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BIPERIODIC FOCUSING FOR HIGH-DENSITY ELECTRON BEAMS

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BIPERIODIC FOCUSING FOR HIGH-DENSITY ELECTRON BEAMS

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A focusing scheme employing two counteracting periodic fields of very short periods is shown to be superior to that which involves only one focusing periodic field. The potential valley formed by the combination of these two counteracting fields is steeper than all previous focusing systems and thus is capable of maintaining a very stable beam flow. The combined field also gives rise to proper cancellation to the space charge field. This field cancellation not only results in an ideal focusing scheme for high-density beams but also compensates for the potential depression inside the beam.

Introduction

The development of beam focusing by periodic electrostatic fields[1,2,3] offered the possibility for devising a light-weight focusing system. At one time this appeared to be the answer for a major simplification of traveling-wave tubes. From the point of view of beam stability and of focusing, periodic electrostatic fields of very short periods have been shown to be superior to those of comparatively long periods.[3] While promising results have been obtained with such focusing systems for thin electron beams of low perveance, no attempts have been made to work with thick beams of high perveance. This is due to the effect of the initial thermal velocity of the electrons on the space charge force. As the current density in the beam becomes larger, the space charge force which is to be balanced by the focusing force becomes more complicated and uncontrollable. If the focusing system is of the "confined-flow" type, where the space charge force is relatively small compared with the external focusing force, a tremendous applied balancing force is required for high density beams. The attainment of this balancing force will not only complicate the focusing scheme but sometimes it is not practicable. Above all, the most serious drawback to previous focusing systems using short-period electrostatic fields is the limitation on the beam size to very thin dimensions. For thick electron beams, special provisions such as a specified non-uniform space charge distribution[1] must be made to maintain a proper force balance in the beam. These provisions usually offset the advantages of simplicity provided by the periodic electrostatic field for focusing.

The following is a discussion of a new focusing method which, while retaining the feature of simplicity possessed by electrostatically focused tubes, offers the additional merit of focusing a thick beam of high current density without any special provision. The method essentially adopts a periodic scheme that is formed by two non-coplanar periodic structures which are characterized by extremely short periods. Between these two structures, an exponential field is formed that varies in an opposite sense around a "null" or zero-field plane. The variation of this exponential field, which is opposed to that of the space charge field, can be approximated to balance the same space charge field within the beam.

In the past, biperiodic fields of very long periods have been used on transverse-field tubes,[2] but their use was limited to very thin beams. Since a periodic field of long periods yields a slowly varying force function instead of an exponential one, this particular biperiodic focusing scheme results in poor stability in the beam flow and thus rules out its use of high density electron beams.

Focusing Model

The focusing model under discussion is shown in Fig. 1. For analysis, a model of cylindrical geometry (r, z) has been choosen with two coaxial periodic ring structures of radii r_1 and r_2 and of periods L_1 and L_2. Suppose a potential difference $2V_1$ is applied to each adjacent pair of rings on the inner structure with an average potential V_0, and a potential difference $2V_2$, on the outer

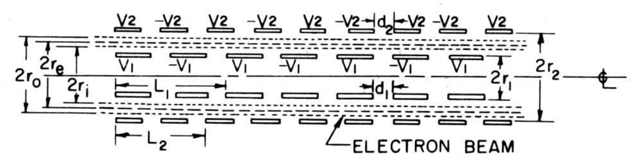

Fig. 1 (a) - Biperiodic structures of unequal periods.

structure with the same average potential V_0. The potential $V(r, z)$ between the two structures can be approximated with the following expression:[4,5]

$$V(r, z) \approx V_0 + \hat{V}_1(r) \cos\left(\frac{2\pi}{L_1}\right) z + \hat{V}_2(r) \cos\left(\frac{2\pi}{L_2} z\right)$$

$$V_0 \gg V_1(r) \qquad (1)$$

$$V_0 \gg V_2(r)$$

$$\hat{V}_1(r) = 4V_1 \frac{\sin \sigma_1 \pi}{\sigma_1 \pi^2} \frac{I_0(\frac{2\pi}{L_1} r) K_0(\frac{2\pi}{L_1} r_2) - I_0(\frac{2\pi}{L_1} r_2) K_0(\frac{2\pi}{L_1} r)}{I_0(\frac{2\pi}{L_1} r_1) K_0(\frac{2\pi}{L_1} r_2) - I_0(\frac{2\pi}{L_1} r_2) K_0(\frac{2\pi}{L_1} r_1)} \qquad (2)$$

