RB-60

KINESCOPE ELECTRON GUNS FOR PRODUCING NON-CIRCULAR SPOTS

RADIO CORPORATION OF AMERICA
RCA LABORATORIES
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In some cathode-ray-tube applications, small spot size is a necessity. This bulletin describes two types of electron guns by means of which particularly small spots can be obtained. One type of gun, using a line crossover rather than the conventional point crossover, is suitable for producing narrow, elongated spots. The other type, employing an electron-illuminated aperture as an electron-optical object, can produce sharply defined spots of any desired size and shape. These guns have been found experimentally to produce spots whose current density equals or exceeds that of the best available guns of conventional design.

Introduction

In some cathode-ray-tube applications the spot size required is quite small. This limitation in spot size is sometimes found to be more severe along one axis than along the other. An example of this is the line-screen color kinescope described by Bond, Nicoll, and Moore in 1951.[1] In such a kinescope, a spot not wider than approximately 10 mils is required; the length of the spot however, can be as large as 30 mils or even more, without loss of color purity and without too poor resolution. In such applications, an elongated spot appears desirable.

It is the purpose of this bulletin to describe the development of electron guns which produce on the screen of a kinescope a rectangular or elongated spot of high current density. More generally, spots of any desired shape and high current density can be obtained by means of the guns with aperture limited object to be described further on.

The spot on the screen of a kinescope is the electron-optical image of a cross section of the beam. This cross-section is called the "object" of the gun; it is usually quite small and is in the vicinity of the cathode. The part of the gun by means of which this object is formed will be called the "object-farming system"; the part of the gun by means of which its image is reproduced on the screen is the "imaging system."

In most conventional electron guns, the object-forming system consists of a cathode, located behind an aperture in the electrically negative grid electrode, and a positive accelerating electrode completing the "triode." The conventional gun has cylindrical symmetry, but in some cases electrode apertures may be of other shapes to attain special effects.

The electron imaging system may use either electric fields or magnetic fields, or a combination of both. The choice of the electron lens system is more often an economic compromise than the result of scientific optimization in production kinescopes.

In one type of gun described here, the object-forming system concentrates the electron beam at the plane of an apertured electrode. Spots of any desired shape can be obtained by using a small aperture of this shape as an object. The part of the cathode current transmitted through it forms the spot on the screen as an image of this aperture. Besides the possibility of producing spots of any desirable shape, a gun with an object-defining aperture has two other important advantages:

(1) The dimensions of the object are independent of current,

(2) The position of the object is independent of current.

Thus, the spot size should be less dependent on current in these guns than in guns of the conventional type. This was indeed found to be true.

Fundamental Limitations

The fundamental limitations on the performance of all types of kinescope guns in general are discussed here in order to show how they influenced the conception of the guns described later

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[1] D. S. Bond, F. H. Nicoll, and D. G. Moore, "Development and Operation of a Line-Screen Color Kinescope," *Proc. I.R.E.*, Vol. 39, pp. 1218-1230, October, 1951.

Because of spherical aberations, the highest spot-current density for fixed spot current is obtained when the product $V_o \sin^2\theta_o$ is minimized, V_o being herein the object voltage and θ_o the angle of beam spread at the object. In this process, the object size is assumed constant.

Thermal electron velocities at the cathode determine the lowest theoretically possible value of the product $V_o \sin^2\theta_o$. Hence, the purpose in designing an object-forming system intended to produce a high spot-current density is to minimize the product $V_o \sin^2\theta_o$, to as near the lower limit determined by thermal electron velocities as possible. A more general demonstration of the advantage of making $V_o \sin^2\theta_o$ small is shown in Appendix I.

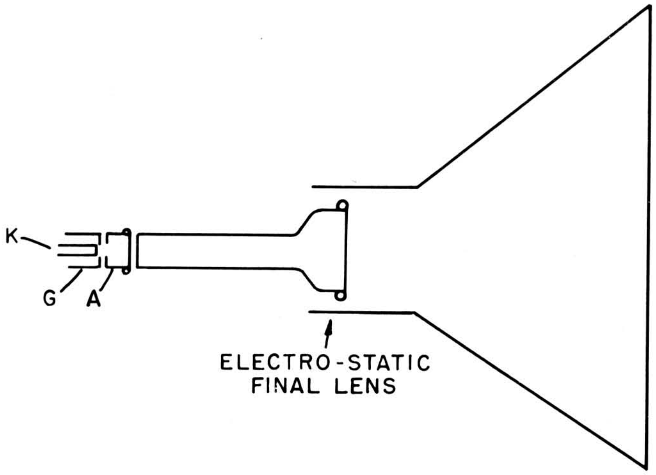
Space charge produces a mutual repulsion of the electrons in the beam; the beam always tends to spread. This process, called "space-charge defocusing," is approximately equivalent in effect to the weakening of the lenses of the imaging system. To compensate for it, a focusing voltage of the imaging system should be readjusted as a function of current.

It can be shown* that space-charge defocusing first occurs in the object space if the object voltage is smaller than the image voltage. As the amount of beam spread is dependent upon the local space-charge density in the beam, space-charge defocusing decreases with increasing beam voltage. Therefore, in order to minimize space-charge defocusing in a kinescope, it is desirable to choose the object voltage, V_o, as high as possible.* If the object voltage cannot be chosen high enough to make space-charge effects negligible, these effects can, to some extent, be compensated for by applying a dynamic focussing voltage to a lens electrode of the imaging system.

