A high efficiency broadcast transmitter can be constructed by employing three power amplifiers and three antennas arranged so that an unmodulated carrier is radiated on a central antenna and the upper and lower sidebands are radiated respectively on two side antennas, which are on a straight line through the central antenna but on opposite sides equidistant from it. The sidebands are generated in much the same manner as described in the preceding section. Only one set of audio phase-shift filters are required, for the upper and lower sidebands can be obtained by simply adjusting the phases of the radio frequencies fed into the two sets of balance modulators.

This system of transmission could be accomplished with the following installed relative power capacity:

<table>
<thead>
<tr>
<th>Component</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier</td>
<td>1.00</td>
</tr>
<tr>
<td>Upper Sideband</td>
<td>0.25</td>
</tr>
<tr>
<td>Lower Sideband</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1.50</td>
</tr>
</tbody>
</table>

When modulating 100 percent, this system will have an overall efficiency of about 66 percent, resulting in a d-c power consumption of 2.25 maximum. This consumption decreases down to 1.5 for zero modulation. For an average modulation of 50 percent the power consumption would thus be about 1.875.

The conventional double sideband plus carrier class-B amplifier must have an installed power capacity of 4.0 for handling the peaks of 100 percent positive modulation and the transmitter runs at 33 percent efficiency, requiring a continuous input power of 3.0 from the d-c supply. Thus the saving in d-c power consumption would be about 1.125 based on the carrier rating, or for a 50-kw transmitter, the saving would amount to 56.25 kw. Besides the savings in power costs, the system would have lower first costs in the water cooling system, power rectifiers and transformers, and in high power vacuum tubes. The replacement tube cost also would be less.

In the operation of emergency communications transmitters it is often desirable to change the radiated carrier frequency a few hundred or a few thousand cycles to avoid either intentional or accidental jamming, yet it is also desirable to retain the benefits of precise frequency control such as is provided by a quartz crystal. One solution is to have available a large number of crystals lying on closely adjacent frequencies selectable by a rotary switch. Another scheme might be to adjust the air gap or shunt capacitance across the crystal. All of these methods have their objectionable features.

**Adjustable Carrier Frequency**

An alternative arrangement is to make use of the single sideband generator already described. By way of example, suppose the assigned frequency of a transmitter were 4,500 kc. One could then use a crystal oscillator on 4,498 kc in conjunction with the balanced modulator and a 2-kc variable frequency oscillator to produce the required 4,500-kc carrier. The oscillator could then be adjusted to other nearby frequencies to avoid jamming, by having it cover a range of roughly 0 to 4 kc and thus be able to change the radiated frequency plus or minus 2 kc about the assigned frequency.

The frequency stability of the system would be quite similar to the master quartz frequency stability and hence could be classified as precise. Wider ranges of control could of course be obtained by utilizing submultiple crystal frequencies or wider oscillator range, within the limits of stability of the final frequency.

**Frequency Shift Keying**

A transmitter equipped as above could be easily converted into a telegraph transmitter with precise frequency shift keying. By way of example, suppose the 4,500-kc transmitter were required to radiate 4,499.9 kc with the key down and 4,500 kc with the key up. All that would be necessary would be to shift the low frequency oscillator from 2.0 kc to 1.9 kc. This shift could be accomplished by keying in a shunt capacitance, and the resulting frequencies would be both precise and stable, maintaining the 0.1-kc difference quite accurately over long periods of operation.

**Other Applications**

The two 90 degree phase angle displaced voltages can be fed into the X and Y deflection plates of an electrostatic deflection type of cathode-ray tube to obtain a circular trace. The pattern will remain circular for wide changes in frequency if one of these phase-shift networks is used. The circular pattern is a useful indication of the closeness to 90 degrees of the phase shift obtained from these circuits and can be used to determine their performance.

By employing the 90 degree phase angle displaced voltages and a reversed Scott connected transformer, three phase power can be developed from a single phase source. Such a circuit could be used to control the speed of a three phase synchronous motor. Varying the frequency of the original source would vary the motor speed.