$$\hat{V}_2(r) = 4V_2 \frac{\sin \sigma_2 \pi}{\sigma_2 \pi^2} \frac{I_0(\frac{2\pi}{L_2} r) K_0(\frac{2\pi}{L_2} r_1) - I_0(\frac{2\pi}{L_2} r_1) K_0(\frac{2\pi}{L_2} r)}{I_0(\frac{2\pi}{L_2} r_2) K_0(\frac{2\pi}{L_2} r_1) - I_0(\frac{2\pi}{L_2} r_1) K_0(\frac{2\pi}{L_2} r_2)} \qquad (3)$$

$$\sigma_1 = \frac{d_1}{L_1} \qquad (4)$$

$$\sigma_2 = \frac{d_2}{L_2} \qquad (5)$$

I_0 and K_0 are the modified Bessel's functions. A ring-type cylindrical beam of radii r_0 and r_i constrained between the two structures is considered in this model. The electron motions in the electrostatic field produced by the periodic structures follow the equations[3]

$$\ddot{r} - \eta \frac{\partial V}{\partial r} = \frac{\eta^2}{4} B_b^2 \frac{r_o^2}{r} \qquad (6)$$

$$\ddot{z} = \eta \frac{\partial V}{\partial z} \qquad (7)$$

Here the derivatives are with respect to time, η is the ratio of electron charge to mass, and B_b is the equivalent Brillouin field which represents the space charge field at the radius r with an outer beam current I_0 which is, as defined in the appendix, the part of the beam current outside the "null" field radius where the space charge field is zero.

In MKS Units,

$$B_b^2 = 0.69 \times 10^{-6} \left(\frac{I_0}{V^{1/2} r_0^2}\right) \left(\frac{r^2 - r_e^2}{r_0^2 - r_e^2}\right) \qquad (8)$$

Approximate solution of the equations of electron motion, Eqs. (6) and (7), can be obtained by assuming that the beam radius r is sinusoidally perturbed to a very small amount around the equilibrium radius r_e, that is,

$$r = r_e + \hat{r}_1 \cos\left(\frac{2\pi}{L_1} z\right) + \hat{r}_2 \cos\left(\frac{2\pi}{L_2} z\right) \qquad (9)$$

where $\hat{r}_1$ is the perturbed peak value radius due to its periodic field of period L_1; and $\hat{r}_2$, due to the periodic field of period L_2. By substituting Eq. (9) into Eqs. (6) and (7), one obtains

$$\dot{r} = -\hat{r}_1 \left(\frac{2\pi}{L_1}\right) \dot{z} \sin\left(\frac{2\pi}{L_1} z\right) - \hat{r}_2 \left(\frac{2\pi}{L_2}\right) \dot{z} \sin\left(\frac{2\pi}{L_2} z\right) \qquad (10)$$

where the axial velocity $\dot{z}$ is

$$\dot{z} = \sqrt{2\eta V} \approx \sqrt{2\eta V_0} \left[1 + \frac{\hat{V}_1}{2V_0} \cos\left(\frac{2\pi}{L_1} z\right) + \frac{\hat{V}_2}{2V_0} \cos\left(\frac{2\pi}{L_2} z\right)\right] \qquad (11)$$

Expand $\hat{V}_1(r)$, $\hat{V}_2(r)$, $\hat{V}_1'(r)$ and $\hat{V}_2'(r)$ and drop high order terms, where primes indicate derivatives with respect to r,

$$\hat{V}_1(r) \approx V_1(r_e) + \hat{V}_1'(r_e) \left[\hat{r}_1 \cos\left(\frac{2\pi}{L_1} z\right) + \hat{r}_2 \cos\left(\frac{2\pi}{L_2} z\right)\right] \qquad (12)$$

$$\hat{V}_2(r) \approx \hat{V}_2(r_e) + \hat{V}_2'(r_e) \left[\hat{r}_1 \cos\left(\frac{2\pi}{L_1} z\right) + \hat{r}_2 \cos\left(\frac{2\pi}{L_2} z\right)\right] \qquad (13)$$

$$\hat{V}_1'(r) \approx \hat{V}_1'(r_e) + \hat{V}_1''(r_e) \left[\hat{r}_1 \cos\left(\frac{2\pi}{L_1} z\right) + \hat{r}_2 \cos\left(\frac{2\pi}{L_2} z\right)\right] \qquad (14)$$

$$\hat{V}_2(r) \approx \hat{V}_2'(r_e) + \hat{V}_2''(r_e) \left[\hat{r}_1 \cos\left(\frac{2\pi}{L_1} z\right) + \hat{r}_2 \cos\left(\frac{2\pi}{L_2} z\right)\right] \qquad (15)$$