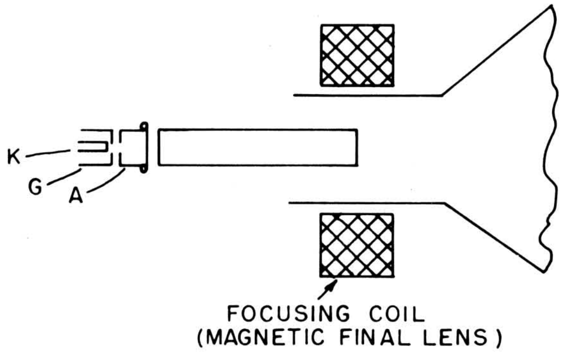
In a gun with an object-defining aperture or other electron intercepting electrode, the heat developed at this electrode due to the intercepted current may also determine the maximum tolerable value of its voltage.

Conventional Kinescope Guns

Typical kinescope guns of what will be referred to as the "conventional" type are shown in Figure 1. The cathode, *K*, grid, *G*, and accelerating electrode, *A*, constitute the object-forming system. The imaging system may be electrostatic as in Fig. 1 (a), or magnetic as in 1 (b). The object-forming system, or "triode," of the conventional gun performs two services – it creates a region of high current density upon which the imaging system can be focused, and it controls the amount of current reaching the phosphor screen. The formation of the "crossover," as this high-density region is called, is possible because of the intense convergent forces produced near the cathode by negative grid potentials. These fields, shown in Fig. 2, form a region of very dense, but not nonuniform flow in fromt of the cathode. The dense region is not at all sharply defined but very **nonuniform** as a result of the absence of perfection in the electron lens and probably to a greater extent the spread in velocities of electron emission.



(a) Electrostatic imaging system



(b) Magnetic imaging system.

Fig. 1 – Conventional kinescope gun.

Despite the fact that at the crossover in a conventional gun, electron flow is highly nonlaminar, this crossover region is small enough to qualify for use as an object in tubes requiring neither an extremely high current density spot, sharpness of the edges of the spot, nor constancy of spot size with current.

The Line-Crossover Gun

The purpose is to produce an elongated spot of high current density. One possibility for this is to alter

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* Appendix II.

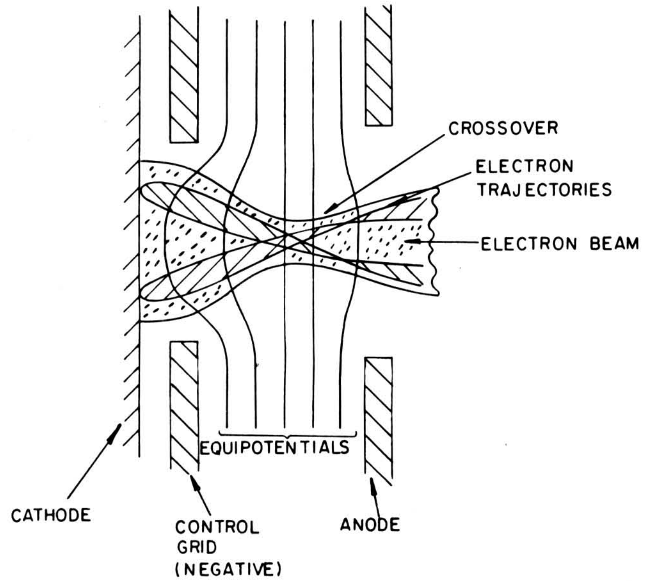

Fig. 2 – Triode system of conventional gun.

the object-forming region of a conventional gun to form, instead of a round crossover, a long linear one. This can conveniently be done simply by changing the round grid aperture to a long slit. Pierce[2] has shown that the current density in such a crossover is slightly greater (other things being equal) than in a round crossover.

By changing to a line crossover, and to optics which are infinite in the direction parallel to the line crossover, an infinite line beam would be obtained. While such a situation offers simplicity of description, a beam of a definite (finite) length is desired. This makes it necessary to (1) limit the crossover length, and (2) provide focusing in the direction of the long axis of the crossover. The second objective, it appears, is most practically met by providing an imaging lens with cylindrical symmetry, since lenses with controlled degree of astigmatism are difficult to build. The function of limiting the length of the line crossover, it has been found, can be performed by the apertured diaphragm of the accelerating anode. With this type of gun structure, shown in Fig. 3, the line length is implicitly controlled by the electrode dimensions and spacings in the "triode" section. The dimensions shown in the figure are suitable to produce a spot of approximately 0.010 X 0.040 inch, with a cathode-to-screen distance of about 21 inches. About 80 percent of the cathode current is intercepted by the first anode. Maximum spot current is about one milliampere.

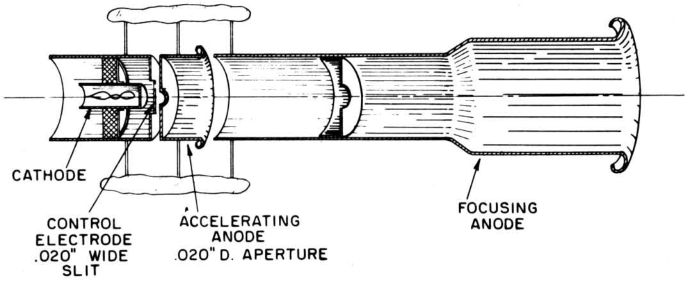

Fig. 3 – Line focus gun.

The Object-Aperture Gun

An alternate possibility of obtaining an elongated spot is to project a very dense beam onto an aperture shaped in the form of the desired spot. The portion of the beam passing the aperture then forms the object of the imaging lens. By this means, spots of any desired shape and having sharply defined edges can be obtained. Elongated spots in particular are conveniently obtained by means of a rectangular object aperture.

This type of gun has a number of advantages: (1) free choice of spot shape, (2) essentially no change in spot size and shape with current, and (3) ability to utilize a larger cathode area, and thus attain high spot current at reasonable cathode loading. The disadvantages are: (1) the attainment of small spot size requires great gun precision (but perhaps no more than an *equivalent* conventional gun), (2) considerable power may be dissipated by the object electrode, from current which does not pass through the aperature, and (3) more electrodes are generally needed. For applications where spot size, shape, and maximum current are of primary importance, this type of gun may be justified.