Using the relation

$$\ddot{r} = \frac{d^2 r}{dz^2}\,\dot{z}^2 + \ddot{z}\,\frac{dr}{dz} \tag{16}$$

and substituting[7] Eqs. (9) through (15) in Eq. (16), equating the constant terms and the coefficients of the cosine terms of Eq. (6) and remembering that $\hat{r}_1$ and $\hat{r}_2$ are small compared to r_e, one obtains,

$$\frac{\hat{V}_1'(r_e)}{V_0}\left[\left(\frac{L_1}{2\pi}\right)^2 \hat{V}_1''(r_e) + \hat{V}_1(r_e)\right] = -\frac{\hat{V}_2'(r_e)}{V_0}\left[\left(\frac{L_2}{2\pi}\right)^2 \hat{V}_2''(r_e) + \hat{V}_2(r_e)\right] + \eta B_b^2\left(\frac{r_0^2}{r_e}\right) \tag{17}$$

$$\hat{r}_1 = -\frac{\hat{V}_1'(r_e)}{V_0}\left(\frac{L_1}{4\pi}\right), \quad \hat{r}_2 = -\frac{\hat{V}_2'(r_e)}{V_0}\left(\frac{L_2}{4\pi}\right) \tag{18}$$

Equation (17) gives the force balance on the electron beam. The term on the left-hand side represents the focusing force contributed by the periodic fields originating from the ring structure of the radius r_1. The restoring forces which are on the right-hand side consist of the balancing force due to the periodic field produced by the ring structure of radius r_2 and the space charge force. The balancing force can be made large enough, in some instances, so that the space charge force may well be neglected. This corresponds to a "confined" electron flow, where the focusing and balancing forces are very strong compared to the space charge effect. It will be shown in the later section that the type of flow resulting from the force balance in Eq. (17) will form a more stable focusing system than all previous ones.

Illustrative Example

For direct application of the above focusing model to helix-type traveling wave tubes, a bifilar helix is used as the periodic structure. Suppose, for convenience, as a special case, both the inner and the outer bifilar helices are wound with the same turns per unit length and are thus applied with a periodic field of the same period L (Fig. 1b). A modification of Eq. (17) for force balance in this case is[3]

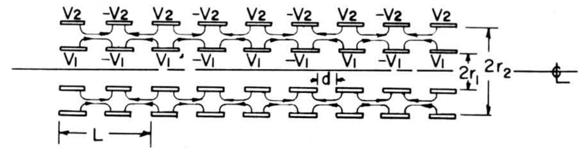

Fig. 1 (b) - Biperiodic structure of equal periods.

$$\left[\left(\frac{L}{2\pi}\right)^2 \hat{V}_1''(r_e) + \hat{V}_1(r_e) + \left(\frac{L}{2\pi r_e}\right)^2 \hat{V}_1(r_e)\right]\frac{\hat{V}_1'(r_e)}{V_0} = -\left[\left(\frac{L}{2\pi}\right)^2 \hat{V}_2''(r_e) + \hat{V}_2(r_e) + \left(\frac{L}{2\pi r_e}\right)^2 V_2(r_e)\right]\frac{\hat{V}_2'(r_e)}{V_0} + \eta B_b^2\left(\frac{r_0^2}{r_e}\right) \tag{19}$$

where

$$\hat{V}_1(r_e) = \frac{4\sin\sigma\pi}{\sigma\pi^2}\;\frac{V_1 K_1(\frac{2\pi}{L}r_2) - V_2 K_1(\frac{2\pi}{L}r_1)}{I_1(\frac{2\pi}{L}r_1)K_1(\frac{2\pi}{L}r_2) - I_1(\frac{2\pi}{L}r_2)K_1(\frac{2\pi}{L}r_1)}\; I_1\left(\frac{2\pi}{L}r_e\right) \tag{20}$$

$$\hat{V}_2(r_e) = \frac{4\sin\sigma\pi}{\sigma\pi^2}\;\frac{V_2 I_1(\frac{2\pi}{L}r_1) - V_1 I_1(\frac{2\pi}{L}r_2)}{I_1(\frac{2\pi}{L}r_1)K_1(\frac{2\pi}{L}r_2) - I_1(\frac{2\pi}{L}r_2)K_1(\frac{2\pi}{L}r_1)}\; K_1\left(\frac{2\pi}{L}r_e\right) \tag{21}$$