Since the work of Law,[3] who was the first to our knowledge to investigate guns of this type in 1937, several techniques have come into use in beam type tubes. One of these is the Pierce[4] gun another the impregnated cathode.[5] Gun construction techniques used for traveling-wave and similar tubes are of importance in achieving the necessary accuracy. It was felt that by putting together some of these advances in gun techniques, an improved object-aperture gun would be possible.

In the following sections, the design considerations and limitations for object-aperture guns are dis-

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[2] J. R. Pierce, "Limiting Current Densities in Electron Beams," *Jour. Appl. Phys.*, Vol. 10, pp. 715-724, October, 1939.

[3] R. R. Law, "High Current Electron Gun for Projection Kinescopes," *Proc. I. R. E.*, Vol. 25, p. 954, August, 1937.

[4] Described in J. R. Pierce, *Theory and Design of Electron Beams*, D. Van Nostrand Co., Princeton, N. J., 1949.

[5] R. Levi, "New Dispenser Type Thermionic Cathode," *Jour. Appl. Phys.*, Vol. 24, p. 233, February, 1953.

cussed in detail and the construction of some experimental guns explained.

Object-Forming System

The object-forming system consists of the cathode the object aperture, and the intervening region. As a consequence of spherical aberrations, the essential aim in the design of an object-forming system is to reduce the product $V_o \sin^2\theta_o$ for given object size and spot current to as near the lower limit determined by thermal electron velocities as possible. How this determines the structure of the object forming systems will be shown in the next section. It is assumed in following sections that cylindrically symmetrical object-forming and imaging systems are to be used; our experience indicates that other types of lenses are almost prohibitively difficult to build. The precise parameters of such an object-forming system are governed to a considerable degree by secondary requirements not apparent in the general relations of Appendix I and Fig. 10. Thus,

(1) The *shape of the object-defining aperture* is determined by the required spot shape. In our experiment, elongated spots were desired; therefore, rectangular object defining apertures were used.

(2) The *size of the object-defining aperture* (parameter y_o in Fig. 10) is chosen as small as possible. Small object dimensions are desirable in order to keep the gun short. (This can be deduced from Eqs. (3) and (5), when y_s, V_s, and V_o are specified.) The minimum practical object dimensions are limited by the mechanical difficulty of making and aligning small apertures. Local bombardment heating must also be taken into consideration.

(3) The *cathode current density*, j_c Eq. (1), indicates it is clearly best to use the maximum cathode current density compatible with good tube life. This is limited by the cathode material, and by the ability of the cathode to replenish its active layer.

(4) The *current I_o transmitted through the object-defining aperature* is approximately equal to the screen current, I_s, when the current lost in the imaging system is kept negligible (as it must be to achieve most efficient operation). The screen current is a quantity which we may choose to attempt to maximize; its minimum usable value is fixed by the minimum picture highlight brightness required in the finished kinescope.

(5) The *object voltage*, V_o, is chosen as high as possible in order to minimize space-charge defocusing (see Appendix II). An upper limit to V_o is usually set by the heat developed at the object-defining aperture due to the intercepted current. Thus the value of V_o is a compromise between the limitations imposed by space-charge defocusing and heat dissipation at the object defining aperture.

(6) The *cathode current*, I_c, is determined by the value of I_o and the transmission ratio I_o/I_c of the object-forming system.

(7) The *transmission ratio* I_o/I_c is made as large as possible in order to keep the intercepted current, the cathode current and the cathode diameter corresponding to given values of j_c and I_o as small as possible. This is normally achieved by making the (round) beam diameter at the object-defining aperture about equal to the largest dimension of this (rectangular) aperture.*

The last condition is usually rather readily satisfied, the simplest method being to focus the beam to a crossover at the plane of the object defining aperture.

Design of the Object-Forming System

Optimum design, as previously indicated, requires that the value of the product $V_o \sin^2\theta_o$ be minimized. It was noted that the value of V_o cannot be chosen arbitrarily; its optimum is usually determined from a balance of power-supply and aperture-heating limitations. Thus, in designing an object-forming system, the direct problem is to minimize the angle of beam spread, θ_o, at the object-defining aperture for given object voltage, V_o, object size y_o, and transmitted current, I_o. Several electrode configurations were tested.

A simple solution might appear to be a convergent Pierce gun with a small angle of convergence, θ_c (essen-

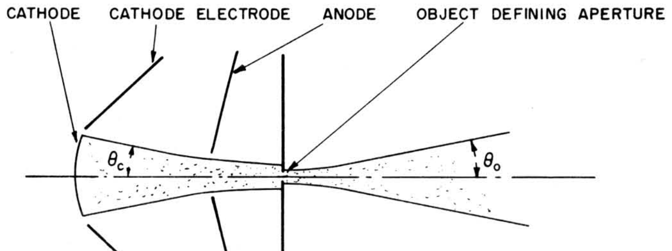

Fig. 4 – Convergent Pierce gun with object-defining aperture.

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*Further improvements might be obtained by means of astigmatic focusing before the object-defining aperture if one of its dimensions appreciably exceeds the other dimension.

tially what Law[3] used). In this case, the smallest beam cross section (crossover) would be placed at the object defining aperture itself, as shown in Fig. 4. Because the flow is not truly laminar, it was found impossible to make $\theta_o < \theta_c$; in all cases $\theta_o \simeq \theta_c$. Thus, one must minimize the angle of convergence, θ_c, of the Pierce gun (parallel flow was ruled out by the excessive cathode current density implied by the desired values of I_c and y_o). In actual designs, with the relatively high cathode current required (in this case of the order of 3 milliamperes), small angles of convergence lead to excessively high object voltages. Further, since the "cutoff voltage"* of a Pierce gun increases with decreasing angle of convergence (for given cathode diameter and beam current), too high cutoff voltages are obtained. Hence, in order to keep the cutoff voltage reasonably small and the object voltage at the desired value, a relatively large angle of convergence, θ_c, appears desirable at the cathode.

A preliminary lens combination used to form a dense beam at the object aperture consisted of the convergent-field Pierce gun followed by a decelerating then an accelerating region. The anode voltage of the Pierce gun and the object voltage were almost the same. The intervening lens, of the Einzel** type, served to overcome the space-charge forces, to extend the cathode-object distance and hence slightly reduce the effective θ_c, and finally to compensate for slight variations in the focusing of the Pierce optics. In this concept, no true crossover was expected within the object-forming optics. The relatively large size of the beam in the Einzel lens region resulted in interception on the low-voltage electrode. Increasing the aperture diameter produced undesirable results as indicated by a less favorable value of the "quality factor," v (defined in Appendix I, Eq. (7).

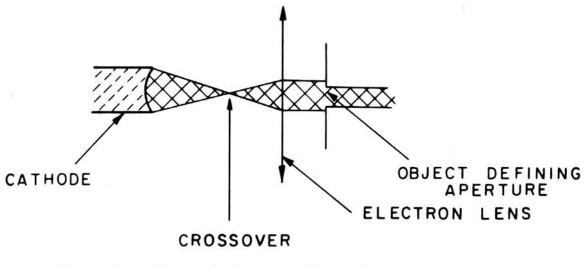

Fig. 5 – Principle of object-forming system.

Study of this type of action, limited as was most of the work, to experiment, was halted when better operation was obtained with a system operating in the mode shown in Fig. 5. The elements used in a structure of the type are physically realizable (convergent lenses only), and successful results have been obtained with guns of that type.

In the object-forming system of Fig. 5, a crossover is formed between the cathode and a lens focused on this crossover. The crossover is considered, in first approximation, as a point source of electrons; thus, the beam emerging from the lens and passing the object defining aperture is essentially parallel. Actually, the finite size of the cross-over and the aberrations of this lens produce some beam spread; the theoretical* minimum angle of beam spread thus obtained is not smaller than the angle obtained with a convergent flow gun converging directly at a small angle on the object aperture. The advantages of the structure of Fig. 5 are the lower object and cutoff voltage possible for given cathode current and cathode diameter.

The crossover, in a gun of the type of Fig. 5, can be produced either by means of an immersion lens** system similar in principle to the systems by which the crossover is formed in conventional guns, or by means of a convergent laminar flow produced, for instance, with a Pierce-type gun. Fig. 6 shown an object-forming system with an immersion lens crossover. This crossover is formed between the first electrode, G_1 (which is negative)

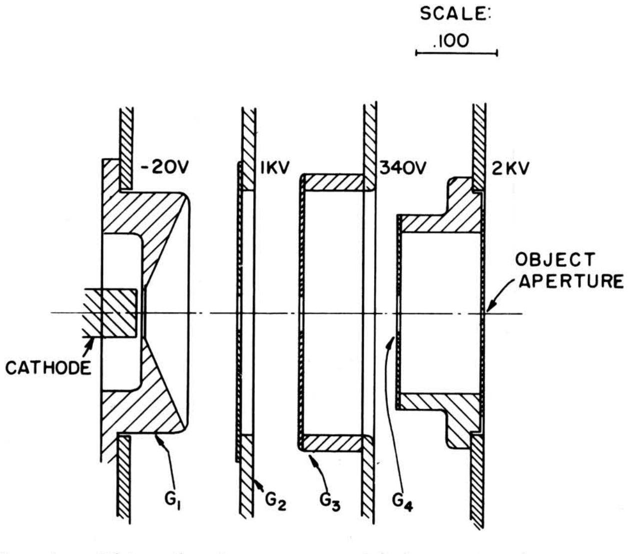

Fig. 6 – Object-forming system with immersion lens cross-over.

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*With all apologies to Dr. Pierce, the beam *was* modulated by the electrode surrounding the cathode. While this destroys the laminar flow, it also effectively cuts off the beam current. Deep electrodes produce cutoff at less negative control voltage than shallow ones, since the accelerating fields do not penetrate to the cathode so readily.

**An Einzel, or unpotential lens, is one consisting of three electrodes, the outermost pair at the same potential. The inner electrode may have either higher or lower voltage.

*The theoretical minimum angle of beam spread is the angle of beam spread obtained in an ideal system limited only by thermal velocities (Equation 1).

**Defined as a lens in which the object (in this case, the cathode) is immersed in the lens proper.

and the second electrode, G_2 (which is highly positive). The three electrodes G_2, G_3, and G_4 constitute a lens focused on the crossover which lies between G_1 and G_2; the lens action is obtained by keeping G_3 at much lower potential than G_2 and G_4. Typical potentials are indicated on Fig. 6.

Another method of forming the crossover between cathode and object aperture is to use a convergent Pierce gun. Fig. 7 shows a design of this type.

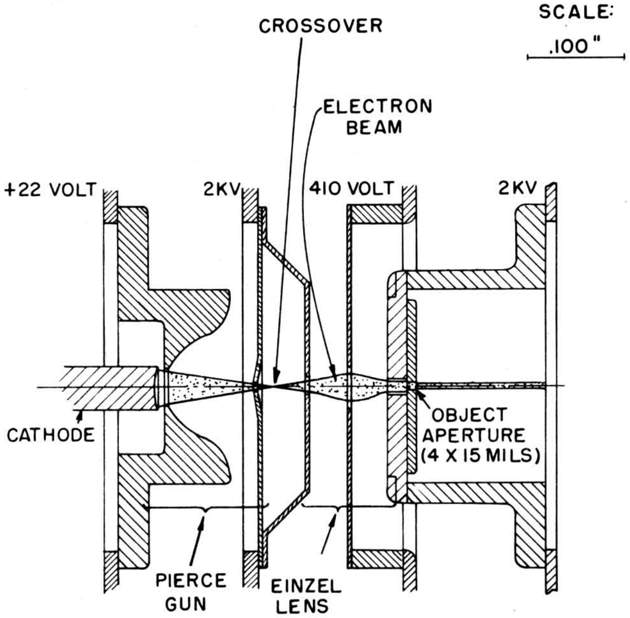


Fig. 7 – Laminar-flow object-forming system.