The terms $(L/2\pi r_e)^2\,\hat{V}_1$ and $(L/2\pi r_e)\,\hat{V}_2$, which are missing in Eq. (17), represent the focusing and balancing forces due to the angular field resulting from the winding pitch of the helix. For very small periods, these forces are very small and can be ignored.

To study the focusing performance of our doubly periodic electrostatic fields, let us plot Eq. (19) in the following way. Assume that a hollow beam is focused in a confined flow condition at an equilibrium radius r_e by these two periodic electrostatic fields alone. The resultant force function versus beam radius in the neighborhood of r_e is plotted as curve (1) in Fig. 2. For comparison, resultant forces in confined flows of previous focusing schemes using a single periodic electrostatic field are also shown with the same potential difference on the periodic structures of the same geometry. Curve 2 illustrates the case in which a centrifugul force[3] is employed as a balancing force in such a single periodic focusing scheme. The case in which a radial electric force[6] instead is used as a restoring force is shown by curve (3). The corresponding potential functions of the

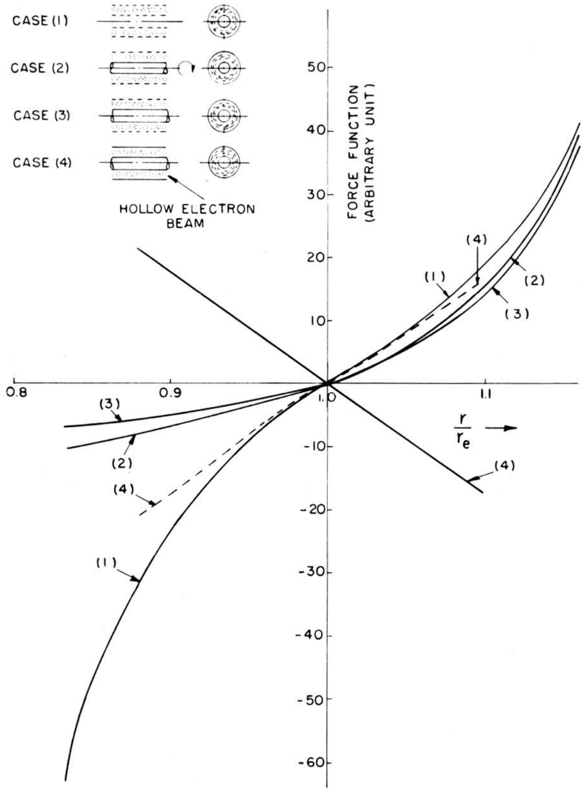

Fig. 2 - Force functions near the equilibrium radius.

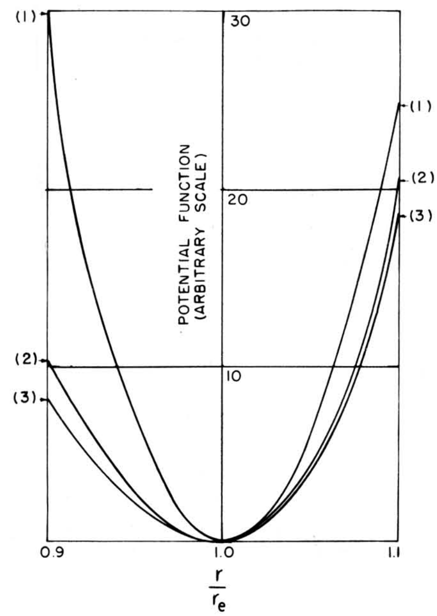

Fig. 3 - Potential valley near the equilibrium radius.

three cases are plotted in Fig. 3. The space charge force inside a hollow beam which has a ratio of the outer radius to the inner one equal to 1.25 in the coaxial structure of Fig. 1 is derived in the appendix and is plotted as curve (4) in Fig. 2. The space charge force is assumed to have a zero field at r_e, and an equal but opposite field to curve (1) at $r = 1.05\ r_e$. The dotted curve which is the negative of the space charge force, is drawn here for the purpose of comparing the degree of compensation for the space charge inside the beam by the three indicated focusing systems.