The Imaging System

Before describing a specific imaging system, let us list the parameters of the imaging systems (left side of Fig. 10) and consider how they may best be chosen.

(1) *Lens type* – Preliminary experience showed that the most adequate lenses within practicality were electrostatic cylinder lenses whose cylinders approached in diameter that of the neck of the envelope. Magnetic lenses were abandoned because of the excessively large magnification which they presented; this would have decreased the object size or increased the tube length, neither of which was practical.

(2) L_s, the *lens-to-screen distance* is determined by deflection requirements and screen size. It is kept as small as possible in order to minimize the tube length, the limitation imposed on the screen current density by thermal velocities (Equation (1)), and space-charge defocusing; indeed, the smaller L_s, the larger the angle of beam convergence at the screen, θ_s.

(3) y_s; the *actual spot dimensions* are determined by picutre requirements. (The maximum tolerable spot size can be determined, e.g., by the minimum tolerable resolution.) y_s is a quantity one may choose to minimize.

(4) Δy_s which measures *spherical aberrations* could clearly have (from Equation (4) any value from 0 to y_s. Due to the loss of current density in the spot for large Δy_s and the difficulty of attaining small Δy_s, a practical compromise of $\Delta y_s \simeq .2\, y_s$ was chosen.*

(5) M, the *magnification*, is determined once y_o and $y_s = y_s - \Delta y_s$ have been chosen. However, in order to keep the gun as short as possible, one must avoid too small values of M. This influences to some extent the choice of y_o; further considerations relevant to this choice have already been expressed in the design procedure of the object-forming system.

(6) V_s, the *screen voltage*, should be as high as possible in order to minimize the effect of thermal velocities (Eq. (1) applied to the screen) and maximize the highlight brightness. The limit is determined by circuit and safety (including X-ray hazard) requirements.

(7) I_s, the *screen current* is determined by the consideration already expressed in the design procedures of the object forming system in relation with I_o.

(8) V_o, the *object voltage*, is determined in designing the object forming system.

It is seen that the screen (image) voltage, the object voltage, the distance from final lens to screen, L_s, and the magnification, M, usually are determined by the various considerations just expressed.

In a *single-lens* system, once the distance from the lens to the screen, the image voltage, and the object voltage are prescribed, the position of the object plane

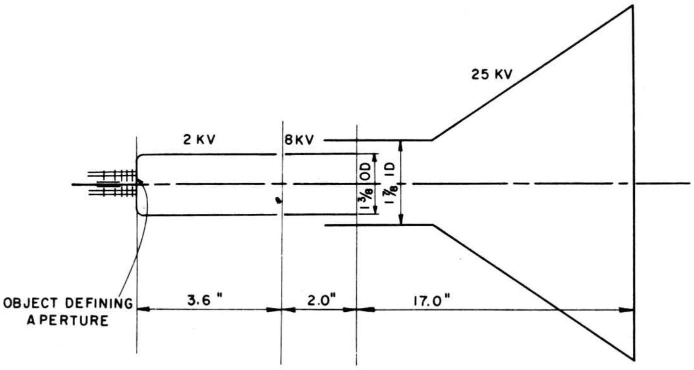


Fig. 8 – Image-forming system.

**An unpublished paper by E. G. Ramberg of these Laboratories shows that, in an elongated spot the maximum current is obtained when $\Delta y_s \simeq .35\, y_s$. Our choice produces slightly less current, but a sharper edged spot.

and the magnification are determined. Usually, the magnification thus obtained does not correspond to the magnification required. In this case, one more degree of freedom is necessary; this is obtained by an additional electron lens.

As an example, a typical electrostatic double-lens imaging system is shown in Fig. 8; this system can be used with the object-forming systems of Fig. 6 or 7.

Test Procedures

Much effort is saved by testing experimental object-forming systems separately from the imaging system. For this purpose, the following quantities have to be measured at a given object voltage:

(a) The cathode current, I_c,

(b) The screen current, I_s,

(c) The angle of beam spread, θ_o, at the object.

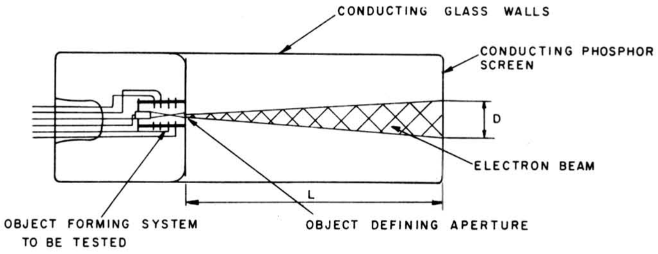
These quantities are conveniently measured by sealing the object-forming part of the gun into a short bulb as shown in Fig. 9. Care must be taken, by proper electrostatic shielding, to prevent secondary electrons from interfering with the measurement of I_c or I_s. (Such secondaries may be emitted from either the screen or the object aperture.)

In the test of an object-forming system by this method, one measures the transmission efficiency, I_s/I_c, and the angle of beam spread, θ_o, for a given object voltage and screen current as a function of electrode voltage, and calculates the ratio v (Eq. (7)), which serves as a "quality factor" for the object-forming structure.

The performance of the imaging system can be tested in a spot magnifier of the type described by Beam.[6] The performance of a complete gun with object forming and imaging system can of course also be conveniently tested by this method.

Experimental Results

As a typical example, results obtained with a complete gun in a 19-inch (screen diameter) kinescope are presented below. This gun consists of an object-forming system of the type shown in Fig. 6, and of an imaging system of the type of Fig. 8. The object consists of a rectangular aperture of 4 X 15 mils; the magnification is about two times.



$\eta = \frac{I_{CATHODE}}{I_{SCREEN}}$; $\tan\theta = \frac{D}{2L}$

Fig. 9 — Test of object-forming systems.

Typical performance data of this gun:

Screen voltage	25 kv
Object voltage	2 kv
Maximum screen current I_s	1 ma
Cathode current density for $I_s = 1$ma	180 ma/cm^2
Cathode current for $I_s = 1$ma	3.2 ma
Spot size, approximately	10 X 35 mils
Object size	4 X 15 mils
Control grid voltage at $I_s = 1$ ma	−40 volts
Control grid voltage at cut off	−150 volts
Angle of beam spread at the object for $I_s = 1$ ma	$\tan\theta_o = 0.045$

It was observed that for screen currents exceeding 0.5 milliampere, space-charge defocusing became apparent, and sharp focus required readjustment of the focusing voltage by 300 volts at 1 milliampere screen current. It should be pointed out that this performance was obtained with the imaging system of Fig. 8, which had not been designed for maximum focusing sensitivity.

Life of guns of this type was found to be short because of over-heating and out-gassing of the object aperture during operation. This was a consequence of the heavy electron bombardment to which it was subjected. Adequate processing and design should overcome this shortcoming, as similar technological difficulties have been successfully solved in previous tubes with bombarded object aperture, as well as in power traveling-wave tubes.

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[6] W. R. Beam, "A New Method for Magnifying Electron Beam Images," *RCA Review*, Vol. XVI, pp. 242-250, June, 1955.

The performance of the gun described above can be compared with the performance of conventional kinescope guns by defining a gun "quality factor," q:

$$q = \frac{j_{\max}}{j_{\text{theor.}}},^{*}$$

where $j_{\max}$ = maximum measured screen current density,

$j_{\text{theor.}}$ = maximum theoretically possible screen current density for the cathode current density at which $j_{\max}$ is measured.

The value of $j_{\text{theor.}}$ can be expressed as a function of the screen voltage, V_{sc} and of the angle of beam convergence, θ_{sc} as follows, assuming thermal velocities to be the factor theoretically limiting.

$$j_{\text{theor.}} \simeq j_{\text{cath}} \times 10 V_{sc} \sin^2 \theta_{sc}$$

for a cathode temperature of 1160 degrees K. Hence

$$q = \sqrt{\frac{j_{\max}}{j_{\text{cath}}}} \times \frac{1}{3.2\sqrt{V_{sc}}\,\sin\theta_{sc}}.$$

With the gun described here, a value of $q = 0.5$ has been obtained. (This is at least as good as with the best gun of conventional design available for test.) Further, up to 500 microamperes, the spot size remained practically constant.

Cathode life problems, and other difficulties encountered in production have not been explored in this developmental program.

Conclusions

In this bulletin the development of guns operating on principles hitherto not fully explored has been described. Their performance has been found comparable with that of some of the best conventional guns except for life. The limitation on life is not inherent and should be straight forward to overcome. The additional advantages of the types of guns described here may bring them into their own in applications where spot size and shape are of primary importance.

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R. Knechtli

Walter R. Beam

Walter R. Beam

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*q differs from v (defined in Appendix I) in being a factor for the *entire* gun, whereas v applies only to the object-forming system.

Appendix I

In this appendix, justification will be given for the paths taken in the design of a gun of the type described in the text.

Basically, the procedure involves working with certain fundamental and limiting relationships. Within these limitations, it was desired to obtain the minimum possible spot-size for a given spot-current, or the maximum possible spot-current in a spot of given size. This objective, while definite enough, does not indicate, per se, the procedure for achieving it. Further, different objectives may also be conceived. Therefore, the general relations between the gun parameters will be established here. The specific applications of these relations to the purpose of minimizing the spot size for given current is carried out as a particular example.

The fundamental relations to be used are listed below:

(a) *Thermal velocities* of the electrons impose the following limitation for the current density j to a point in the beam at potential V where electrons from the cathode coverage with half angle θ:

$$j \leq \left(1 + \frac{eV}{kT}\right) \sin^2\theta \times j_c,$$

where j_c is the cathode current density, k Boltzmann's constant, and T the cathode temperature in $^{\circ}$K. This expression was first derived by D. B. Langmuir.[7] In the usual case where $eV/kT \gg 1$, this reduces to

$$j \leq \frac{eV}{kT} \sin^2\theta \times j_c. \tag{1}$$

(b) The spherical aberrations of an electron lens, which are the only ones of major importance in a well-constructed kinescope gun, result in every point of the object being reproduced as a circle at the image. The result is an increase in the size of the image. Expressing the increase in any dimension of the spot (due to spherical aberrations) as Δy_s, and the lens aperture as d_a,

$$\Delta y_s = C \times d_a^3 \tag{2}$$

C is a constant depending on the lens type and other dimensions Ramberg[8] has analyzed spherical aberrations in certain types of electron lenses. In general, however, the value of Δy_s is best obtained experimentally.

(c) The *magnification*, M, of an electron lens is defined by

$$M = y_s'/y_o, \tag{3}$$

where y'_s and y_o are equivalent dimensions of the spot and the object in the absence of aberrations. If y_s is the actual spot dimension, one has

$$y_s = y_s' + \Delta y_s. \tag{4}$$

The magnification in a distortionless imaging system can also be expressed by the relation.

$$M^2 = \frac{V_o \sin^2\theta_\sigma}{V_s \sin^2\theta_s} \tag{5}$$

where V_o and V_s are object and image (screen) voltages, respectively; θ_o and θ_s are the angles that a given ray (electron path) intersecting the axis at the object makes with the lens axis at the object and screen, respectively. When Eq. (5) is used, the outermost ray contributing to the formation of the spot will implicitly be considered. Thus θ_o is equal to the angle of beam spread at the object when no current is intercepted on apertures in the imaging system, and θ_s is the angle of beam convergence at the screen.