It is interesting to note from Figs. (2) and (3) that the doubly periodic fields not only give the steepest potential valley of all but also offer the approximate cancellation to the space charge force everywhere inside a thick hollow beam. This is extremely important for focusing a thick beam of very high current density. The single periodic electrostatic field, as shown by curves (2) and (3) in Fig. 2, deviates very much in magnitude from the space charge field and thus limits its use to the focusing of very thin electron beams near the equilibrium radius.

In this numerical example, with a periodic voltage variation of 5 percent[3] on the beam, it is possible to focus an electron beam of perveance in the order of 10^{-5} amperes per volt $^{3/2}$.

Conclusion

The focusing scheme employing two counteracting periodic fields of very short periods is shown to be superior to that which involves only one focusing periodic field. The potential valley formed by the combination of these two counteracting periodic fields is steeper than all previous focusing systems, and thus is capable of maintaining a very stable beam flow.

The most outstanding feature of the new focusing system is the proper cancellation between the focusing field and the space charge field. This field cancellation not only results in an ideal focusing for the beam but also compensates for the potential depression inside the beam so that all electrons in the beam travel with the same average velocity. This has never been true in previous focusing system unless special provisions have been made.

Equation (19) reveals that the force balance on the beam in the confined-flow condition is independent of the average potential. This may extend the application of the new focusing scheme to the accelerating region where the average electron velocity varies with distance. In the drift region where electrons interact with the r-f delay line, if the beam focusing is not dependent upon the d-c velocity of electrons, defocusing caused by r-f interaction may be avoided.

Kern K. N. Chang

Appendix

Suppose the electron beam shown by Fig. 1(a) has been focussed to a hollow cylindrical ring with radii r_0 and r_i with a uniform space charge density ρ. Then the Poisson's equation for the space charge is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{\rho}{\epsilon_0} \tag{A-1}$$

Integration of Eq. (A-1) yields

$$\frac{dV}{dr} = -\frac{\rho r}{2\epsilon_0} + \frac{1}{r}C_1 \tag{A-2}$$

where C_1 is an arbitrary constant determined by the boundary conditions at $r = r_0$ and $r = r_i$. In the space-charge free region, the electric fields are:

$$\frac{dV}{dr} = \frac{\phi_0 - \phi_2}{r\ ln\frac{r_2}{r_0}} \qquad for\ \ r_2 > r > r_0 \tag{A-3}$$

and

$$\frac{dV}{dr} = \frac{\phi_1 - \phi_i}{r\ ln\frac{r_i}{r_1}} \qquad for\ \ r_1 < r < r_i \tag{A-4}$$

where ϕ_i is the potential at $r = r_i$; ϕ_1 is the potential at $r = r_1$, etc. To satisfy the boundary conditions, one obtains for the space charge field

$$\frac{dV}{dr} = -\frac{1}{r}\frac{\rho}{2\epsilon_0}\left[r^2 - \frac{1}{ln\frac{r_2}{r_1}}\left(r_0^2\ ln\frac{r_2}{r_0} + r_i^2\ ln\frac{r_i}{r_1}\right) - \frac{r_0^2 - r_i^2}{2\ ln\frac{r_2}{r_1}}\right]$$

$$= \frac{-1}{r}\frac{\rho}{2\epsilon_0}\left[r^2 - r_e^2\right] \tag{A-5}$$

where r_e is the equilibrium radius at which the space charge field is zero. According to Eq. (A-5).

$$r_e^2 = \frac{1}{ln\frac{r_2}{r_1}}\left[\left(r_0^2\ ln\frac{r_2}{r_0} + r_i^2\ ln\frac{r_i}{r_1}\right) + \tfrac{1}{2}(r_0^2 - r_i^2)\right] \tag{A-6}$$

If the beam current in the region between the radius r_e and r_0 is I_0, then Eq. (A-5) becomes

$$\frac{dV}{dr} = \frac{\sqrt{2}\ I_0}{4\pi\epsilon_0\eta^{1/2}V_0^{1/2}}\ \frac{1}{r}\left(\frac{r^2 - r_e^2}{r_0^2 - r_e^2}\right) \tag{A-7}$$

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