(d) Let L_s be the distance from the second principal plane of the final lens to the screen. The lens aperture d_a of Eq. (2) is equal to the maximum beam diameter at that plane. Thus, the angle of beam convergence, θ_s, at the screen is given by

$$\tan\theta_s = \frac{d_a}{2L_s}. \tag{6}$$

The interrelation of the principal parameters of the gun through Eq. (1) to (6) and through the design of the object forming system are shown in Fig. 10.

On the left side of the figure are parameters of the imaging system, on the right, parameters of the object-forming system. The entire design of the object-

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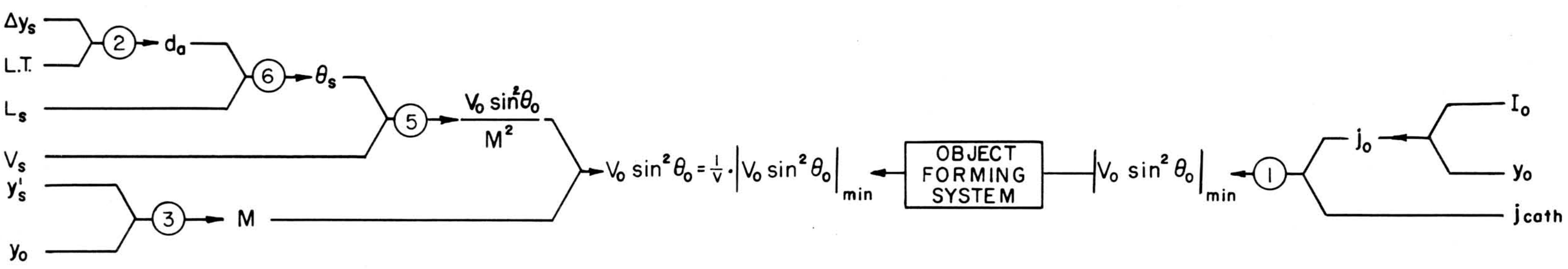
[7] D. B. Langmuir, "Theoretical Limitations of Cathode-Ray Tubes," *Proc. I. R. E.*, Vol. 25, p. 977, August, 1937.

[8] E. G. Ramberg, "Variation of Axial Aberrations of Electron Lenses with Lens Strength," *Jour. Appl. Phys.*, Vol. 13, pp. 582-594, September, 1942.



LEGEND

PARAMETERS OF IMAGING SYSTEM

y_s = SPOT SIZE

Δy_s = CONTRIBUTION OF SPHERICAL ABERRATIONS TO SPOT SIZE

y'_s = $y_s - \Delta y_s$ = SPOT SIZE IN THE ABSENCE OF ABERRATIONS

y_o = OBJECT SIZE

L_s = LENS-TO-SCREEN DISTANCE

V_s = SCREEN (IMAGE) VOLTAGE

L.T. = LENS TYPE

PARAMETERS OF THE OBJECT FORMING SYSTEMS

y_o = OBJECT SIZE

I_o = CURRENT TRANSMITTED THROUGH THE OBJECT DEFINING APERTURE

j_{cath} = CATHODE CURRENT DENSITY

SECONDARY PARAMETERS

d_a = APERTURE OF FINAL LENS

M = MAGNIFICATION

θ_o = ANGLE OF BEAM DIVERGENCE AT THE OBJECT

θ_S = ANGLE OF BEAM CONVERGENCE AT THE SCREEN

j_o = OBJECT CURRENT DENSITY

$$v = \frac{|V_o \sin^2\theta_o|_{min}}{V_o \sin^2\theta_o} = \text{PERFORMANCE FACTOR OF OBJECT FORMING SYSTEM } (0 < v < 1)$$

EQUATIONS: FOR EQ. (1a) TO (6a), cf APPENDIX I

Fig. 10 — Fundamental relations for the design of a kinescope gun.

forming system is considered in this figure as a single operator determining the ratio

$$v = \frac{(V_o \sin^2 \theta_o)_{\min}}{V_o \sin^2 \theta_o} \qquad (7)$$

of the minimum theoretically possible value of $V_o \sin^2\theta_o$ (fixed from I_o, y_o, j_c by Eq. (1)) to the actual value of this product.

When the design of the object-forming system has been chosen (v determined) and all of the parameters on the left side of Fig. 10 except one, have been prescribed, this last parameter is determined through the relationship shown. It is seen that *best gun performance is obtained* quite generally when *the object-forming system is designed to make $v \rightarrow 1$* (its theoretical maximum value).

For example, when all parameters except the spot size y'_s are prescribed, the minimum spot size is obtained when $v = 1$. This is shown as follows:

The value of $V_o \sin^2\theta_c = \frac{(V_o \sin^2\theta_o)_{\min}}{v}$ is determined by the parameters on the right of Fig. 10 and by the design of the object-forming system. The value of $\frac{V_o \sin^2\theta_o}{M^2}$ is fixed (call it C) by the parameters on the left of Fig. 8 (excluding y'_s). Thus, the magnification M is found to be

$$M = \frac{(V_o \sin^2 \theta_o)_{\min}}{Cv}$$

and $y'_s = My_o$. The smallest value of y_s is obtained when M is minimized; this is done by maximizing v (q.e.d).

One may conclude that for best performance, an object-forming system has to be designed to make $v \rightarrow 1$. Once the principal parameters I_o, j_o, and j_c (right of Fig. 10) of the object forming system have been selected, this amounts to minimizing the actual value of the product $V_o \sin^2\theta_o$.

Appendix II

The electron trajectories in a converging or diverging beam can be computed, taking space charge into account, under the following assumptions:

(1) Field-free region

(2) Paraxial rays (small angles between electron trajectories and axis of the beam),

(3) Laminar flow (no crossing of electron paths),

(4) Round beam cross section,

(5) No ion neutralization.

With these assumptions, the equation of the beam envelope is found to be[9]

$$r_o = 32.3 \frac{V^{3/4}}{I^{1/2}} \int_1^R \frac{dR}{\sqrt{\ln R}} \qquad (8)$$

where $r =$ beam radius at the abscissa z,
$r_o =$ minimum beam radius occurring at the abscissa $z = 0$,
$R = \frac{r}{r_o} \geq 1$,
$I =$ beam current in milliamperes,
$V =$ beam voltage in kilovolts.

Space-charge limitation may occur in either the object space or the image space. The object space is the part of the imaging system in which the object is located. Its potential is the object potential, V_o. Let the average angle of beam divergence in the object space be θ_o. One defines similarly the image space with the image potential, V_i, and an average angle of beam convergence, θ_i.

Further, let the imaging system be represented by an equivalent imaging system with a single thin lens, as shown on Fig. 11. Let the sum $L_1 + L_2$ be equal to the actual distance L between object and image:

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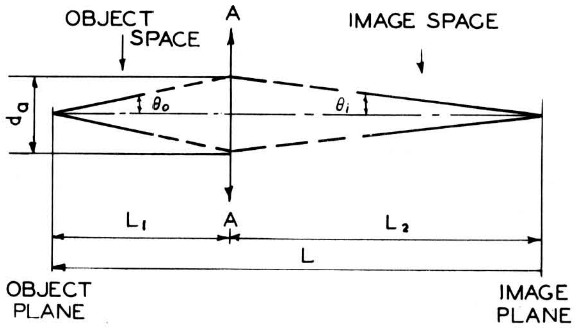
[9] B. J. Thompson and L. B. Headrick, "Space Charge Limitations on the Focus of Electron Beams," *Proc. I.R.E.*, Vol. 28, pp. 318-324, July, 1940.

$$L_1 + L_2 = L.$$

The angles θ_o and θ_i being specified together with L, the maximum beam diameter d_a in Fig. 11 is also specified.

Let d_o = object diameter (assuming for convenience a round object),

d_i = image diameter.



AA= PRINCIPLE PLANE OF EQUIVALENT SINGLE THIN LENS.

Fig. 11 – Dimensions of equivalent imaging system.

For $d_i \ll d_a$, the maximum possible current through the object space is obtained when the minimum beam cross section is approximately at the object. Thus, applying Eq. (8) to the object space one obtains

Max current which can be passed through $d_a = I_{omax} =$

$$\frac{(32.3)^2 V_o^{3/2}}{\left(\dfrac{2L_1}{d_o}\right)^2}\left[\int_1^{\frac{d_a}{d_o}} \frac{dR}{\sqrt{\ln R}}.\right]^2 \qquad (9)$$

Let us express I_{omax} as a function of the image diameter:

$$d_o^2 = \frac{d_i^2}{M^2},$$

$$\frac{1}{M^2} = \frac{V_i \sin^2 \theta_i}{V_o \sin^2 \theta_o} = \frac{V_i}{V_o}\,\frac{(L_1)^2}{(L_2)^2}.$$

Introducing these relations into Eq. (9) one obtains for, I_{omax},

$$I_{omax} = \left(\frac{d_i}{2L_2}\right)^2 \left[32.3 \int_1^{\frac{d_a}{d_o}} \frac{dR}{\sqrt{\ln R}}\right]^2 V_i \sqrt{V_o}. \qquad (10)$$

In a kinescope, L_2 is approximately the distance from the screen (image plane) to the final lens. Little freedom usually exists in the choice of this parameter; therefore, we may consider it as prescribed. The image diameter d_i and the image voltage V_i (same as screen voltage) also are prescribed. The maximum beam diameter at the lens, d_a, is determined by spherical aberrations, as shown in Appendix I. Once the magnification has been chosen, the integral in the bracket of Eq. (10) is also determined. Thus, the only parameters left to choice are the object voltage, V_o, and the object distance L_1, both being related by the condition.

$$\frac{V_o}{L_1^2} = \frac{M^2}{L_2^2} V_i. \qquad (11)$$

From Eq. (10) it appears that the maximum possible current in the object space increases with increasing object voltage:

$$I_{omax} \propto \sqrt{V_o}.$$

Therefore, in order to reduce space-charge effects in the object space, one has to increase the object voltage (q.e.d.).

Computing the maximum possible current I_{max}, in the image space, one finds, in a similar fashion,

$$I_{imax} = \left(\frac{d_i}{2L_2}\right)^2 \left[32.3 \int_1^{\frac{d_a}{d_i}} \frac{dR}{\sqrt{\ln R}}\right]^2 V_i^{3/2}. \qquad (12)$$

Numerical evaluation of Eq. (10) and (12) shows that for magnification of the order of 2 and $V_o/V_i < 1/2$, $I_{omax} < I_{imax}$. Thus, under the usual conditions corresponding to those figures, space-charge effects are more important in the object space than in the image space (q.e.d.).

In the case of object and image shapes other than a circle, the preceding derivations still hold approximately if equivalent object and image diameters are defined, such that the object and image areas of the equivalent round beam are the same ones as in the actual beam. This has been experimentally found to hold for rectangular objects and spots.

In practice, the assumptions of laminar flow and absence of ions appear the least likely to be satisfied, while the other assumptions upon which this analysis relies are usually well satisfied. However, non-laminar flow and ion neutralization has opposite effects. This may explain why the relations indicated in this appendix have been fairly well confirmed by our measurements, at least qualitatively.